Substituting in the above the formulae (5) and (6) of Section 3, then after some computation the following expressions are obtained

$$\Delta P_x = \frac{f_r \cdot R}{\beta_t \cdot D} \cdot \sin (\Omega - \alpha + 45°)$$

$$\Delta P_y = \frac{f_r \cdot R}{\beta_t \cdot D} \cdot \cos (\Omega - \alpha + 45°).$$

These, rather simple, formulae (4) are easily solved by the help of simple nomograms.

Conclusion

A systematic procedure for affine rectification with a Zeiss-SEG V Rectifier may be as follows:

1. Scale and orient on diagonal 1.3; remove discrepancies in 2 with the table tilts.
2. Distribute the residual R_4 over the points 4 and 2 with the table tilts.
3. Compute (use nomograms) ΔP_x and ΔP_y and set these values in the rectifier.
4. Repeat step 1.
5. If necessary: Repeat step 2 etc.

Section 5. The above method has been developed in the National Cartographic Centre of Iran for the production of large-scale (1:5,000) controlled mosaics to be used for the planning of the canals etc. in irrigation projects.

With a minimum of ground-control—traverses perpendicular to the photo-strips every 20 kilometers—a reasonable absolute accuracy ($m_p = 20$ m) is obtained by slotted templet triangulation, while with a minimum of time spent at the rectifier, a very high relative accuracy is obtained by application of the described method of systematic affine rectification.

The necessary amounts of P_x and P_y are found from nomograms (see Figure 7). Because in the SEG V the sines of the table tilts instead of the table tilts themselves are read on the dials, the latter are used as the variables in the nomograms.

References

(1) The expressions "perspective" and "affine" rectification were introduced by Dr. R. Burkhardt in "Ueber Notwendigkeit and Möglichkeit der Affin-Entzerrung," *Bildmessung und Luftbildwesen*, 1956, pp. 10.

(2) Dr. C. A. Traenkle: "Affine Bildumformung mittels Entzerrungsgerät," *Zeitschrift für Instrumentkunde*, 1944, pp. 90–96.*

(3) Dr. C. A. Traenkle: "Die Bestimmung der räumlichen Lage von Flugzeugen mittels Luftbildmessung," *Bildmessung und Luftbildwesen*, 1943, Heft ½.

(4) It will be clear to the reader of the more recent paper of Dr. Traenkle: "Affinity Transformations in Photogrammetric Rectifiers," Photogrammetric Engineering, 1956, pp. 750–763, that, after some elaboration, the same formulae could be obtained from section 6 of that paper.

* The summary of part of the publication of Dr. Traenkle is given above by the author in order to make understanding of the present paper possible and also for those who do not read German.

Abstracts of Papers Read at 38th Annual Meeting of Highway Research Board

Editor's Note: A very large number attended the January 5–9 meeting. Among the papers on Highway Design, seven papers discussing the application and use of photogrammetry were given on January 7. These and the other 170 papers delivered at the meeting will later be published by the Board and a copy can be purchased by those interested. In the meantime, to assist the Society and resulting from the cooperation of Bill Pryor, the Director of the Board has given permission to reprint the abstracts of the photogrammetric papers and also the entire paper by Mr. Pryor. Thanks are extended and credit is given to the Board for this cooperation and assistance to the readers of this Journal. The abstracts of six papers follow. Due to insufficient space publishing the Pryor paper had to be postponed probably until the June issue.

Photogrammetric Profiles Developed from Stereo Aerial Vertical Photographs Aid in Highway Location Studies

D. J. Olinger, Aerial Engineer,
Wyoming Highway Department

Many areas of the United States still lack the U. S. Geologic Survey quadrangle maps with the smaller contour intervals (of 20 or 40 ft.) necessary for developing profiles on highway locations during area and route reconnaissance studies. In such areas reliable profiles can be developed from stereo air photo coverage through parallax measurements between selected image points on the photographs.

Discussed are the following:

1. Use of instruments and tools usually required, such as mirror stereoscope, 50 scale, parallax bar, millimeter scale, altimeter, and barograph.

2. Safeguards used to establish a reliable profile that will preclude detrimental errors in grade that would result from unrecognized and/or uncorrected tilt in the photography.

3. The miles per day output that can be expected from a two-man party.

A Photogrammetric Approach to Highway Route Location and Reconnaissance

Arthur C. Quinnell,
Montana State Highway Comm.

In Montana all available data from PMA and Forest Service photography, at scales of 1:20,000 and 1:63,000, United States Geological Survey quadrangle maps, and planimetric maps made by photographic projection, are utilized as guides to highway reconnaissance. However, this material is limited, in that no accurate comparisons of alternate routes in extremely rugged terrain where minor horizontal or vertical projections might double construction costs, can be made. Recency of this photography and mapping, and possible future changes in types of development, are taken into consideration together with traffic data and a good understanding of the objectives of the highway in eliminating all but a few, or perhaps one, route possibility.

The general problem is to keep all these factors in mind, and make the most economical, as well as rapid, use of various additional photogrammetric processes and equipment in selecting the best possible route.

Once several alternates have been chosen, the highway engineer must choose the most feasible one. Photogrammetry has been found to be the most rapid and economical method. Strip photography at a scale of 1:12,000 gives a fairly accurate difference in elevation for use in planning gradients and profile for cost comparison in eliminating all but the most feasible route. An alternate for rugged terrain is to use strip photography at appropriate scales to prepare topographic maps for use in comparing grades and computing earthwork quantities in arriving at a final location.

There are advantages and disadvantages in both of these methods. The first is usually rapid as well as economical; however, when applied to extremely rugged terrain where alternates are located very close together, it is not accurate enough. The second attains the accuracy required, but is more expensive and time consuming. Montana has adopted the following procedure, which utilizes parts of each of these methods.

Step 1. Reconnaissance of area to determine route possibilities using U.S.G.S. and Forest Service photography, at a scale of 1:21,000 to 1:63,000, and U.S.G.S. quadrangle maps and planimetric maps.

Step 2. Photographing route possibilities as selected in Step 1 at a scale of 1:12,000.

Step 3. Comparing the alternate routes by mirror stereoscope and parallax methods.

Step 4. Comparing final alternates by actual cost estimates (a) of entire project where required, and (b) of problem sections where a high degree of accuracy is required (as is the case in most instances).

This is accomplished by means of a small economical contact-print stereoplotter, acquiring ground control from highway construction plans, U.S.G.S. maps, and previously mapped areas.

By actual comparisons of field methods for comparing route alternates and the aforementioned method, it has been found that the accuracy attainable in earthwork quantities and other pertinent data falls within ± 5 per cent. The method also appears to be approximately four times as fast as a comparable field survey, and considerably more economical.

Photogrammetry in Highway Planning

David S. Johnson, Assistant Chief of Planning
Connecticut State Highway Dept.

Photogrammetry has two main fields of usefulness in highway planning—preliminary engineering and public relations.

In preliminary engineering, which in Connecticut is a function of planning, photogrammetry is used for determining and weighing alternate lines, and for transmitting the recommended schematic layout for processing through the survey and design stages.

Further, it is used as a base for portraying proposed highway improvements at the public hearings required by federal and state statutes.

Adjustment of Photogrammetric Surveys

L. L. Funk, Photogrammetric Engineer
Calif. Div. of Highways

A previously reported experimental project indicated the possibility of increased accuracy of earthwork quantities by adjusting photogrammetric surveys to an accurate field profile. This paper is a report on the effect of such adjustments on earthwork quantities obtained under actual field conditions on portions of three construction projects.

In each case construction plans were developed, and design quantities obtained, from photogrammetric maps at a scale of 1 in. = 50 ft. with 2-ft. contours. Field cross-sections for determining pay quantities were taken as the projects were slope staked. Comparisons were made for ten sections, each approximately 1 mile in length, from the three projects. The difference between field survey quantities and photogrammetric survey quantities for the ten sections ranged from 0.3 per cent to 5.4 per cent in excavation and from 0.9 per cent to 9.7 per cent in embankment.

The photogrammetric surveys were then adjusted by raising or lowering the entire terrain at each cross-section by an amount equal to the error in the centerline elevation. After adjustment the differences in excavation quantities were reduced to amounts ranging from 0.0 per cent to 0.7 per cent in eight of the ten sections and to 1.4 per cent and 1.8 per cent in the other two. In embankment one of the sections showed a difference of 1.8 per cent after adjustment, with the remaining nine having 0.1 per cent to 0.7 per cent differences. Comparisons were also developed to show differences before and after adjustment for large individual cuts and fills and for sections 1,000 ft. in length.

Data concerning the relationship of the accuracy of photogrammetric surveys in measuring the elevation of discrete points and the accuracy of earthwork quantities, also developed in the study, show a close relation between the arithmetic mean of a centerline profile and the accuracy of earthwork quantities before adjustment. They also indicate a less direct but apparent relation between the standard deviation of errors in the elevation of discrete points and the accuracy of earthwork quantities after adjustment. Further research may develop a method of expressing the accuracy of earthwork quantities after adjustment in terms of probability.

The study indicates that adjustment of photogrammetric surveys by means of an accurate field-profile will (a) materially reduce large localized errors in earthwork quantities, and (b) result in over-all quantities which are within limits generally considered tolerable for purposes of payment.

Terrain Data for Earthwork Quantities

L. L. Funk, Photogrammetric Engineer,
Calif. Div. of Highways

Earthwork quantities for a 2,000-ft. experimental section were measured by two field surveys and six photogrammetric surveys. One of the field surveys was made with sufficient precision to serve as a yardstick for measuring the accuracy of the other surveys. Three of the photogrammetric surveys were made by reading spot heights along cross-section lines in the stereomodel. Two-ft. contour interval maps at a scale of 1 in. = 50 ft. were the end product of the other three photogrammetric surveys.

Analysis of the errors in measuring the elevations of centerline stations and slope stakes by the photogrammetric surveys showed a wide variation in accuracy. The arithmetic mean of the errors in the centerline profile indicated a tendency towards systematic errors in four of the surveys. All but one of the photogrammetric surveys were well within the usual specification requirements for 2-ft. contour interval maps.

Earthwork quantities for the 3,000-ft. section were approximately 63,000 cu.yd. of excavation and 29,000 cu.yd. of embankment. Errors in quantities from the photogrammetric surveys ranged from 0 to 2.5 per cent in excavation and from 0.8 to 7.9 per cent in embankment. These errors in quantities were equivalent to a mean vertical difference over the entire area between slope stakes of from +0.02 to +0.27 ft. In four of the photogrammetric surveys this equivalent vertical error was 0.10 ft. or more.

Analysis of the results showed that the arithmetic mean of the errors in the centerline profile was closely related to the errors in earthwork quantities in each of the six photogrammetric surveys. This relationship indicated the possibility of adjusting the earthwork quantities by raising or lowering the entire terrain at each cross-section by an amount equal to the error at centerline. These adjustments were made by machine computation.

The adjustments reduced the errors in earthwork quantities from the photogrammetric surveys to amounts ranging from 0 to 0.8 per cent in excavation and from 0.1 to 1.3 per cent in embankment. The maximum equivalent vertical error after adjustment was +0.04 ft. There were no significant differences in accuracy, either before or after adjustment, between quantities from cross-sections taken from a photogrammetric contour map and those from spot heights read directly in the stereoplotter.

Both excavation and embankment quantities from the second field surveys were in error by 1.7 per cent. The equivalent vertical error of this survey was +0.11 ft., or almost three times as great as that of any of the adjusted photogrammetric surveys.

Data concerning costs of the various steps involved in obtaining earthwork quantities by several different methods are included and various methods of obtaining earth work quantities are discussed. Conclusions indicated by the study are presented in the last section.

The most important fact developed by the study is that adjusted earthwork quantities from all of the photogrammetric surveys were within limits generally considered tolerable for payment. They were more accurate than quantities obtained by a field survey made by commonly accepted methods.

The method of adjusting photogrammetric quantities by use of a centerline profile appears to have considerable potential value as a means of obtaining pay quantities with a minimum expenditure of manpower. Further tests are being made of adjustments on projects scheduled for early construction.

Relationship of Topographic Relief, Flight Height, and Minimum and Maximum Overlap

William T. Pryor, Chief of Aerial Surveys Branch, Bur. of Public Roads

The effects of topographic relief on overlap in aerial stereoscopic photography become acute when flight height must be sufficiently low for taking aerial photographs suitable for large-scale mapping by photogrammetric methods for highways. Although these same effects are present in small-scale photography used to compile small-scale maps, their consequences are not acute because the large flight height permits a proportionally greater relief height. For the double projection photogrammetric instruments commonly used, the ratio of relief height to flight height (h/H) varies from 0.36 to 0.21.

Principles governing the design of endlap (overlap in line of flight) and sidelap (overlap of one strip of photographs on another) are presented. Considerations that must be made when determining the minimum flight height that can be utilized according to the relief height existing in the area to be photographed and mapped at large scale with small contour interval are outlined, and their effects on the maximum map scale attainable are pointed out. Whenever large-scale mapping for highway surveys is to be undertaken by precise photogrammetric methods, the specific relationship between relief height in the area to be mapped and the photography flight height must be fully considered. Graphs are provided to serve as aids in ascertaining limiting conditions.

Errata

Paper by James B. Case in December 1958 issue. As prepared by the author Fig. 4(a) and Fig. 4(b) on page 819 were one figure. A separation was found necessary. The result is that the bar scale is not correct for the map. The author advises that $2\frac{1}{8}$ inches on Fig. 4(a) equals 2000 meters. The approximate scale for the map is 1:37,400.