is in quantitative form. It is important that definitive rather than ambiguous extraction techniques be incorporated. Some of the problems to which a contribution can be made are stated as questions, as follows: What procedures should be used to locate certain types of complexes from a mass of image data, within certain time limits, such as might be expected from an "open sky" program or a "sputnik" program? How little image information can I tolerate in order to identify certain complexes with certain probabilities? How can I increase information outputs to a significant degree without changing the information gathering system? How much must the ground resolution be changed to provide significantly greater information outputs? How much will I gain in information outputs by adding another specific type of sensing system? Will there be a significant difference in information outputs between two specific systems? In conclusion, consideration of fundamental extraction techniques assists in providing more definite results in the operations analysis of such problems.

*Determination of Sand Grain Sphericity by Stereo Photomicrography**

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An index property of particulate materials is their component shapes which may be described by analogy to geometric forms. Because sand grains tend to approach spheres in the course of time and wear, an expression for grain shapes is furnished by *sphericity*. It is the purpose of this paper to show how measurement of sand grain sphericity may be performed photogrammetrically.

Sphericity is a function of the relative lengths of particle intercepts and is measured according to the equation of Wadell (1932):

$$S = \sqrt[3]{\frac{V_p}{V_{cs}}}$$

where

S = sphericity;
V_p = particle volume;
V_{cs} = volume of circumscribing sphere.

A platy particle has a sphericity near zero whereas with a true sphere the value is unity. Values of sphericity are independent of the particular shape class into which the particle falls, thus particles with decidedly different appearances may have equal sphericity values. Also, sphericity is independent of *roundness* which measures the departure from angularity of outline. Examples A and B show different degrees of roundness but comparable values of sphericity.

Sphericity is not a significant index for water transported quartz particles smaller than 200 mesh (0.074 mm.). A film of water is omnipresent on these grains and protects them from attrition in transport; thus an angular and fresh appearance is preserved even after considerable transportation.

Data about the sphericity of sand deposits can be of service to the engineer and the geologist. A study of this sort can be helpful to the placer prospector for he can specify transporting agents and decide whether or not his specimens came from the traction load or the suspension load; this is often an important distinction. In addition, he can unravel age relations among stream terraces using the guide that, in general, the most nearly spherical grains have been the most travelled and are hence occupying the lowest (most recent) terraces. For the same reasons, sphericity studies can be of service to the glacial geologist or geomorphologist attempting to interpret the geomorphic history of an area. Geochemical prospecting can also be served by sphericity analyses, the results of studies helping to distinguish residual from transported soils and, as above, indicating the nature of the transportation. The petroleum geologist, interested in aquifer storage potential and permeability can make more accurate pro-

* This paper was submitted in the 1957 competition for the Bausch and Lomb Photogrammetric Award of the American Society of Photogrammetry.

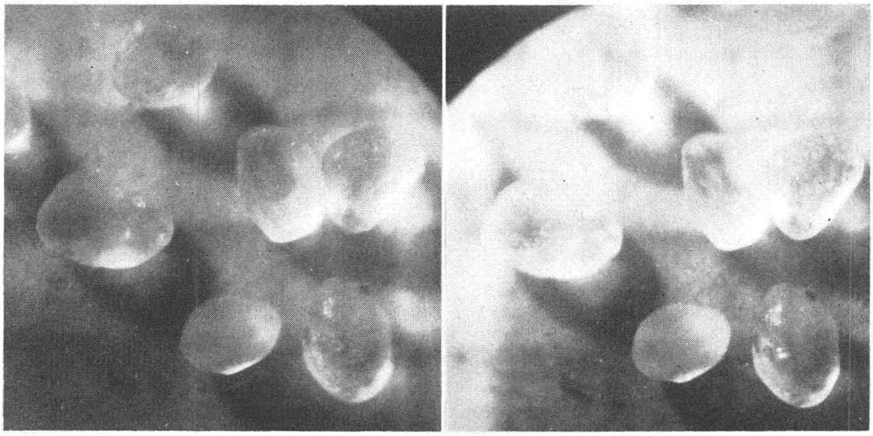

FIG. A. Beach sand; the glue used to tack down the grains interferes with view of the base grid. ×20.

jections into unknown areas if he can accurately predict variations in particle shapes; for example, if a sandstone formation has decreasing sphericity values in a westerly direction, the formation to the west might be expected to have lower porosity and fewer interconnected pores, thus lower storage capacity. Particle shape is an important consideration to the soils engineer in predicting values for maximum density, compressibility, and shearing strength, although there is not at present an established relationship between these properties and precise values of sphericity. The highway engineer might be able to utilize sphericity information in the future preparing specifications for mineral filler and asphalt sand and asphalt mix designs.

For all the uses mentioned above, research may take the form of evaluating the dependence of values of sphericity on such factors as grain size, mineralogy, transporting agent, or behavior during transportation.

For this study, sand grains were photographed under the microscope in "flight lines" to obtain overlapping stereo coverage. The sands selected were almost pure quartz and represent beach and alluvial types. The specimens were washed and sieved to the #20 to #40 U. S. sieve size (0.4 to 0.8 mm.), and were then quartered repeatedly to obtain the most minute possible representative sample. The grains were placed on a grid of 0.25 mm. spacing and photographed using reflected light through a microscope at twenty power. A piece of sheet metal of known thickness was photographed along with the grains to provide vertical control. The resulting stereo-photomicrographs were studied with an Abrams Height Finder and the cardinal dimensions obtained for each grain.

The value of sphericity is determined from the cardinal dimensions by computing the volume of the particle, dividing by the circumscribing sphere, and extracting the cube root. For computation of particle volume the shape is assumed to be ellipsoidal and the volume equals $\pi abc/6$. The volume of the circumscribing sphere utilizes the greatest dimension of the grain (a) and equals $\pi a^3/6$.

Sphericity values determined by the author for the grains shown in A and B ranged from 0.70 to 0.90. The results were reliable to two decimal places. However much greater accuracy is obtainable with better photography and a more even distribution of vertical control.

Problems encountered in the work involved splitting the sample, obtaining an accurate grid, and controlling movement of grains between photographs. Having a representative sample is supremely necessary; this involves painstaking quartering technique. It was necessary to have a base grid of the order of magnitude of the smallest specimens (0.3 mm.); this was accomplished by photographic reduction. In future work of this kind, however, diapositives would be made so as to use refracted rather than reflected illumination; the shadows interfered with accurate determination of grain boundaries, and the requirement of having reflected illumination from several different directions created an unwieldly array of lamps which complicated photographic technique. An additional problem was controlling the movement of particles between

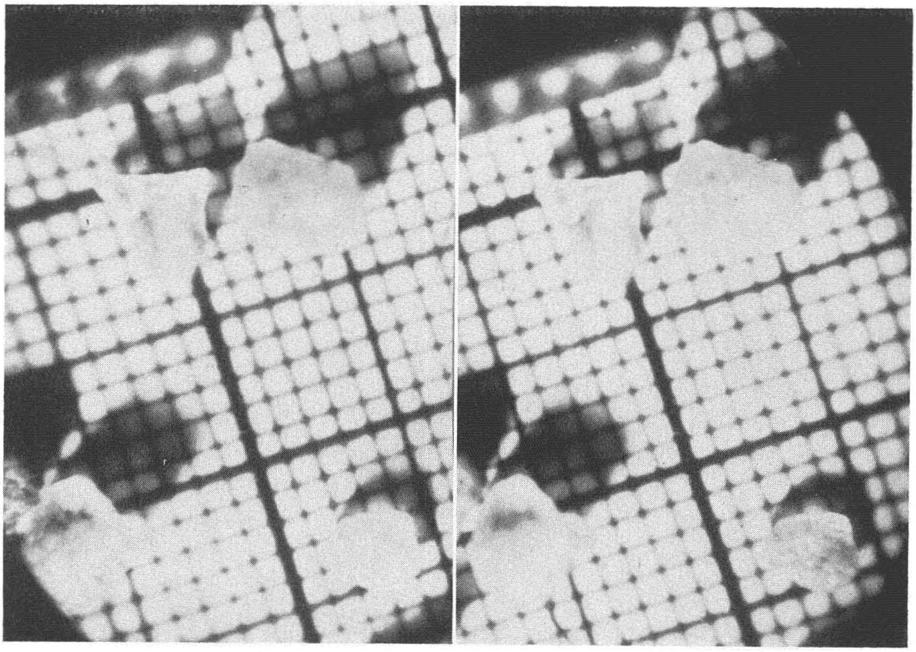

FIG. B. Alluvial sand; the base grid here is much smaller than in Figure A. (here 0.25 mm. spacing). Though much more angular, the sphericity of these grains approximates the average value in *A* of 0.79. The pictures are reversed to yield pseudoscopic vision; it was found that greater accuracy could be obtained this way. ×20.

photographs. In order to preserve the grain orientation both glue and static electricity were utilized; neither was completely satisfactory.

For the purpose of making statistically reliable studies of sphericity photogrammetrically, determinations should be made for a large number of grains. The method practiced by the author is clearly too involved and slow to be practical. However, it would not be difficult to mass produce stereophotomicrographs by streamlining the procedure. Most important in effecting greater efficiency would be a microscopic stereo camera with built-in base grid, vertical-control, and container for sand grains. A camera along these lines was developed and utilized by Aschenbrenner (1955).

Other methods of determining sphericity of small particles are: 1) obtaining the *c* dimension under the two dimensional microscope by focusing at high power on the top and then on the bottom of the grain; 2) estimating the *c* dimension with a binocular microscope; 3) estimating sphericity from two dimensional appearance. With larger particles, sphericity is determined by measuring the volume by displacement in a liquid. With small particles, all of these methods are inferior to the photogrammetric for purposes of accuracy and certainty.

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