The Automatic Map Compilation System

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ABSTRACT: *The Automatic Map Compilation System, currently under development by PRA for USAERDL, is designed to produce a true orthographic photomap output from each conjugate pair of aerial diapositives without establishing a stereo model. Groups of such outputs can be pieced together to make up a composite photomap, corrected for relief, aircraft tilt, scale change, adjustment to control, etc. The system operates by scanning electronically those conjugate points on the two diapositives which correspond to points along a straight line in the stereo model. The scanning is controlled by a high-speed digital computer, and the resulting picture coordinates and ground elevations are tape recorded. The recorded numbers then control a rectifying print-out to produce the orthographic photomap. The operation is fully automatic, and should effect a considerable time saving compared to conventional compilation methods. A preliminary laboratory model of the scanning and matching section indicates satisfactory tracking capability.*

THE analog approach to the design of stereo mapping equipment has been successfully applied since mapping instruments were first made, and today it is still the only basic approach used by instrument designers. A stereo model, which is an exact analog of the ground terrain, is created by two projectors, each of which is so oriented and positioned as to exactly simulate the aerial camera orientation at the time each aerial photograph was taken. By this means aircraft tilts, drifts and altitude changes are easily compensated. Relief displacements appearing upon the projected photographs produce the three dimensional effect of the stereo model, and since drafting is done from the model rather than from the photographs, true horizontal as well as vertical coordinates are recovered.

These important advantages are lost when the strictly analog approach is abandoned. It is interesting to note that several attempts to automatize particular aspects of the analog approach to compilation have met with significant success: the Orthophotoscope[1] and the automatic contour plotters[2] are well known examples. Nevertheless, it is desirable to give up the analog approach to further the development of a completely automatic mapping system. Without the analog approach, recognizing and correcting for the several causes (tilt, relief, scale change, etc.) of unwanted image displacements on each aerial photograph appears difficult, since only the total image displacement and not its several components is recognizable. Therefore attention has been directed to the analytic equations relating photographic and ground coordinates and possible means for instrumenting them. These relations, derived in terms of the Eulerian angles of the exposure stations, are given in Eqs. (1) and (2) of the Appendix. These matrix equations are point transformations between the photographs and the stereo model, and suggest a point-by-point scanning of the aerial photographs. Such a scanning system has been developed.[3] When used as one component of a computer controlled automatic solution to Eqs. (1) and (2), the scanning units become an integral part of The Automatic Map Compilation System.

This mapping system is one of several studied and devised by Paul Rosenberg Associates during the course of their work for the U. S. Army Engineer Research and Development Laboratories.[4,5] It a high-speed, fully automatic compiling system, designed to produce an orthographic photo-map print corrected for scale change, aircraft tilt, relief and adjustment to control. It is expected that its operating time will be measured in hours instead of the days characteristic of conventional compilation methods. Accuracy is to be maintained by precise scanning of aerial diapositives and the use of a very fast digital computer to control the over-all operation. This computer operates at arithmetic speeds in the microsecond range, and will allow a complete scanning of a diapositive pair in approximately one hour. The system requires as inputs the space resection coordinates and the orientation matrix elements of each exposure station. (See Appendix.) These input parameters are usually adjusted to the known ground-control first. They can be obtained in a number of ways, but it is anticipated that the analytic triangulation procedures recently developed[6] will supply them most easily and rapidly.

The Automatic Map Compilation System is designed to produce a corrected photomap by tracking profiles on the diapositives and then printing in corrected form the video information encountered. It therefore contains both a scanning and matching section, and a printing section. An experimental laboratory model of the scanning and matching section has been built and operated to test the feasibility of tracking profiles by this method. A laboratory model of the complete system is currently under development. In the description that follows a profile in the y direction is assumed, although a profile in any other direction can be tracked as easily.

Description of the System

The symbolic and block diagram of Figure 1 shows the major components, the main information flow, and the control connections of the Automatic Map Compilation System currently being developed and constructed. The heart of the system is the computer, in which most of the control signals originate. The computer not only acts as an equation solver (the computer aspect of its operation), but also provides the timing and programming for all the other operations involved. The computer programs the compilation process by choosing a seemingly arbitary succession of (X, Y, Z) ground coordinates for points in the stereo model, computing the corresponding picture coordinates (x_1, y_1) and (x_2, y_2), and then directing the flying spot scanners to scan these areas on the diapositives. It is interesting to note that by scanning the diapositives only to check on the accuracy of previously assumed ground point elevations, this procedure reverses the ordinary flow of information from diapositive to stereo model in conventional compilation.

The Automatic Map Compilation System can be divided into two parts—a scanning and matching section in the upper half of Figure 1, and a printing section shown in the lower half of Figure 1. In the scanning and matching section, a straight profile is cut through the stereo model (X held constant) by automatically scanning and matching the corresponding conjugate profile points on the two diapositives. Correct elevations for all profile points are recorded on magnetic tape, along with the (x_1, y_1) coordinates for these points on one diapositive. In the printing section,

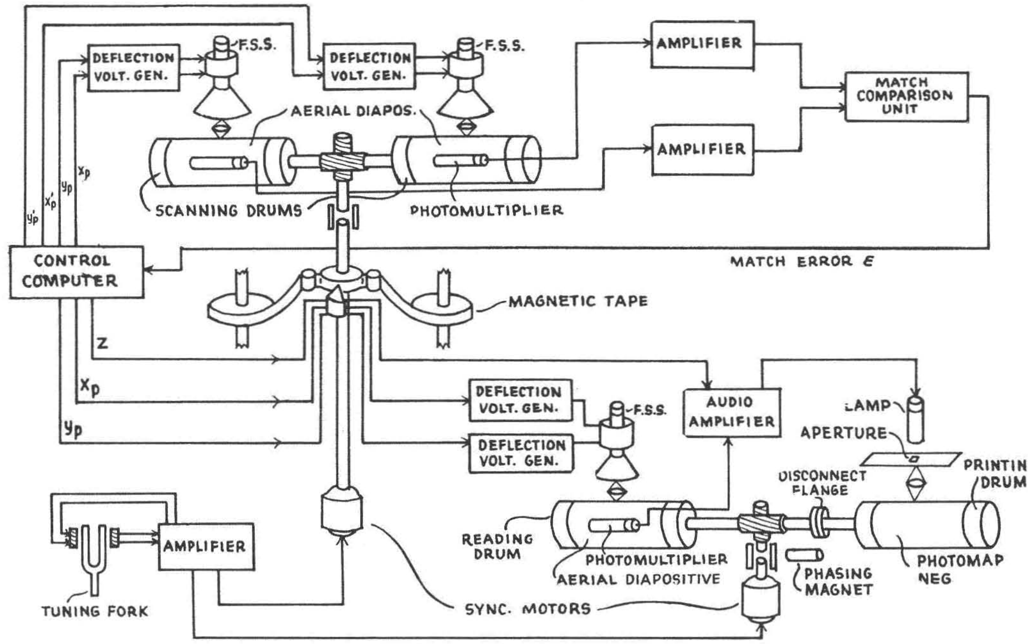

FIG. 1. Block diagram of the Automatic Map Compilation System.

this diapositive is again scanned along the irregular line made up of the sequence of recorded (x_1, y_1) values, and simultaneously the video signals are printed out along a straight line on a photomap.

The photomap should be a true orthographic projection of the terrain, corrected for ground relief and aircraft tilt, yaw, drift and altitude change. The photomap is not necessarily an end product, although in itself it will find valuable application. It can also be used as a master manuscript from which overlays can be made in the normal map production manner.

Initially a set of (X, Y, Z) coordinates for a ground control point (one whose coordinates are known from survey) are set into the computer. The computer—a fast, transistorized, digital type—solves Eqs. 1 and 2 for the conjugate picture coordinates (x_1, y_1) and (x_2, y_2) for this ground-control point. The diapositive coordinates, which are not available in digital form, must be converted by the deflection amplifiers shown in Figure 1 to a proportional set of currents. A set of four such currents is generated (one each for x_1, y_1, x_2, and y_2). When applied to the appropriate deflection coils of the flying spot scanners shown in Figure 1, these currents cause the spots to take positions on the faces of the tubes, such that images of the spots formed by the two lenses fall on those two points on the diapositives which correspond to the point (X, Y, Z) in the stereo model. Both diapositives are carefully registered upon rotating, transparent glass drums. Actually a large part of the motion of the spots relative to the diapositives (which is dictated by the solutions x_1, y_1, x_2 and y_2 of Eqs. 1 and 2) is produced by the accurately controlled rotational motion of the drums, rigidly joined upon a common axis. This rotational motion causes displacements of the spot images which are only approximations to the desired Y profile cuts. The deflection coil currents act as very sensitive vernier controllers to introduce the necessary corrections for tilt, ground relief, etc.

The lenses demagnify the flying spot-images to a diameter of approximately 0.001 inch on the diapositives, thereby preventing appreciable loss of resolution due to spot size. The two illuminated points on the overlapping diapositives must be compared to verify their correspondence, and for this purpose the spots are caused to move linearly along line segments 0.05 inch in length. Ten such line segments, each separated by 0.005 inch, make upon square scanning rasters 0.05 inch on a side, centered on the points of the diapositives whose coordinates, as specified by the computer, are (x_1, y_1) and (x_2, y_2). The intensities of the spots, which are transmitted through the emulsions and glass drums, and focused by condensing lenses onto photomultiplier tubes mounted on the drum axis (see Figure 1), are modulated by the densities of the emulsions. Hence the photo-multipliers, which convert the time-varying light signals to time-varying voltages, produce output waveforms which are equivalent to plots of emulsion density along the scan lines.

The voltage waveform resulting from each of the ten linear scans making up a scanning raster on one diapositive is compared with its corresponding and simultaneous linear scan in the raster on the other diapositive. The matching circuit (Figure 1) determines the difference, or error signal, from the matching of all ten corresponding short scans and adds them together to form a total match error "ϵ." The match error ϵ is a measure of the accuracy with which the ground elevation Z has been predicted for the point (X, Y). Since this first point of the profile is a control-point whose elevation is known, the match error ϵ should be very small. (This will not be true in general for succeeding points.)

After verification of the control-point images on the two diapositives, represented by the operating cycle just completed, the computer begins the same operation for the next point on the profile. In the time of this cycle (approximately 0.025 seconds), the drums have rotated enough to provide the approximate profile displacement required for the next point. The computer programs for the next profile point by holding the same X value as for the last point and increasing Y (merely a number as far as the computer is concerned) by the increment .02/12 S ft., where S is the scale of the photography (e.g. $S=1/40{,}000$).

This increase in Y corresponds to an approximate change in y_1 or y_2 of 0.02 inch. For this new set of (X, Y) values there is some Z value such that the new point (X, Y, Z) lies on the stereo model. The computer can make an intelligent guess at this new Z value by extrapolating from known Z values for preceding points on the profile. Specifically, the new Z is found from Eq. (3) of the Appendix, where the subscripts indicate Y coordinates along the profile, superscript "0" indicates an estimated Z value, and supercript "cor" indicates a known or corrected Z for earlier profile points. The α is a non-linearity parameter introduced to account for the natural, "eroded" character of actual terrain profiles. When two previous profile points do not exist, the value of Z is merely held constant from the preceding single point which does exist.

The computer again solves Eqs. 1 and 2 for (x_1, y_1) and (x_2, y_2) which correspond to the new $(X, Y, Z)^\circ$. In addition, the computer finds a set (x_1, y_1) and (x_2, y_2) for elevations $(Z^\circ \pm \Delta Z)$ and $(Z^\circ \pm 2\Delta Z)$, where $\Delta Z = .01/12S$ ft. For each of the five elevations, the flying spot scanners are directed to the appropriate points on the diapositives and match errors ϵ are obtained by comparing the scan wave forms. The ϵ's are designated ϵ°, ϵ^+, ϵ^{++}, ϵ^- ϵ^{--} to correspond to Z°, $(Z^\circ + \Delta Z)$, $(Z^\circ + 2\Delta Z)$, $(Z^\circ - \Delta Z)$ and $(Z^\circ - 2\Delta Z)$ respectively. Plotting match errors ϵ against elevations Z, a curve such as that shown in Figure 2 is obtained. This is known as a match error curve, and the one shown in Figure 2 is typical of those obtained with the laboratory equipment.

The curve of Figure 2 can be represented by a fourth order polynomial, and a quartic interpolation can be used to arrive at the elevation Z^{cor} which produces minimum match error. ϵ. (The computer can evaluate Z^{cor} by rapid approximation methods without actually constructing the curve of Figure 2.) The true elevation X^{cor}, in conjunction with its (X, Y) values, leads to a new computer solution of Eqs. 1 and 2 for (x_1, y_1). A solution is found for the picture coordinates on only one diapositive, and this solution is not verified by further scanning and waveform matching. Instead, the new (x_1, y_1) values are to be stored in digital form on two channels of magnetic tape (Figure 1), from which they are read later to control the printing of the photomap. Similarly, the Z^{cor} values will be recorded in digital form on a third channel of the same tape drive mechanism. The tape drive can be geared with the scanning drums during scanning operations so that the x_1, y_1 and Z^{cor} values are recorded synchronously with the advance along the profile produced by the rotating drums and deflected flying spots.

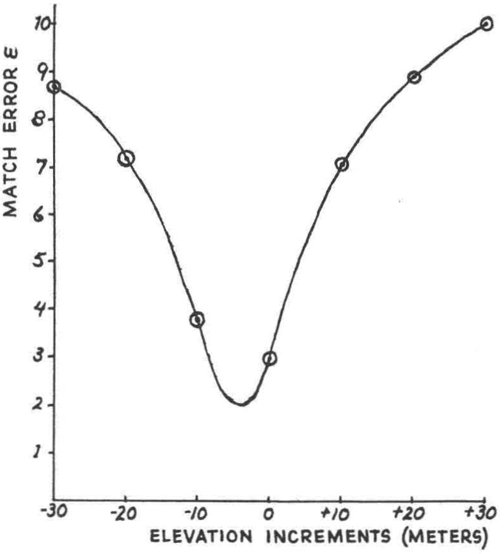

FIG. 2. Typical match error curve obtained with the experimental laboratory equipment shown in Fig. 3.

The system proceeds with successive points along the Y profile in the stereo model in exactly the same fashion. The computer holds the previous X value, advances Y by the increment .02/12 S, and extrapolates according to Eq. 3 to a new Z°. Choosing four additional elevations centered about Z°, the computer solves Eqs. 1 and 2 for a quintet of picture coordinates (x_1, y_1) and (x_2, y_2), and match errors ϵ°, ϵ^+, ϵ^{++}, ϵ^-, ϵ^{--} are found for each picture coordinate set. The correct elevation Z^{cor} for the profile point in question is then found by the interpolation just described. Thus, although a stereo model is never created, the assumption of successive sets of numbers (X, Y, Z) in the computer is equivalent to profile tracking through three dimensional space. A slow and constant axial drive of the two scanning drums provides the required X motion. The diapositives can be mounted upon the drums in a slightly skewed fashion, so that X actually changes value between Y profiles rather than during them.

Thus far it is evident that most of the signals generated by the components of Figure 1 are unstored time-varying quantities, e.g. the scan waveforms produced by the photomultipliers, the extrapolated elevation Z°, and the flying spot deflection currents generated from (x_1, y_1) and (x_2, y_2). Temporary storage occurs only within the control computer, where rapid access matrix storage is required for the intermediate steps in the solution of Eqs. 1 and 2, and in the match comparison unit, where the error signals resulting from matching individual scan lines are temporarily stored as charge on a condenser until the total error signal (or charge) ϵ for a complete scanning raster has been obtained. Permanent storage is required only when x_1, y_1 and Z^{cor} values are recorded on magnetic tape in digital form.

The first phase in the operation of the system is completed when all aerial negatives of a flight strip have been scanned, and the quantities x_1, y_1 and Z^{cor} entered on magnetic tape in digital form. The second phase, called printing or reproduction is entirely unsynchronized with the first, and may start any time after the first model of the first phase is completed. The printing phase has been designed as shown in the lower part of Figure 1. One of the diapositives, that for which (x_1, y_1) values have been tape-recorded, is to be placed with careful registration upon the reading drum of a facsimile printer. The printing drum, rigidly attached to the same shaft as the reading drum, carries a photosensitive emulation which will become a negative for a portion of the final orthographic print.

The design of the facsimile operation differs from the usual type by incorporating a flying spot scanner which scans the diapositive on the recording drum in a deliberately nonlinear fashion. The scanning motion is dictated by the (x_1, y_1) values recorded on the tape, so that video information is removed from the diapositive along the irregular lines which correspond to straight lines in the (nonexistent) stereo model. The signals produced by this irregular motion of the flying spot, after amplification, intensity modulate a fixed light-source, which exposes the photosensitive emulsion on the printing drum along straight, parallel, evenly spaced lines.

To describe the printing process in slightly greater detail, reference is made to the bottom part of Figure 1 where the proposed printing operation is shown. A synchronous motor, controlled by a tuning fork and amplifier, drives the two drums at a very constant angular speed (approximately 90 r.p.m.). The motor and amplifier are so designed that instantaneous angular shaft position errors can be limited to ±0.1 degrees, and reduction gears between the motor and drum drive shafts can further reduce the angular error encountered. The drive motor is mounted between the two drums to eliminate any torsional twists of the two shafts which might result from an end drive.

Finely threaded screw drives cause both drums to move in the X direction at very slow and constant linear speeds. If the spot on the flying spot scanner were not deflected, it would be focused by the demagnifying objective lens so as to trace out a set of parallel straight scans on the diapositive, because of the rotation and slow linear motion of the drums. Instead, the tap-stored (x_1, y_1) numbers are detected at the reading heads and sent, still in digital form, to the two deflection amplifiers. The deflection amplifiers convert the digital (x_1, y_1) numbers to analog currents, which in turn are applied to the X and Y deflection coils of the flying spot scanner to deflect the spot image on the diapositive in the desired fashion. Note that the deflection is only the incremental deflection (in x_1 and y_1) which must be added to the linear scan (provided by the motion of the drums) to arrive at the irregular scan line that is desired.

The same tuning fork which controls the synchronous motor driving the drum shafts is also designed to control the motion of the magnetic tape, and hence the rate of removal of (x_1, y_1) values. The two motions are carefully synchronized so that the complete set of (x_1, y_1) values corresponding to one Y scan through the stereo model will correspond to one rotation of the reading and printing drums. The proper phase between the drums and tape reel can be set by a magnetic phasing clutch.

The reading drum is a hollow glass cylinder containing an axis-mounted photomultiplier tube. The photomultiplier amplifies the light signal transmitted through the negative and drum, and converts it to a time varying voltage. A simple low-frequency amplifier of conventional type, except for low-frequency compensation and the presence of a thyrite expander, amplifies this signal before applying it to the lamp modulator. The expander feature partially compensates for the fact that the density of the diapositive on the reading drum is proportional to the logarithm of its original exposure, rather than to the exposure directly. The output voltage of the amplifier is then more nearly representative of the original aerial negative exposure than of its logarithm, although the exact relationship also involves the connection between the printing lamp brilliance and the modulating voltage applied to it.

Variations of the modulator voltage produce corresponding variations in the amount of current through the printing lamp. The latter is to be an R 1130B crater lamp which produces a brilliance proportional to the modulating voltage at frequencies up to 20 kc. A square aperture is located immediately in front of the printing lamp, where it lies in the object plane of a demagnifying lens forming an image of the aperture upon the printing emulsion. To avoid overlap or underlap during printing, the aperture image will be square, and the butting of aperture images lying adjacent to each other in the axial direction (aperture images appearing on suc-

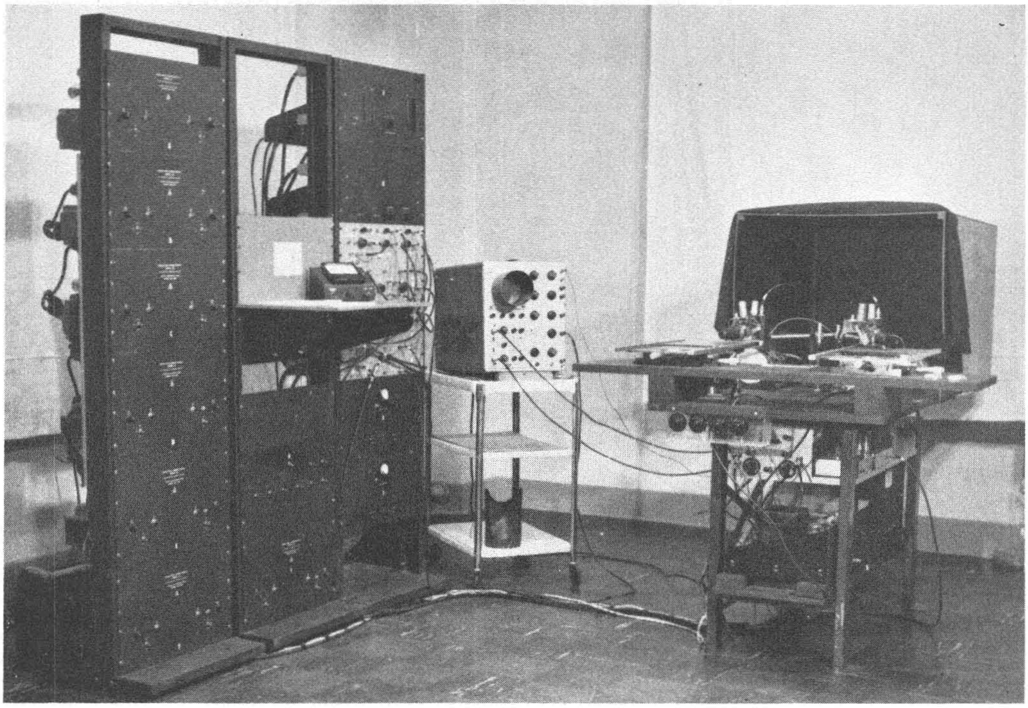

FIG. 3. Experimental laboratory equipment for preliminary study of match error curves and profiles.

cessive revolutions of the drum) held to an accuracy of better than 0.001 inches.

At the same time that (x_1, y_1) values are read off the magnetic tape, the stored Z^{cor} information is also sensed. Whenever a stored Z^{cor} has the proper value for the elevation of a contour line, a pulse will be sent to the modulator of the printing light to cause a spot to appear at that point on the photomap emulsion. A series of such spots, all for the same Z^{cor} value, will constitute a contour line.

A change of scale in the orthographic print can be made easily merely by replacing the printing drum by another of different diameter. The entire printing drum and drive shaft can be replaced by disconnecting from the motor drive at the bolted flange plates.

After it is developed, the photosensitive emulsion on the printing drum will hold a negative image, from which positive photomap prints can be made. These positive prints are true orthographic projections of ground images, corrected for scale change, rectification, relief, and adjustment to control. Prints for successive exposure stations along the flight path can be pieced together to make a composite orthographic mosaic.

EXPERIMENTAL RESULTS

An experimental laboratory model of the scanning and matching section (top part of Figure 1) has been constructed to make preliminary tests of the ability of the system to track profiles. This apparatus is shown in the photograph of Figure 3. Hand operated micrometer drives for glass diapositive plates were used instead of rotating scanning drums, and a simple desk calculator, which followed the same program, was used instead of the fully automatic digital computer. Otherwise, the electronic circuitry and all operations involved in taking experimental data were similar to those planned for the complete system.

Results of profile tracking with this experimental equipment are shown in Figure 4. The solid curve represents the profile as found[7] using conventional stereo plotting equipment (Wild A7 Stereoplotter). The circular points distributed along this curve were found using the laboratory model of the scanning and matching section of the system. The profile is taken from Aviogon RC-5 Arizona Test Area photography at a scale of 1:40,000; it contains several abrupt elevation changes and runs through areas containing no cultural features and fairly sparse topographic detail.

Although some difficulty was experienced in interpolating to a corrected elevation at four points of this profile, these experimental results were considered quite encouraging at the time they were obtained, and have led to

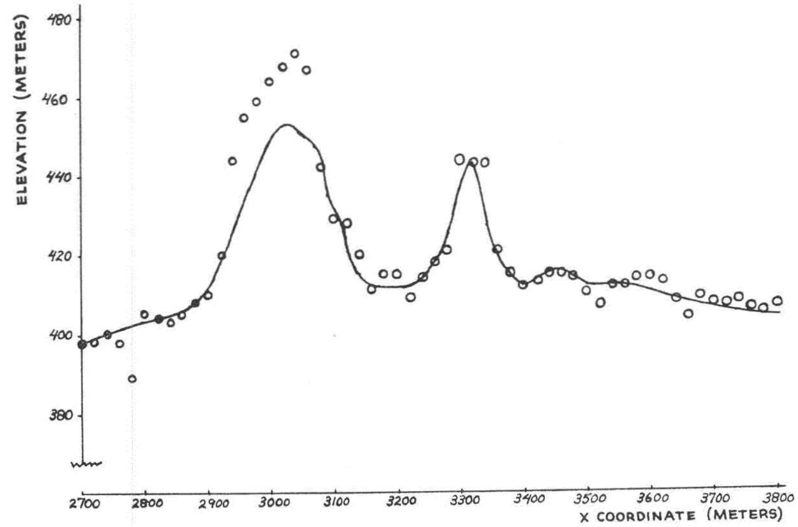

FIG. 4. Preliminary profile taken with the experimental laboratory equipment shown in Fig. 3. Solid curve represents the true profile; circles show experimental points.

modifications in the computer program which are expected to provide even closer automatic tracking of the true profile. A laboratory model of the complete Automatic Map Compilation System (including the printing section) is now being constructed and developed to test over-all performance.

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APPENDIX

Coordinates (X, Y, Z) of points on the ground can be related to photographic coordinates (x_1, y_1) or (x_2, y_2) of the corresponding image points by a matrix equation.[8]

$$\begin{pmatrix} x_{1\text{ or }2} \\ y_{1\text{ or }2} \\ -f \end{pmatrix} = \begin{pmatrix} A_{11}{}^{1\text{ or }2} & A_{21}{}^{1\text{ or }2} & A_{31}{}^{1\text{ or }2} \\ A_{12}{}^{1\text{ or }2} & A_{22}{}^{1\text{ or }2} & A_{32}{}^{1\text{ or }2} \\ A_{13}{}^{1\text{ or }2} & A_{23}{}^{1\text{ or }2} & A_{33}{}^{1\text{ or }2} \end{pmatrix} \begin{pmatrix} X - X_n \\ Y - Y_n \\ -H^{1\text{ or }2} + Z \end{pmatrix} \frac{f}{\left[-A_{13}{}^{1\text{ or }2}(X - X_n) - A_{23}{}^{1\text{ or }2}(Y - Y_n) + A_{33}{}^{1\text{ or }2}(H^{1\text{ or }2} - Z)\right]}$$

where

$x_{1 \text{ or } 2}, y_{1 \text{ or } 2}$ = photographic coordinates
X, Y, Z = ground-coordinates
X_n, Y_n = ground-radii-coordinates
f = camera focal-length
$H^{1 \text{ or } 2}$ = flying altitude
$A_{ij}^{1 \text{ or } 2}$ = orientation matrix element involving Eulerian angles at the exposure station 1 or 2

Solving for x and y in normal algebraic form,

$$x_{1 \text{ or } 2} = f\left[\frac{A_{11}^{1 \text{ or } 2}(X - X_n) + A_{21}^{1 \text{ or } 2}(Y - Y_n) - A_{31}^{1 \text{ or } 2}(H^{1 \text{ or } 2} - Z)}{-A_{13}^{1 \text{ or } 2}(X - X_n) - A_{23}^{1 \text{ or } 2}(Y - Y_n) + A_{33}^{1 \text{ or } 2}(H^{1 \text{ or } 2} - Z)}\right] \quad \text{Eq. (1)}$$

$$y_{1 \text{ or } 2} = f\left[\frac{A_{12}^{1 \text{ or } 2}(X - X_n) + A_{22}^{1 \text{ or } 2}(Y - Y_n) - A_{32}^{1 \text{ or } 2}(H^{1 \text{ or } 2} - Z)}{-A_{13}^{1 \text{ or } 2}(X - X_n) - A_{23}^{1 \text{ or } 2}(Y - Y_n) + A_{33}^{1 \text{ or } 2}(H^{1 \text{ or } 2} - Z)}\right] \quad \text{Eq. (2)}$$

The expression for an extrapolation to an expected elevation (Z°) at the next profile point, based upon previously known elevations (Z^{cor}), is given by 8:

$$Z_{Y+1}^{\circ} = (1 + \alpha)Z_Y^{\text{cor}} - \alpha Z_{Y-1}^{\text{cor}} \quad \text{Eq. (3)}$$

where α is a constant ≤ 1.

Some Notes on the Displacement of Photographic Images Caused by Tilt and Relief

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ABSTRACT: *Formulas are developed to express the relationships between angles measured about the principal-point, isocenter, and nadir-point in the plane of a tilted photograph and corresponding angles on the ground. These formulas are then analyzed to determine errors of photographic directions under selected conditions of tilt and relief.*

BOTH graphical and analytical principal-point radial triangulation are based on the assumption that the aerial photographs are absolutely vertical, because angles between rays emanating from the principal-point of a vertical photograph are precisely equal to corresponding angles on the ground, regardless of ground relief. Thus, in theory, there is a direct analogy between photogrammetric radial triangulation and triangulation conducted by the usual ground-survey methods.

In actual practice, of course, the aerial photographs are tilted up to a maximum of about three degrees and, except for lens distortion, the principal-point has no geometric significance as a radial-center, because relief displacements radiate from the nadir-point of a tilted photograph and tilt displacements radiate from the isocenter. In fact, if the photographic images are displaced both because of tilt and relief, there is no single point on a photograph which can correctly be used as the radial-center. Accordingly, until some means is provided for keeping the axis of the aerial camera absolutely vertical, the accuracy of photogrammetric radial triangulation will be limited by these image displacements regardless of whether the principal-point, the nadir-point, or the isocenter is used. However, the amount of error introduced is dependent upon the radial-center