then be interpolated in the same manner, in reference to these points, as contours are interpolated with reference to control elevations on a map. See Figure 4.

The correction-graph overlay may be used as a means of determining corrections to the observed parallax measurements of any number of random points on the overlap of a stereo-pair by simply matching the overlay to the photograph and visually interpolating the correction to be applied to the parallax measurement of each point. Figure 3 suggests a tabulation form for calculating the corrections to be used in the construction of the correction graph.

A line graph, as illustrated in Figure 5, may be constructed from the data shown in Figure 3, and may be used to read elevations directly without application of the parallax formula. Using the *corrected* parallax measurement values from column 10 for the abscissa and the H_x values from column 2 for ordinate, the several control points listed may be plotted to any suitable scale.

Since this plotting utilizes the corrected parallax measurements for points of known elevation, a line drawn through the plotted points expresses the relationship between elevation and parallax measurement. Thus, by the use of *corrected* parallax measurement as argument, such a graph provides a simple way of securing elevations for any number of desired points distributed over the stereoscopic overlap.

Application of this procedure affords a simple way of increasing the accuracy of parallax measurements from aerial photographs, thereby improving the determination of elevation.

*Systematic Procedure for Affine Rectification**

J. VISSER, *United Nations,*
P. O. Box 1555, Teheran

SECTION 1. The *subject* of this paper is the production of large-scale controlled photomosaics, based on control obtained from slotted-templet triangulation in approximately negative scale.

Despite the limited accuracy of this control, a *high relative accuracy* (a good fitting) of the mosaic can be achieved if all five degrees of freedom of the optical-mechanical rectifier are employed instead of the three used in normal perspective rectification (1). Through affine rectification, however, the projection can be deformed in such a manner that its four corner passpoints are brought into perfect coincidence with those on the control sheet. Because the neighboring photographs of the same and adjacent strips will be rectified while using the same points, no discrepancies will be evident in those points when fitting neighboring rectified photographs to each other.

The need for affine rectification is especially felt when dealing with large-scale mosaics because their scale is usually three or four times larger than the negative scale. After perspective rectification on three points, the residual in the fourth point—caused by the deformation of the control net—will be 0.5 to 1.0 mm. when the mosaic scale is about equal to the negative scale; this residual will be 2 to 4 mm. when a four-times enlargement is used. In the first case the residual might be easily distributed over the four pass-points by some changes in scale and tilt. In the latter case, however, this would leave intolerable discrepancies; consequently another method must be sought.

Such a method can be found through employing the two, yet unused, degrees of freedom of the rectifier, viz. both components of displacement of the negative carrier, relative to the rectifying lens. Because the result is an *affine* deformation of the projection, the procedure of using all five degrees of freedom of the rectifier is called "affine rectification."

In the following, a practical systematic procedure for affine rectification is developed in which, step-after-step, the five settings of

* This paper was presented and distributed at the Second United Nations Regional Cartographic Conference for Asia and the Far East (Tokyo, Japan, Oct. 20 to Nov. 1, 1958.

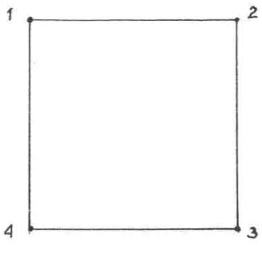


FIG. 1

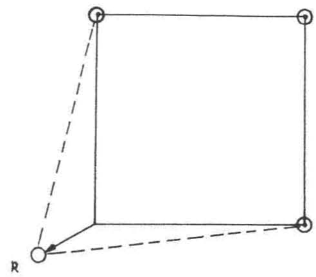


FIG. 2

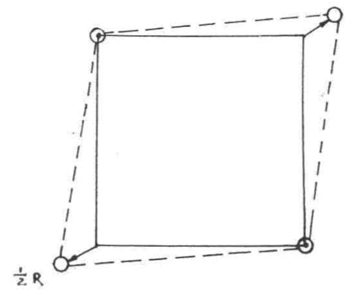


FIG. 3

the rectifier are solved. The method is based on using the Zeiss SEG V—Rectifier but can be applied—after a slight modification—to any other optical-mechanical rectifier having five degrees of freedom.

It goes without saying that in affine rectification the automatic vanishing point control of the SEG V cannot be used and that consequently the mechanism for this control must be uncoupled.

SECTION 2. Following the usual procedure, first by scaling (foot-pedal) and by moving the control sheet on the easel, the projections of the points of a diagonal (the points 1 and 3 in Figure 1) are brought into coincidence with the corresponding points of the control sheet.

The next step—elimination of the discrepancy in point 2—is facilitated if in what follows, the automatic vanishing point control is uncoupled. In practice, the automatic vanishing point control would be uncoupled at the beginning and would not be used at all. Then, half the x-discrepancy in point 2 is eliminated by tilting the easel about the x-axis; the control sheet is slightly moved until points 1 and 3 of projection and control sheet are in coincidence again; and half the y-discrepancy in point 2 is eliminated by tilting the easel about the y-axis. By a slight movement of the control sheet, its points 1, 2 and 3 can now be brought into perfect coincidence with the corresponding projection points; point 4 however will show a discrepancy.

For point 4 repeating the above procedure as described for point 2 but with the difference that only a quarter of the x and y discrepancies is now eliminated, the situation shown in Figure 3 is obtained; a typical affine deformation.

SECTION 3. Analytically, any affine transformation may be described as a linear transformation with six elements: a scale change ($\Delta\lambda$), a change in azimuth ($\Delta\psi$), a translation in x-direction (C_x), a translation in y-direction (C_y), an elongation (or contraction) of the figure in an arbitrary direction and a shear deformation in a direction perpendicular to the latter.

Let the elongation (in the direction of L, at an angle α with the y-axis) be: μ, and the shear deformation (perpendicular to the latter): ρ. See Figure 4.

For an arbitrary point, P is: $\Delta l = l\cdot\mu$ and $\Delta m = l\cdot\rho$; thus

$$\Delta x = \Delta l\cdot\sin\alpha + \Delta m\cdot\cos\alpha = l(\mu\sin\alpha + \rho\cos\alpha)$$
$$\Delta y = \Delta l\cdot\cos\alpha - \Delta m\cdot\sin\alpha = l(\mu\cos\alpha - \rho\sin\alpha)$$

Further:

$$l = y\cdot\cos\alpha + x\cdot\sin\alpha,$$

thus

$$\Delta x = (y\cos\alpha + x\sin\alpha)(\mu\sin\alpha + \rho\cos\alpha)$$
$$\Delta y = (y\cos\alpha + x\sin\alpha)(\mu\cos\alpha - \rho\sin\alpha)$$

In our case, the elements of the necessary affine transformation will be small because the residuals ($\frac{1}{2}R$) are small (some millimeters only), and consequently the total change in x and y may be found as the algebraical sum of the individual small changes:

$$\Delta x = (y\cos\alpha + x\sin\alpha)(\mu\sin\alpha + \rho\cos\alpha) + x\cdot\Delta\lambda + y\cdot\Delta\psi + C_x$$
$$\Delta y = (y\cos\alpha + x\sin\alpha)(\mu\cos\alpha - \rho\sin\alpha) + y\cdot\Delta\lambda - x\cdot\Delta\psi + C_y.$$

Substituting in these equations the known values x, y, Δx and Δy for the points 1, 2 and

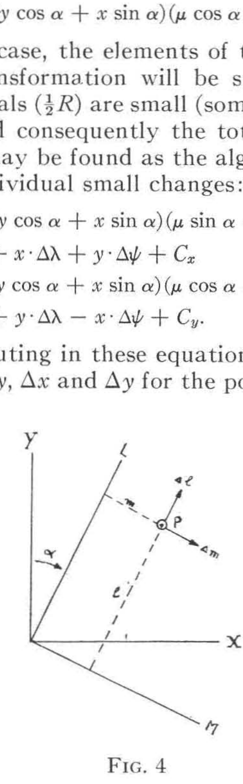


FIG. 4

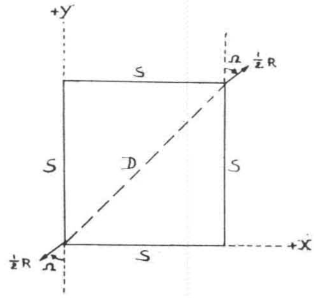


FIG. 5

3 (which—together with point 4—approximately form a square with side lengths S; see Figure 5), after some computation the following result is found:

$$C_x = -\tfrac{1}{2}R \sin \Omega$$
$$C_y = -\tfrac{1}{2}R \cos \Omega$$
$$\Delta\lambda = R/D \sin(\Omega - \alpha) \cdot \cos(\alpha + 45°)$$
$$\Delta\psi = -R/D \cos(\Omega - \alpha) \cdot \cos(\alpha + 45°)$$
$$\rho = \tfrac{1}{2}R/S\{\sin(\Omega - 2\alpha) + \cos(\Omega - 2\alpha)\}$$
$$\mu = \tfrac{1}{2}R/S\{\cos(\Omega - 2\alpha) - \sin(\Omega - 2\alpha)\}.$$

Substituting the above values, as a check, in the formulae for Δx_4 and Δy_4, the expected values

$$\Delta x_4 = +\tfrac{1}{2}R \sin \Omega$$

and

$$\Delta y_4 = +\tfrac{1}{2}R \cos \Omega$$

are found.

SECTION 4. The affine transformation must be performed in the rectifier. Dr. C. A. Traenkle (2) derived the formulae for the settings ΔU and ΔR—these respectively are the components of displacement of the negative carrier in the direction of maximum tilt, and in the direction perpendicular to the latter—as functions of the required elongation (μ) and shear deformation (ρ) for rectifiers with only one easel axis.

Dr. Traenkle based his formulae on the known equations:

$$x \approx \frac{h}{f_a} x^1 + i \frac{h}{f_a^2} (x^1)^2$$
$$y \approx \frac{h}{f_a} y^1 + i \frac{h}{f_a^2} x^1 y^1$$

in which x, y are the coordinates in the horizontal projection plane x^1y^1 the coordinates in the negative plane, while in both planes the principal point acts as the origin of the coordinate-system, and the x- respectively x^1—axis is in coincidence with the direction of maximum tilt. Further: h = flying height (in scale of projection), f_a = focal distance of the taking camera and i = phototilt.

Differentiation gives:

$$\Delta x = \frac{h}{f_a} \cdot \Delta x^1 + 2i \cdot \frac{h}{f_a^2} \cdot x^1 \cdot \Delta x^1$$
$$\Delta y = \frac{h}{f_a} \cdot \Delta y^1 + i \cdot \frac{h}{f_a^2} \cdot y^1 \cdot \Delta x^1 + i \cdot \frac{h}{f_a^2} \cdot x^1 \cdot \Delta y^1.$$

After reduction for scale, azimuth and translations, this appears to correspond with an elongation in x-direction for which

$$\mu = i \cdot \frac{\Delta x^1}{f_a}$$

and a shear deformation for which

$$\rho = -i \cdot \frac{\Delta y^1}{f_a} \cdot$$

Because the pencil of rays in the rectifier has the same perspective properties as the original pencil of rays of the exposure (3), the above derivation applies after substitution of

$$i = \frac{f_a}{f_r} \cdot \beta_t$$

(the known relation between the table tilt β_4 and the phototilt i where f_r = focal distance of the rectifying lens) and $\Delta x^1 = \Delta U$, $\Delta y^1 = \Delta R$. We have Dr. Traenkle's[2] result:

$$\Delta U = \frac{f_r}{\beta_t} \cdot \mu \quad \text{and} \quad \Delta R = -\frac{f_r}{\beta_t} \cdot \rho \qquad (2).$$

For rectifiers with two easel axes (like the Zeiss SEG V) and corresponding components of displacement of the negative carrier ΔP_x and ΔP_y, where the azimuth of maximum tilt has a random value (α), the above formulae can be easily converted to

$$\Delta P_x = \Delta U \cdot \sin \alpha - \Delta R \cdot \cos \alpha$$
$$\Delta P_y = \Delta U \cdot \cos \alpha + \Delta R \cdot \sin \alpha$$

(see Figure 6)
Thus

$$\Delta P_x = \frac{f_r}{\beta_t} \cdot (\mu \sin \alpha + \rho \cos \alpha)$$
$$\Delta P_y = \frac{f_r}{\beta_t} \cdot (\mu \cos \alpha - \rho \sin \alpha).$$

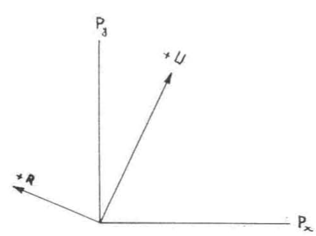
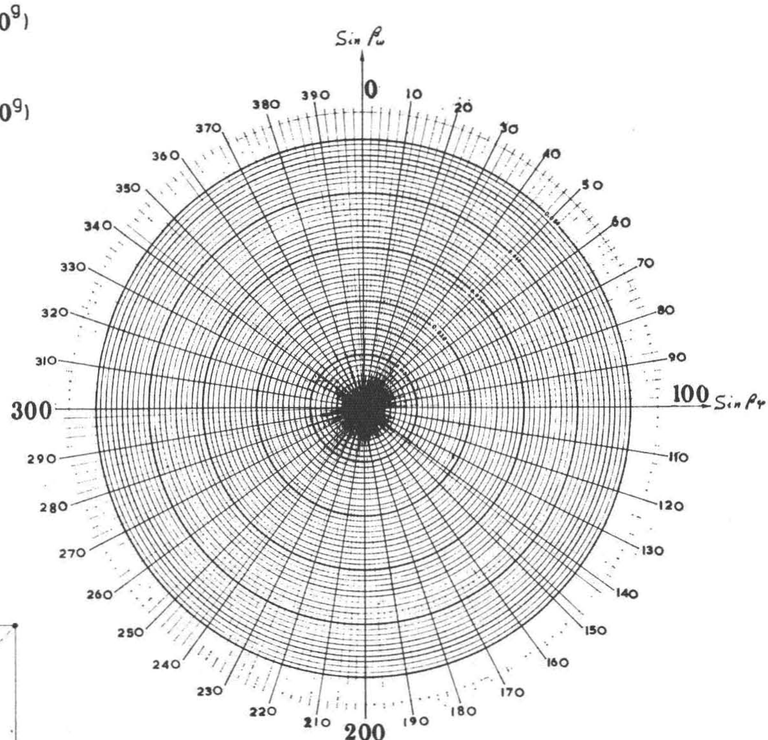


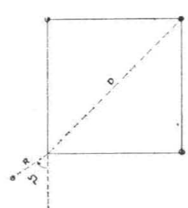
FIG. 6

NOMOGRAMS FOR AFFINE RECTIFICATION

$$P_X = \frac{f_r \cdot R/D}{\rho_t} \operatorname{Sin}(\Omega - \alpha + 50^g)$$

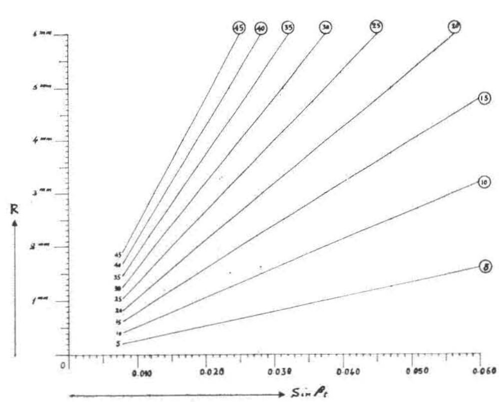
$$P_Y = \frac{f_r \cdot R/D}{\rho_t} \operatorname{Cos}(\Omega - \alpha + 50^g)$$





Nomogram for $\alpha = tg^{-1} \frac{\sin \rho_\varphi}{\sin \rho_\omega}$

$\operatorname{Sin} \rho_t = \sqrt{\sin^2 \rho_\varphi + \sin^2 \rho_\omega}$



Nomogram for $\frac{f_r\, R/D}{\rho_t}$ While D is assumed to be 960mm. For other values of D (D′) the result has to be multiplied with $\frac{960}{D'}$

Fig. 7

Substituting in the above the formulae (5) and (6) of Section 3, then after some computation the following expressions are obtained

$$\Delta P_x = \frac{f_r \cdot R}{\beta_t \cdot D} \cdot \sin(\Omega - \alpha + 45°)$$

$$\Delta P_y = \frac{f_r \cdot R}{\beta_t \cdot D} \cdot \cos(\Omega - \alpha + 45°).$$

These, rather simple, formulae (4) are easily solved by the help of simple nomograms.

Conclusion

A systematic procedure for affine rectification with a Zeiss-SEG V Rectifier may be as follows:

1. Scale and orient on diagonal 1.3; remove discrepancies in 2 with the table tilts.
2. Distribute the residual R_4 over the points 4 and 2 with the table tilts.
3. Compute (use nomograms) ΔP_x and ΔP_y and set these values in the rectifier.
4. Repeat step 1.
5. If necessary: Repeat step 2 etc.

Section 5. The above method has been developed in the National Cartographic Centre of Iran for the production of large-scale (1:5,000) controlled mosaics to be used for the planning of the canals etc. in irrigation projects.

With a minimum of ground-control—traverses perpendicular to the photo-strips every 20 kilometers—a reasonable absolute accuracy ($m_p = 20$ m) is obtained by slotted templet triangulation, while with a minimum of time spent at the rectifier, a very high relative accuracy is obtained by application of the described method of systematic affine rectification.

The necessary amounts of P_x and P_y are found from nomograms (see Figure 7). Because in the SEG V the sines of the table tilts instead of the table tilts themselves are read on the dials, the latter are used as the variables in the nomograms.

References

(1) The expressions "perspective" and "affine" rectification were introduced by Dr. R. Burkhardt in "Ueber Notwendigkeit and Möglichkeit der Affin-Entzerrung," *Bildmessung und Luftbildwesen*, 1956, pp. 10.

(2) Dr. C. A. Traenkle: "Affine Bildumformung mittels Entzerrungsgerät," *Zeitschrift für Instrumentkunde*, 1944, pp. 90–96.*

(3) Dr. C. A. Traenkle: "Die Bestimmung der räumlichen Lage von Flugzeugen mittels Luftbildmessung," *Bildmessung und Luftbildwesen*, 1943, Heft ½.

(4) It will be clear to the reader of the more recent paper of Dr. Traenkle: "Affinity Transformations in Photogrammetric Rectifiers," Photogrammetric Engineering, 1956, pp. 750–763, that, after some elaboration, the same formulae could be obtained from section 6 of that paper.

* The summary of part of the publication of Dr. Traenkle is given above by the author in order to make understanding of the present paper possible and also for those who do not read German.

Abstracts of Papers Read at 38th Annual Meeting of Highway Research Board

Editor's Note: A very large number attended the January 5–9 meeting. Among the papers on Highway Design, seven papers discussing the application and use of photogrammetry were given on January 7. These and the other 170 papers delivered at the meeting will later be published by the Board and a copy can be purchased by those interested. In the meantime, to assist the Society and resulting from the cooperation of Bill Pryor, the Director of the Board has given permission to reprint the abstracts of the photogrammetric papers and also the entire paper by Mr. Pryor. Thanks are extended and credit is given to the Board for this cooperation and assistance to the readers of this Journal. The abstracts of six papers follow. Due to insufficient space publishing the Pryor paper had to be postponed probably until the June issue.

Photogrammetric Profiles Developed from Stereo Aerial Vertical Photographs Aid in Highway Location Studies

D. J. Olinger, Aerial Engineer,
Wyoming Highway Department

Many areas of the United States still lack the U. S. Geologic Survey quadrangle maps with the smaller contour intervals (of 20 or 40 ft.) necessary for developing profiles on highway locations during area and route reconnaissance studies. In such areas reliable profiles can be developed from stereo