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*About the Character of Errors in Spatial Aerotriangulation**

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"We must take into account the errors as they are, not as one would like them to be."
—M. ZELLER

ABSTRACT: *This paper deals with the character of errors in spatial aerotriangulation, and investigates the possibility of separating the different categories of errors. A practical example is worked out in some detail; it shows how the systematic, quasi-systematic, accidental and pseudo-accidental errors are related to each other. A proposal is made to classify the errors in aerotriangulation according to their effect on the results, without regard to their origin or nature. Thus the errors could be classified as errors with systematic effect and errors with accidental effect, and could be treated accordingly.*

FOR about 15 years there has been increasing uneasiness and endless debating among the photogrammetrists dealing with aerotriangulation, because of a big and still unanswered question, brought up by Professor Bachmann, Lausanne, Switzerland, about the character of errors in spatial aerotriangulation. He was soon joined by Professor Roelofs, Netherlands, who gave a paper at the 1948 Congress of the International Society of Photogrammetry, showing that the error propagation in an aerotriangulation is such that the influence of purely accidental errors might give the impression that a systematic error exists. Some of Roelofs' examples show also that the result of accidental errors has a similar effect, as if breaks (jumps or cassures) are present.

Many photogrammetrists have seemed unconvinced by either the Bachmann's or Roelofs' articles and publications and have retained the opinion that a considerable part of the discrepancies and irregularities that they found in their practical work were due to local influences of the photographs—such as lack of flatness of film, local distortions, instrumental errors, etc.—or to changing operators during the triangulation on the stereoplotter.

There still exists a great variety of opinions on this subject. Many photogrammetrists consider this problem of vital importance because one cannot expect to obtain a fruitful development of aerotriangulation if there is lacking a deep and correct insight in the character of errors with which one deals.

In aerotriangulation, as in any other measurement, one expects to have accidental as well as systematic errors. The ideal solution of the problem of the adjustment of aerotriangulation would therefore be based on a separate treatment of each of these categories. The whole difficulty is how to separate the systematic from the accidental errors. Unfortunately, the theory of errors in photogrammetry does not give enough information for solving this problem.

Nowadays, the adjustment of aerotriangulation is made by adopting one of two principles: (1) either one must neglect the acci-

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dental errors and adopt an interpolation method to treat all the errors as systematic; or (2) one must assume that all the errors follow the so called Gauss' Law of Propagation of Errors and apply the method of least squares for the adjustment. Pioneers of the first method are Zeller of Switzerland, and Zarzycki of Canada. Pioneers of the second principle are Bachmann of Switzerland, and Roelofs of the Netherlands.

An attempt to put some light on this problem included triangulating one and the same strip of photographs several times, practically under the same conditions throughout (the operator, the instrument, the temperature, the method of relative orientation, the method of triangulation, etc.). It was thought that keeping all these factors practically constant would lead to a better understanding of the character of errors, and thus give at least an idea of how to separate the different categories.

The chosen strip was photographed in 1954 using the Wild RC5 camera with plate adapter and the aviogon lens (f = 115 mm.). The flight height was 4,600 meters above ground (about 15,000 ft.). The strip was 30 kilometers long and contained 14 photos (13 models). The terrain photographed was fairly flat. The triangulations of the strip were carried out on the Wild A7 Autograph of the Institute of Photogrammetry of the Swiss Federal Institute of Technology, Zurich, in 1955 by the author, who was then preparing his doctoral thesis (advisors were Prof. Dr. Max Zeller and Prof. Dr. Fritz Kobold).

The strip was triangulated six times as an aerial polygon (i.e., neither statoscopic nor horizon data were taken into consideration) and four times as aerial levelling (i.e., taking the statoscopic or altimetric data into consideration). Another triangulation was executed under the same conditions and the same average base-ratio using grid plates instead of actual negatives. In all these triangulations, no absolute orientation was done on the first pair. The first plate was always kept horizontal, and the triangulation was carried on. There were 23 control points scattered over the strip. Of these 23 points, 6 were chosen near or on the axis of the strip and could be considered practically axial. The deviations (errors) from the terrestrially and precisely determined values for the elevation and coordinates of the various control points were determined. To analyze the errors, three longitudinal profiles: upper (U), axial (A), and lower (L) were chosen (see Figure 1). The errors in the three profiles proved to have

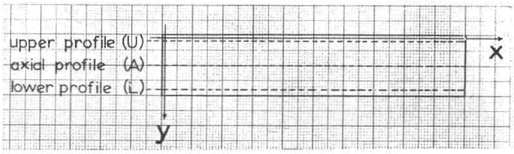

FIG. 1

more or less the same tendency. Due to lack of time, and to avoid unnecessary repetition, the author will show and discuss the errors of the axial points only (see Figure 2).

Figure 2 shows the actual errors in X, Y and H in the different aerial polygons and aerial levellings. The curves and lines shown in the figure are only connections of the points on the graphs (i.e., they are not constructed curves with a certain order or degree). The results obtained are shown.

Before going further, some definitions and simple theory will be briefly discussed.

First, a distinction should be made between systematic, accidental, quasi-systematic, and pseudo-accidental errors. *Systematic errors* are errors following certain conditions that could in some cases be expressed by mathematical formulas. Under the same conditions, systematic errors will always be of the same size and sign. These errors are caused by mechanical and optical errors of the camera and the plotting apparatus, as well as by operational errors.

Accidental errors are unavoidable personal errors that affect the measurements purely accidentally with regard to size and sign, following the Gauss' law of distribution.

Quasisystematic errors, include those which lead to a systematic falsification of a model or results; their magnitude and sign, however, may not be the same under the same conditions. These errors are accidental in origin but systematic in effect. They are caused mainly by personal errors of the operator, as well as by eventual image errors present. A simple example for this category of errors could be shown by repeating several times the simple taping of a long distance by a tape. Due to the accidental errors in alignment, there will always be a negative error in the measurement. This however, is not always of the same size (see Figure 3).

Pseudo-accidental errors include the errors with systematic origin but of accidental effect. These are caused by residual errors in the plotting apparatus and by errors in the photographic acquisition system (camera, film, plates, etc.). An example of this category of errors is the effect of the irregular film shrinkage which sometimes differs in the transversal and in the longitudinal directions. Another example is the effect of the residual errors of the plotter, which act differently in different positions of the space rods for example.

Now, on the basis of the experiments done in this field, the author finds, it advantageous to reclassify the errors in the following main categories:

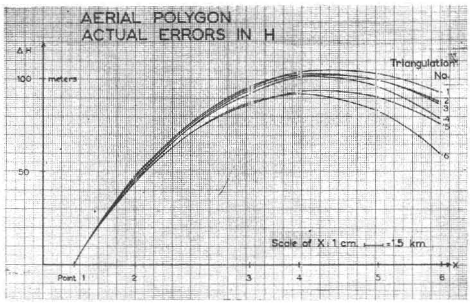

FIG. 2a

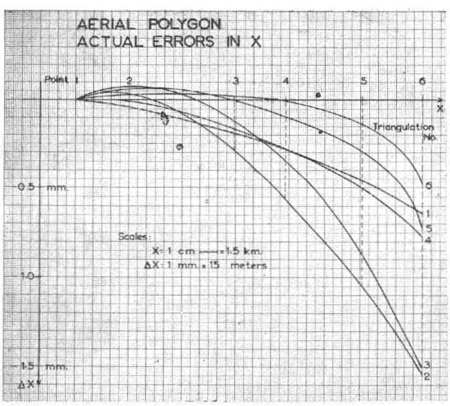

FIG. 2b

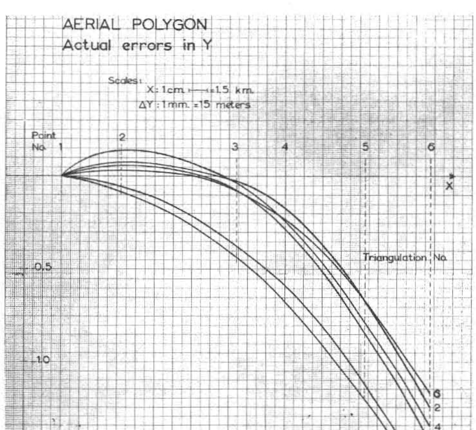

FIG. 2c

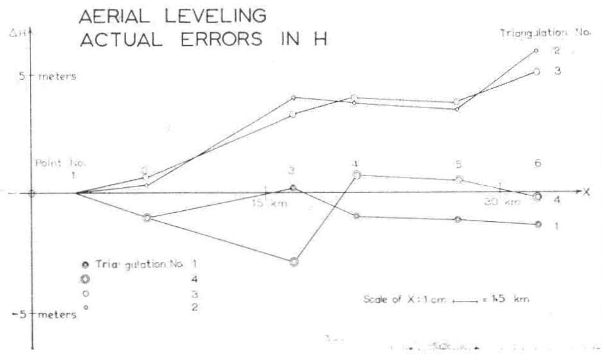

FIG. 2d

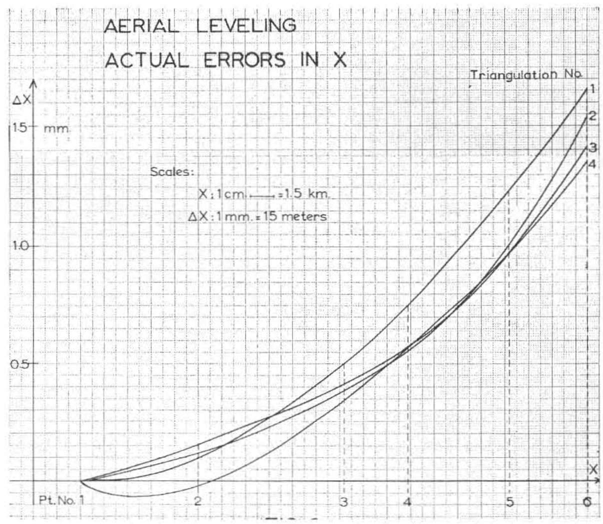

FIG. 2e

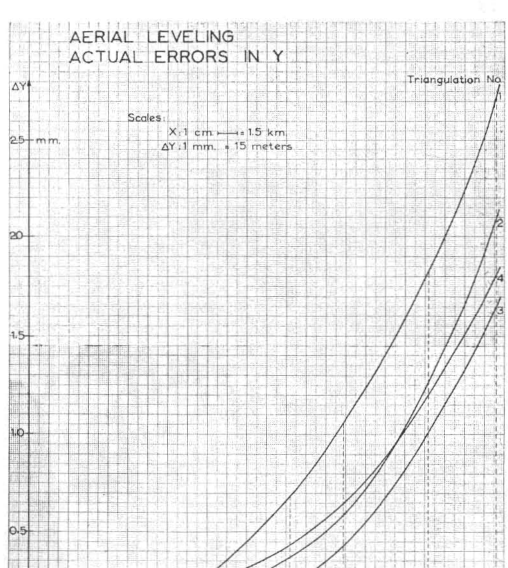

FIG. 2f

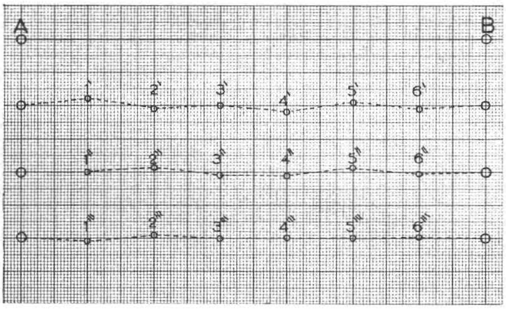

FIG. 3

	AERIAL POLYGON			AERIAL LEVELLING			REMARKS
	ΔX	ΔY	ΔH	ΔX	ΔY	ΔH	
R	3	3	2	2	2	1	
P	4	4	3	3	3	2	for a profile with constant Y
P	6	6	5	6	6	4	for points with variable Y (not on a long profile)

TABLE 1

1. *Errors with systematic effect:* This category includes the systematic as well as the quasi-systematic errors; i.e., all the errors that affect the result systematically without regard to their nature. Such a category would affect our triangulations systematically, but not necessarily with the same amount and sign each time.
2. *Errors with accidental effect:* including the strictly accidental as well as the pseudo-accidental errors.

The relation between the different categories and classifications of errors could then be given as follows:

TOTAL ERROR

With Systematic Effect	With Accidental Effect
Systematic	Accidental
Quasisystematic	Pseudo-accidental

SEPARATION OF THE DIFFERENT CATEGORIES OF ERRORS

It has been shown that the propagation of the different errors with systematic effect in an aerotriangulated strip follows laws which can be represented by polynomials of different orders. The following table gives the degree and order of such polynomials for different cases. A convention adopted in Table 1 should be first explained. A polynomial of the Rth order containing p coefficients is termed here a (R_p) polynomial. For example, $z=a+bx+cx^2+dxy$ is hence termed a (2_4) polynomial.

Now if there are n points to determine an (R_p) polynomial, the best polynomial could be determined by taking into consideration the $(n-p)$ redundants. If consideration is being given to the propagation of errors in elevations (H), so the best polynomial drawn with the help of the n values would represent the errors with systematic effect, while the deviations from this best curve (kept to a minimum through the least squares treatment) would represent the errors with accidental effect. (Figure 4)

If the same strip is triangulated q times, there will be $(q \cdot n)$ values for the determination of the (R_p) polynomial that would represent the propagation of the systematic errors. This best polynomial could be determined by taking into consideration the $(q \cdot n - p)$ redundants. In the meantime, the limits of scatter or deviation from this best curve (or, in other words, the effect of the quasi-systematic errors), as well as their probability of occurrence, could be determined. In Figure 5 the bold line represents the best polynomial through the $q \cdot n$ points, while the other six dotted curves give the probabilities of scatter or deviation from this best curve (the probability that the curve of the errors with syste-

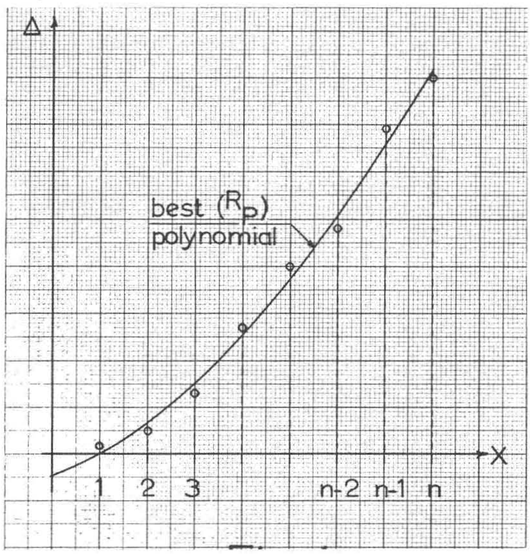

FIG. 4

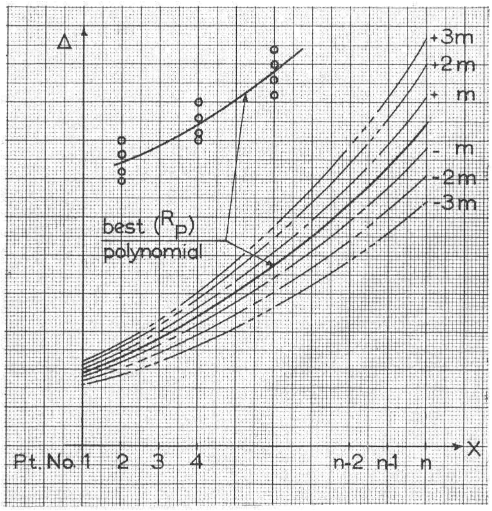

FIG. 5

the strictly accidental errors from the pseudo-accidental errors. Still more research is needed in this point. The author hopes to report on this matter as soon as some complementary experiments have been accomplished.

Now, to return to the given example. The errors in X, Y, and H for the three different longitudinal profiles have been treated as outlined above. The curves representing the propagation of the systematic errors, the quasi-systematic errors, as well as the errors with accidental effect, have been drawn. Because of the lack of time, and to avoid unnecessary repetition, there will be shown and discussed the curves representing the different categories of errors in H in the central profile only (Figure 6). All the other curves have shown more or less the same tendency. On Figure 6 the results of one grid triangulation are also shown.

matic effect lies outside the area bounded by the curves of:

$+m$ and $-m$ is 1:22

$+2m$ and $-2m$ is 1:370

$+3m$ and -3 is 1:5800).

So it has been seen how one could theoretically separate the errors with systematic effect from the errors with accidental effect, and also the systematic errors from the quasi-systematic errors. It remains then to separate

CONCLUSIONS:

The above shown curves, together with the curves that have not been shown, show that:

1. In the afore-mentioned experiments, the errors in X, Y, and H have shown a similar systematic tendency.
2. In all these experiments, the effect of the so-called quasi-systematic errors is remarkable.
3. In these experiments, the errors with accidental effect have a practically neg-

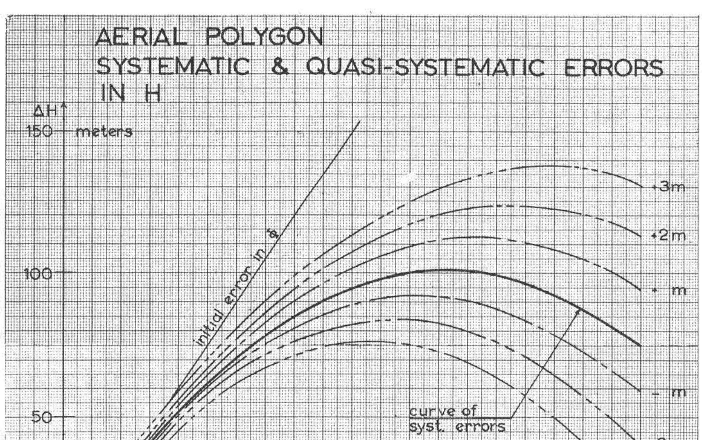

FIG. 6a′

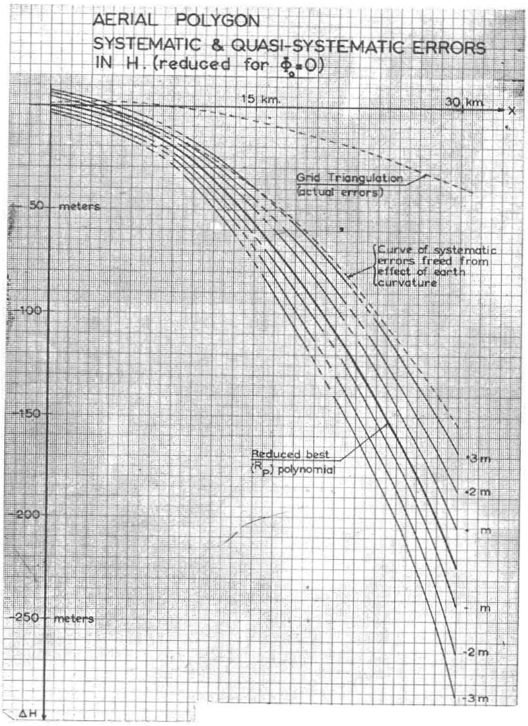

FIG. 6a

ligible effect, compared with the errors with systematic effect.

4. In the 11 afore-mentioned experiments, virtually in all the experiments carried out at the Zurich ETH, no sign of breaks or jumps has been seen or detected. The author sees no reason at all why such jumps should be expected if all the outer conditions in the experiment (the operator as well) remain practically the same during the whole triangulation. The jumps or breaks that are sometimes mentioned in the photogrammetric literature are—in the author's opinion—due mainly to shift work and the change of operators during the triangulation.
5. A comparison between the behavior of the errors in the triangulations carried out by real photographs, and the triangulation using the grid plates, shows clearly the effect of refraction and the eventful errors in the optics and the photographic material used.

More experiments in this field are still needed before generalizing the conclusions and facts mentioned above. The author plans to conduct an experiment that includes independent repeated triangulations of different strips taken at different altitudes. Each strip is to be triangulated 25–30 times to secure more reliable results.

The results of the above mentioned limited experiment permits, however, a comparison of the two principles used nowadays for the adjustment of aerotriangulations. Adjusting simply by the so-called interpolation curves, one takes into consideration only the errors with systematic effect. The errors with accidental effect that are neglected in the interpolation methods have been shown to have a practically negligible effect compared with the other errors. According to the second principle, all the errors are assumed to be acci-

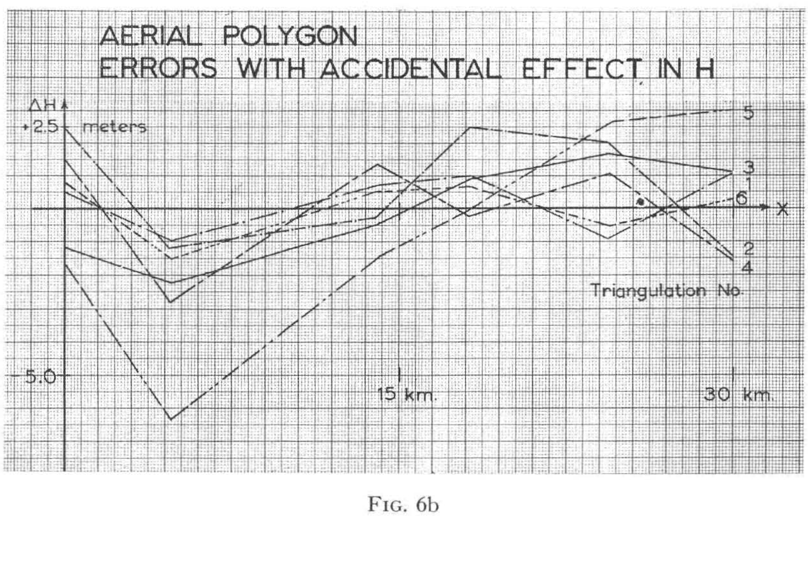

FIG. 6b

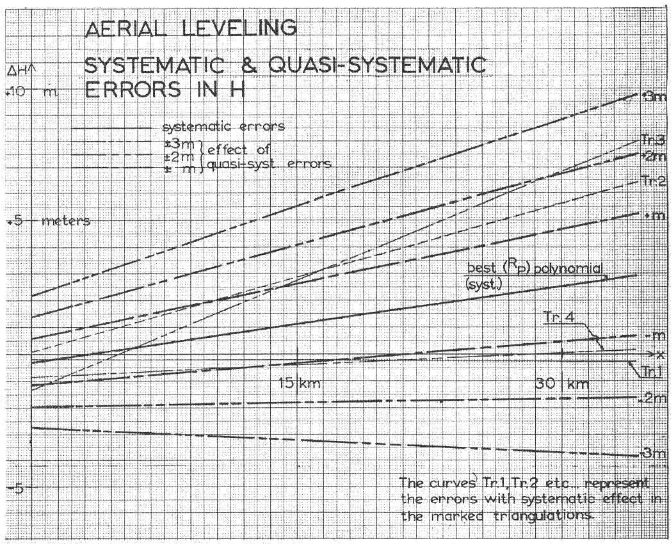

FIG. 6c

dental and to follow the Gauss' Law of Propagation. In this case, a strip triangulation is adjusted by the least square method based upon the closing errors. One has to be clear that something is done which is not in accordance with the theory of accidental or random errors. All know that the systematic errors *do not* follow either the Gauss' Distribution or the Gauss' Law of Propagation.

Research effected in this connection has shown that both principles yield practically the same accuracy. The least square method needs more computations and thus more time and money. Both principles, by the way, could be treated by fast electronic computers.

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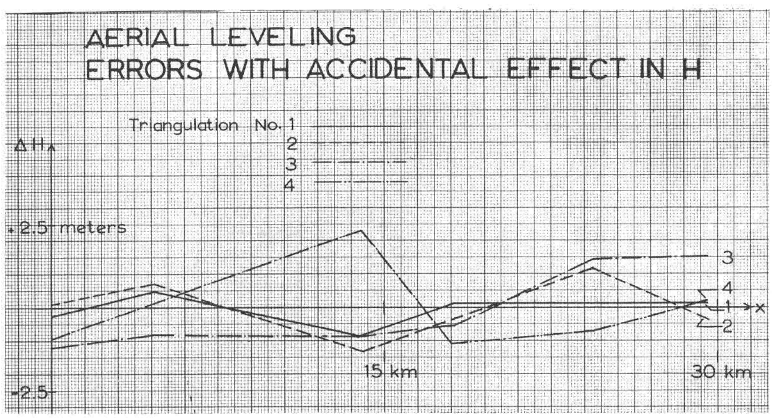

FIG. 6d