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Differential Elevation by Adaptation of the Parallax-Correction Graph to Parallax Measurements on Aerial Photographs

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ABSTRACT: *This paper describes a method which will assist the photogrammetrist in attaining greater accuracy in determining elevations from aerial photographs. Through the use of simple and inexpensive equipment and the application of correction factors, the effects of distortion and mis-orientation on parallax measurement are reduced to a considerable degree, thereby improving the accuracy of the values used in the determination of elevation.*

DISTORTION in aerial photographs has long been appreciated as an active source of error in parallax measurements taken on aerial photographs for the purpose of determining elevation. Likewise, systematic errors may be introduced by tilt and mis-orientation of the photographs in mounting them for stereoscopic viewing. The purpose of this paper is to describe a method of applying corrections to observed parallax measurements which will substantially improve the quality of the measurements.

This method contemplates the use of a lens or mirror stereoscope with a parallax ladder (for description and use see Figure 6).* These, when skillfully employed, enable the photogrammetrist who is limited in experience and equipment to improve his accuracy in determining elevations from aerial photographs.

A stereo-pair of photographs is carefully oriented and mounted securely so that the flight line of each photograph is superimposed in extension with that of the other, and separated from each other the correct amount for the creation of a clear stereoscopic model when viewed with a stereoscope.

Parallax measurement between points of corresponding imagery is accomplished by means of the parallax ladder.

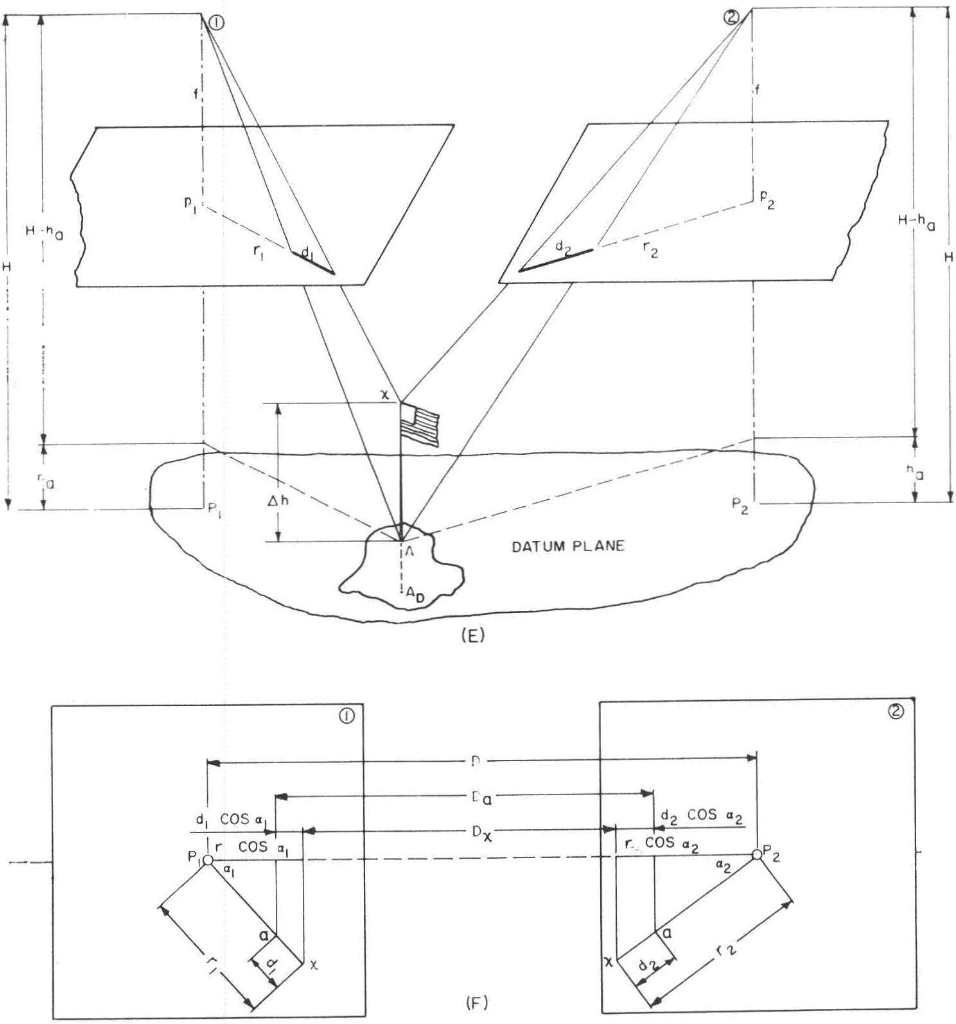
The formula used to convert parallax measurement into difference in elevation varies somewhat from the conventional form in that the photo-base, employing the average measurement between the principal and conjugate principal-points of each photograph, is dispensed with and the total separation distance between the principal points of the stereo-pair is employed.

Referring to Figure 1 it is shown that

* With the lens stereoscope the author employs a simple mirror accessory which permits the mounting of a stereo-pair without overlapping, and simplifies the application of the parallax-correction graph.

$$\Delta h = \frac{(H - h_a)(D_a - D_x)}{D - D_x}$$

$$\text{Transposing } D_a - D_x = \Delta p = \frac{(D - D_x)\Delta h}{H - h_a} \qquad \text{II}$$



FOR PHOTO ① $d_1 = \frac{r_1 \Delta h}{H - h_a}$ FOR PHOTO ② $d_2 = \frac{r_2 \Delta h}{H - h_a}$

$$\Delta h = \frac{(H - h_a) d_1}{r_1} = \frac{(H - h_a) d_2}{r_2} = \frac{(H - h_a)(d_1 \cos \alpha_1 + d_2 \cos \alpha_2)}{r_1 \cos \alpha_1 + r_2 \cos \alpha_2}$$

$$(d_1 \cos \alpha_1 + d_2 \cos \alpha_2) = D_a - D_x = \Delta p \qquad (r_1 \cos \alpha_1 + r_2 \cos \alpha_2) = D - D_x$$

$$\text{(I) } \Delta h = \frac{(H - h_a)(D_a - D_x)}{D - D_x} = \frac{(H - h_a)\Delta p}{D - D_x} \qquad \text{(II) } \Delta p = \frac{(D - D_x)\Delta h}{H - h_a}$$

FIG. 1 (E). Relief displacement.
FIG. 1 (F). Parallax measurement.

Δh = difference in elevation between point a (elevation known) and point x (elevation sought).

Δp = difference in parallax measurement between the two points.

H = flight altitude above datum (usually mean sea level).

h_a = elevation of point A above datum.

D_a = parallax measurement between images of a on photographs 1 and 2.

D_x = parallax measurement between images of x on photographs 1 and 2.

D = separation distance principal points of photos 1 and 2.

These equations are based upon the assumption that the photographs are perfect in all respects, and are free from distortion from any cause. Under this assumption the measurements made between identical photo-images on a stereo-pair may be utilized to secure elevational difference. Accuracy of measurements is limited only by the visual acuity of the observer and the precision of the measuring devices employed.

The various causes of possible distortion which may result in the displacement of the position of photo-detail, thereby causing error in observed measurements, may be noted as lens imperfection, tilt of the camera axis, error in elevation of camera station, unequal expansion and contraction of negatives and prints in processing, and improper or careless orientation of photographs in mounting them for stereoscopic viewing.

The following procedure presents a means of detecting much of the distortion and determining corrections to be applied to observed photo measurements.

CASE I. In Figure 2, assuming that no distortion exists in prints 1 and 2 due to the various causes previously noted.

D_a would equal the correct parallax measurement for point A, elev. 500 ft.

D_b would equal the correct parallax meas-

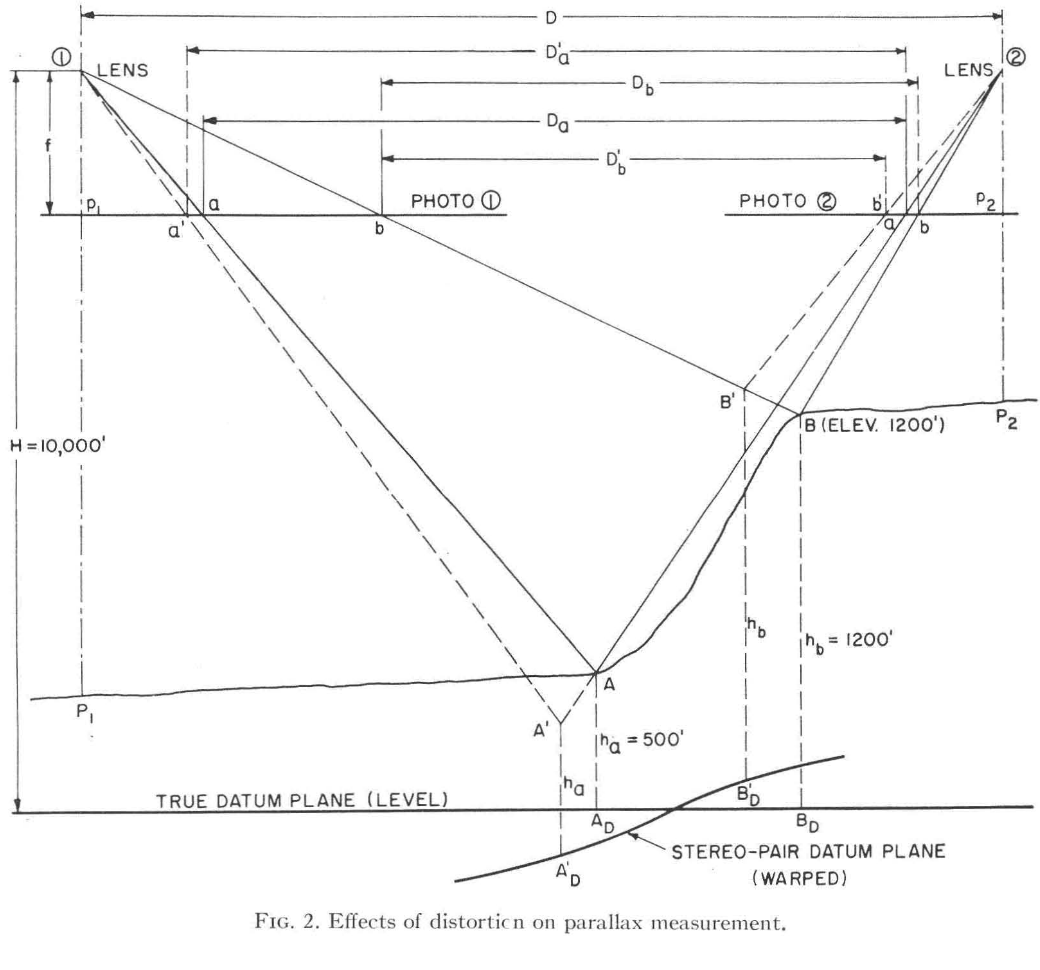


FIG. 2. Effects of distortion on parallax measurement.

urement for print B, elev. 1,200 ft.

Let $D_a = 50.70$ mm and $D_b = 44.59$ mm. Separation distance $D = 127.50$ mm.

Using formula (II) and computing the parallax difference to datum, D_d:

For point A

$$\Delta p_a = \frac{(127.50 - 50.70)500}{10,000} = 3.84 \text{ mm.}$$

For point B

$$\Delta p_b = \frac{(127.50 - 44.59)1200}{10,000} = 9.95 \text{ mm.}$$

If the datum positions of points A and B could be observed, the parallax readings for the points would be:

For point A

$$D_d = D_a + \Delta p_a = 50.70 + 3.84 = 54.54 \text{ mm.}$$

For point B

$$D_d = D_b + \Delta p_b = 44.59 + 9.95 = 54.54 \text{ mm.}$$

In this case the datum measurement D_d as computed for points A and B are equal fulfilling the ideal condition that for points of equal elevation the parallax measurement will remain constant.

CASE II. Assuming distortion to be present in prints 1 and/or 2 due to one or more of the causes previously mentioned. Thus in Fig. 2:

$$\Delta p_x = \frac{(D - D_x)\, h_x}{H}$$

Flight Altitude "H" = 10,000 Ft.
Separation of P_1 and P_2 = D = 127.50 mm

(1) Point	(2) Elev. H_x Feet	(3) Parallax Meas. D_x mm.	(4) $(D - D_x)$ mm.	(5) $\frac{h_x}{H}$	(6) Δp_x to Datum mm.	(7) Par. Rdg. to Datum $D_x + \Delta p_x = D_d$ mm.	(8) Assumed Correct D_d mm.	(9) Corr. to Observed Parallax Meas. mm.	(10) Corr. Parallax Meas. mm.
1	500	50.80	76.70	0.0500	3.84	54.64	55.00	+0.36	51.16
2	452	51.28	76.22	0.0452	3.44	54.72	55.00	+0.28	51.56
3	395	51.96	75.54	0.0395	2.98	54.94	55.00	+0.06	52.02
4	532	50.62	76.88	0.0532	4.09	54.71	55.00	+0.29	50.91
5	483	51.16	76.34	0.0483	3.69	54.85	55.00	+0.15	51.31
6	420	51.58	75.92	0.0420	3.19	54.77	55.00	+0.23	51.81
7	300	52.65	74.85	0.0300	2.25	54.90	55.00	+0.10	52.75
8	346	52.20	75.30	0.0346	2.60	54.80	55.00	+0.20	52.40
9	385	51.85	75.65	0.0385	2.87	54.72	55.00	+0.28	52.13
10	405	51.53	75.97	0.0405	3.08	54.61	55.00	+0.39	51.92
11	472	51.20	76.30	0.0472	3.60	54.80	55.00	+0.20	51.40
12	536	50.90	76.60	0.0536	4.10	55.00	55.00	0.00	50.90
			127.50 - Col. 3	Col. 2 / 10,000	(Col. 4)(Col. 5)	Col. 3 + Col. 6		Col. 8 - Col. 7	Col. 3 + Col. 9

FIG. 3. Correction Graph Tabulation Sheet.

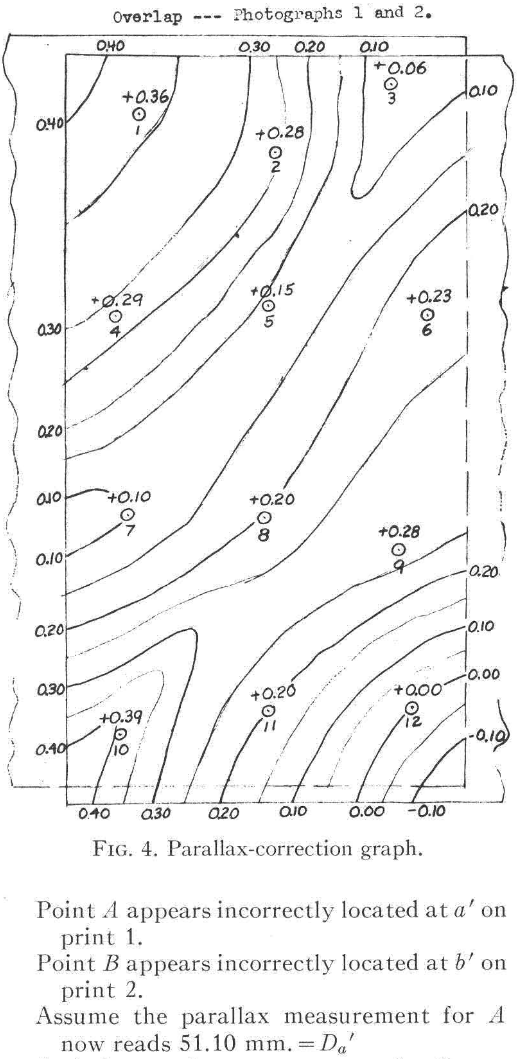


FIG. 4. Parallax-correction graph.

Point A appears incorrectly located at a' on print 1.

Point B appears incorrectly located at b' on print 2.

Assume the parallax measurement for A now reads 51.10 mm. $= D_a'$

And the parallax measurement for B now reads 44.25 mm. $= D_b'$

Apply formula (II) as before:

For point A

$$\Delta p_a' = \frac{(127.50 - 51.10)500}{10,000} = 3.82 \text{ mm.}$$

For point B

$$\Delta p_b = \frac{(127.50 - 44.25)1,200}{10,000} = 9.99 \text{ mm.}$$

Computing parallax readings to datum as before:

For point A

$$D_d = D_a' + \Delta p_a = 51.10 + 3.82 = 54.92 \text{ mm.}$$

For point B

$$D_d = D_b' + \Delta p_b = 44.25 + 9.99 = 54.24 \text{ mm.}$$

It is apparent that the computed parallax measurements to datum are not now constant; indicating the equivalent of a warped datum plane. An extension of this examination to additional points within the stereoscopic overlap provides a measure of the extent of the apparent warpage of the datum plane within the boundaries of the overlap.

Thus: a correction made to the observed parallax measurements of points a and b, would have the effect of restoring these points to their correct relative position on the photograph.

To illustrate the practical application of this principle, consider the following:

Select an arbitrary value for D_d in Case II such as 55.00 mm. (Any value would do since it is the *difference* in parallax measurement we are correcting, and these differences would be the same regardless of the value of D_d selected.) Comparing this assumed value with computed D_d for point A in Case II, it is noted that the difference in datum measurement is equal to:

$$55.00 - 54.92 = +0.08 \text{ mm.}$$

A like treatment for point B yields a difference in datum measurement equal to:

$$55.00 - 54.24 = +0.76 \text{ mm.}$$

Applying these differences to the observed parallax measurements of points A and B respectively results in corrected parallax measurements as follows:

For point A:

$$51.10 + 0.08 = 51.18 \text{ mm.}$$

For point B:

$$44.25 + 0.76 = 45.01 \text{ mm.}$$

$$\text{Corrected } \Delta p = 6.17 \text{ mm.}$$

$$\text{Observed } \Delta p = 6.85 \text{ mm. Case II}$$

Substituting the *corrected* Δp in formula (I):

$$\Delta h = \frac{(10,000 - 500)6.17}{127.50 - 45.01} = 710 \text{ feet.}$$

Substituting the *observed* Δp in formula (I):

$$\Delta h = \frac{(10,000 - 500)6.85}{127.50 - 44.25} = 782 \text{ feet.}$$

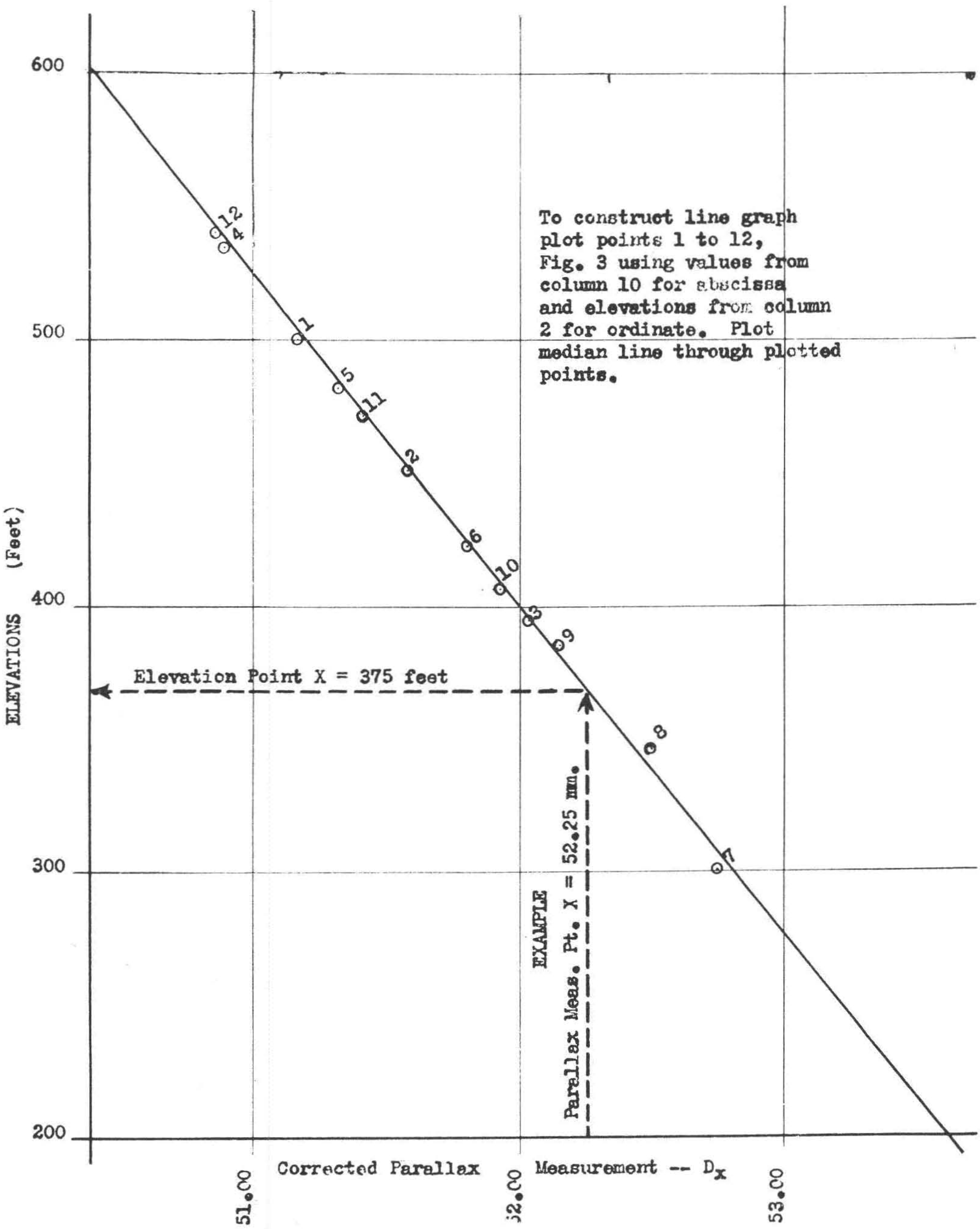


FIG. 5. Line graph for spot elevations.

Obviously the corrected parallax measurements, when applied in the formula, yield a more accurate difference in elevation than do the uncorrected measurements. By selecting additional points of known elevation falling within the overlap of a stereo-pair, and calculating the value of D_d for each, it can be seen that the apparent warp of the datum plane can be given mathematical expression. This can be expressed graphically by making a transparent overlay of the selected points of known elevation, plotted in their proper relative positions, and noting for each point the difference between the calculated datum parallax measurement and the assumed datum measurement. Correction lines can

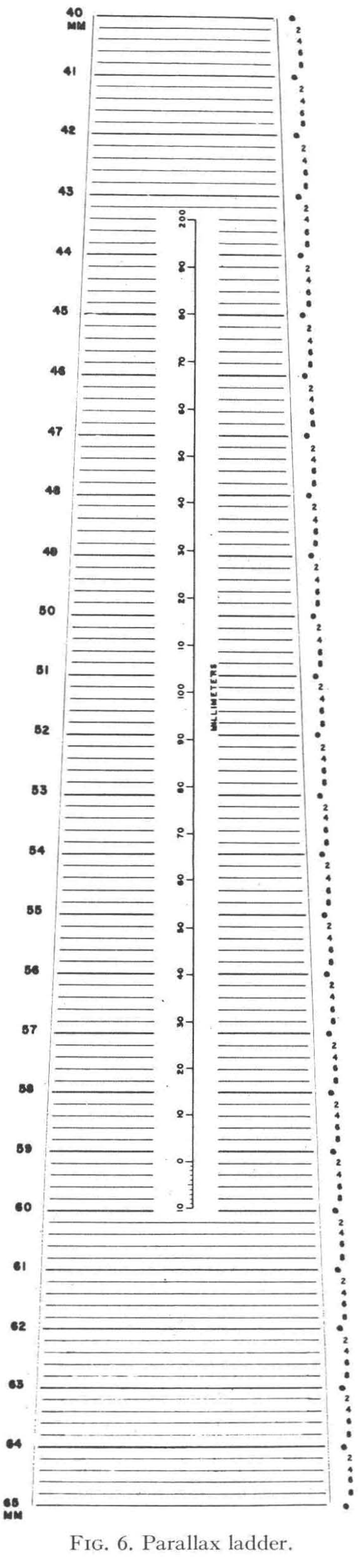


FIG. 6. Parallax ladder.

Description and Use: The parallax ladder is a device for reading parallax measurement between identical images on the left and right-hand photographs of a stereo-pair. The separation of the rails of the ladder can be varied to fit the type of stereoscope used. The example in Fig. 6 is designed for use with the lens type stereoscope. Obviously, a mirror stereoscope would require a greater distance between the rails of the ladder to accommodate the greater separation of the photographs.

To read parallax measurements the ladder is placed over a properly mounted stereo-pair and adjusted so that the left-rail will contact some detail on the left-photograph; the right-rail will contact the identical detail on the right-photograph; the graduations on the left and right-hand rails will be the same and a measure of the distance between the points.

When viewed through the stereoscope the rails of the ladder appear merged as one line sloping downward towards the observer and piercing the stereo-model at the point for which parallax measurement is sought. With practice it is possible to read a parallax measurement to the nearest 0.2 mm. and estimate to the nearest 0.05 mm.

The dimensions on this illustration are 78.5% of those on the original drawing.

then be interpolated in the same manner, in reference to these points, as contours are interpolated with reference to control elevations on a map. See Figure 4.

The correction-graph overlay may be used as a means of determining corrections to the observed parallax measurements of any number of random points on the overlap of a stereo-pair by simply matching the overlay to the photograph and visually interpolating the correction to be applied to the parallax measurement of each point. Figure 3 suggests a tabulation form for calculating the corrections to be used in the construction of the correction graph.

A line graph, as illustrated in Figure 5, may be constructed from the data shown in Figure 3, and may be used to read elevations directly without application of the parallax formula. Using the *corrected* parallax measurement values from column 10 for the abscissa and the H_x values from column 2 for ordinate, the several control points listed may be plotted to any suitable scale.

Since this plotting utilizes the corrected parallax measurements for points of known elevation, a line drawn through the plotted points expresses the relationship between elevation and parallax measurement. Thus, by the use of *corrected* parallax measurement as argument, such a graph provides a simple way of securing elevations for any number of desired points distributed over the stereoscopic overlap.

Application of this procedure affords a simple way of increasing the accuracy of parallax measurements from aerial photographs, thereby improving the determination of elevation.

*Systematic Procedure for Affine Rectification**

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SECTION 1. The *subject* of this paper is the production of large-scale controlled photomosaics, based on control obtained from slotted-templet triangulation in approximately negative scale.

Despite the limited accuracy of this control, a *high relative accuracy* (a good fitting) of the mosaic can be achieved if all five degrees of freedom of the optical-mechanical rectifier are employed instead of the three used in normal perspective rectification (1). Through affine rectification, however, the projection can be deformed in such a manner that its four corner passpoints are brought into perfect coincidence with those on the control sheet. Because the neighboring photographs of the same and adjacent strips will be rectified while using the same points, no discrepancies will be evident in those points when fitting neighboring rectified photographs to each other.

The need for affine rectification is especially felt when dealing with large-scale mosaics because their scale is usually three or four times larger than the negative scale. After perspective rectification on three points, the residual in the fourth point—caused by the deformation of the control net—will be 0.5 to 1.0 mm. when the mosaic scale is about equal to the negative scale; this residual will be 2 to 4 mm. when a four-times enlargement is used. In the first case the residual might be easily distributed over the four pass-points by some changes in scale and tilt. In the latter case, however, this would leave intolerable discrepancies; consequently another method must be sought.

Such a method can be found through employing the two, yet unused, degrees of freedom of the rectifier, viz. both components of displacement of the negative carrier, relative to the rectifying lens. Because the result is an *affine* deformation of the projection, the procedure of using all five degrees of freedom of the rectifier is called "affine rectification."

In the following, a practical systematic procedure for affine rectification is developed in which, step-after-step, the five settings of

* This paper was presented and distributed at the Second United Nations Regional Cartographic Conference for Asia and the Far East (Tokyo, Japan, Oct. 20 to Nov. 1, 1958.