

winds, a splicer and an action editor are important accessories for the selection and preparation of footage for projection. Very rarely is the entire 100 feet of a film run us-

able so it should be edited to save time in projection. This equipment is also necessary if films are to be prepared for class or meeting use.

## *A Note on Recent Developments in Analytical Aerial Triangulation*

D. W. G. ARTHUR,

*Photogram. Group, Ordnance Survey, Chessington, England*

RECENT literature on analytical aerial triangulation published in the U.S.A.\* shows some interesting trends which are deserving of comment since they involve details of technique which have yet to be clarified. The principal features of the new methods are (1) new observation techniques, some of which are linked to 3-table comparators, and (2) new computation methods with greater generality and a more rigorous approach to the handling of the data. Both of these methods seem to imply somewhat greater costs in the instrumentation, observations and computations. The question thus arises, for organizations which are cost-conscious, whether these innovations will be found worth while.

In these new methods considerable importance is attached to the identification of the same point on several photographs. It is widely realized, of course, that certain difficulties exist when more than two photographs are involved. These arise from the essential character of stereoscopic vision as a one-to-one correspondence process between *two* image arrays. If stereoscopic observers had as many eyes as are required to scan simultaneously all the photographs on which the point is registered, then the problem would not exist. However, the photogrammetrist must take things as they are,

and adapt his techniques to the limitations of binocular vision.

The classical method of stereo-observation achieves a solution within these limits by marking the point on one of the photographs involved. This is illustrated as follows. Suppose the point is imaged on photographs  $a, b, c \dots m \dots$  and is marked on photograph  $m$ . The point may be co-ordinated on  $m$  by simple monocular pointing not involving stereoscopy. It is co-ordinated on all the other photographs by pairing them in turn with the marked photograph and by using stereoscopic transfer. Thus the pairs  $ma, mb, mc \dots$  are observed for this point, but such pairs as  $ab, ac, bc$ , are not, and in fact these cannot be observed. In strip triangulation a pass point marked on photograph 2 is observed in the pairs 12 and 23, but not in the pair 13. If, in addition, this point is a tie registered on the photographs of the adjacent strip then photograph 2 is paired with these in turn so that the point is co-ordinated on the photographs of the other strip. The classical solution thus appears to be adequate, and can be achieved with the normal 2-table comparator.

The 3-table instrument makes possible observing the alternate pair 13. Schmid has suggested that this is advantageous. Putting aside his suggestion for the moment, there will be considered the problems of using the pairings 12, 23 and 13, for a pass-point registered on the consecutive photographs 1, 2, 3. The point may be marked on 2, or its position may merely be indicated by the half-mark of the telescope system associated with photograph 2. First, in the pair 12, the parallaxes may be removed by movements of photograph 1. Then switch-

\* (a) Schmid, H. Panel Discussion, "The Future of Analytical Aerial Triangulation." PHOTOGRAMMETRIC ENGINEERING, XXIV, No. 1, p. 91, March 1958.

(b) Zurlinden, R. "Conditions to be fulfilled by a Rational System of Analytical Aerial Triangulation." PHOTOGRAMMETRIC ENGINEERING, XXIII, No. 4, p. 659, September 1957.

ing to the pair 23, the parallaxes at the same point are eliminated by movements of photograph 3, photograph 2 being undisturbed in this second setting. The observer now switches to pair 13 and hopes to find that the images are still in correspondence. If his first two settings were errorless, then the setting 13 will be parallax-free, and there will be no point in making this third setting. If, however, he finds slight parallaxes in 13—and this is far more likely—what does he do about them? He can remove them by adjustments of photographs 1 and 3, but in doing this he discards his first two settings. No details have been published yet concerning this type of observation, and the author does not desire to speculate further on the actual observation procedure, but it seems to him that certain problems are present which have yet to be discussed.<sup>1</sup>

Returning to Schmid's suggestion that the observation of pass-point images on alternate photographs has advantages because of the doubling of the base, the author remarks that it is possible to have too much base. He has noticed with wide-angle (90°) photography that observations of alternate photographs are often accompanied by much visual discomfort, particularly when the terrain is mountainous. The shadows of trees and other asperities are so dissimilar that frequently there is some difficulty in obtaining fusion. Therefore, in general the author would say that the facility to switch to alternate pairs is not likely to become an important feature in 3-table instruments.

Has the 3-table instrument any advantage over the older form of comparator? The author thinks it has, but only in relation to certain analytical approaches to the problem of forming the strip or block model. It is well known that the surface of the stereoscopic surface is somewhat woolly or indefinite; accordingly the floating mark may be set to different depths in a layer. Now the 3-table comparator, with its facility for switching rapidly from one pair to the next, will be helpful, as it will make it easier for the observer to set the mark at the same depth in the layer of indefiniteness. This feature is important, but only for the newer methods of computation in which the

orientation errors are sensitive to the  $x$ -errors of the measurements.

Schmid\*(a) suggests a stereoscopic technique in which the measurement is monocular and is divorced from the stereoscopic transfer. In this case the function of the transfer instrument is to place a measurable mark on the photograph and this is subsequently measured monocularly. The author cannot see any advantage in this approach. It seems impossible that the marking operation can sustain the accuracy of the optical pointing which precedes it. In fact the whole idea boils down to the incorporation of an unnecessary stage in the observation process.

Indeed the required result of simultaneously co-ordinating a point on all photographs on which it appears does not seem feasible, unless there be accepted the limitations of the classical method in which one photograph assumes unique importance for the point concerned.<sup>2</sup> This is not to say that the problem cannot be solved by a non-stereoscopic method, for example, an application of the blink principle. Transfer-marking techniques and 3-table stereocomparators do not appear to be very helpful for the general problem.

Turning now to the computation techniques, the principal feature of the newer methods is the incorporation of what may be called the coincidence conditions. These are additional to the correspondence conditions and their employment alters the character of error propagation in the strip, as already noted by Schut. The coincidence condition states that the perspective rays passing through the adjusted image positions are to intersect in a single model point. An asymmetric application of this principle, as in Bartorelli's method, in which the added photograph is sited so that its rays intersect pass-points already fixed, cannot lead to good results. This has already been noted by other writers. The correct application of the coincidence conditions involves the simultaneous treatment of the strip as a whole. This contrasts sharply with the older methods depending solely on the correspondence principle, in which each model may be computed separately. The introduction of the coincidence conditions thus has a very adverse effect on the economics of the computations, and implies calcula-

<sup>1</sup> R. Zurlinden's criticisms of the classical instrument in the opening paragraph of (2) are much too strong, and the author does not entirely accept them. He implies an assumption in the classical method which is certainly not present in the method as now known by the author.

<sup>2</sup> The observation on this photograph will have a greater precision than those on the other photographs.

tions which may fairly be described as massive.

Turning to the question of error-propagation it is evident that problems exist here which require specific research. In the computation methods which use only the correspondence condition, the propagation of errors is tied almost entirely to the  $y$ -errors of the measurements. That is, the propagated errors of orientation are almost completely insulated from the  $x$ -errors. The same is largely true of the scale errors if several points are used for each scale transfer. This last remark, by the way, explains why the 3-table instruments have no strong advantages for the older methods of computation. The newer methods of strip formation employing the coincidence principle are certainly vulnerable to the  $x$ -errors of the measures, for the relative orientations are directly affected by these. On the face

of it, the new techniques with their additional conditions should give the better answer, but since new errors are introduced, the author would like to see this proved.

For most survey organizations, aerotriangulation is a substitute for the more expensive ground survey, so that almost all aerotriangulation processes are partly governed by economic considerations. Any innovations implying higher costs or greater complexity therefore require careful examination before they are brought into use by such organizations. If the author appears to be unduly critical of the recent developments, it is because of his experience with aerotriangulations in which costs had always to be considered along with accuracy. The views expressed here are those of the author, and are not necessarily those of the Ordnance Survey.

## *Comments on Stereographs*

G. S. DRUHOT.

*U. S. Geological Survey, Menlo Park, Calif.*

THE article, "An Application of Models and Stereo Images to Teaching Photographic Geometry," by Raymond M. Nelson interested me.\* One can see that stereophotos in color of these models would be very effective in teaching photographic geometry to students not accustomed to visualizing three dimensions while looking at a drawing which, perforce, is limited to two dimensions.

But why limit the idea to photographic geometry? I have often wondered why the publishers of our text books on solid geometry and related subjects have never seen fit to utilize the stereograph to illustrate the more complex figures; and I almost have been moved, at times, to propose the idea. As a matter of fact, I have gone so far as to prepare stereoscopic drawings of an ellipsoid for my own use. I have hesitated to report this, because the idea has, I believe, been in use for many years, particularly, by European photogrammetrists; but Mr. Nel-

son's article provides a good opportunity to mention my experiment in a comment.

I learned that stereographs are easily constructed; particularly, if the lines and planes involved are not curved. Fair drafting ability, a slight acquaintance with descriptive geometry, and a ratio to use in computing the separation of conjugate images—these, along with common drafting instruments and materials, are all that is needed.

For my experiment, I used a ratio of 2.6 to 14. Why that particular ratio? Well, I decided that 2.6 was about the normal pupillary distance, and that I would use that distance for the separation of points appearing on the plane of the paper. Then I took 14 inches as a good distance for normal vision and assumed that rays from a pair of points in the picture plane would merge at that distance. It was then a simple matter to construct the two converging lines and actually measure the separation at varying distances from the picture plane. However, it is preferable to reverse the figure so that the picture plane is at the vertex. Then the separation of the lines is applied as a correction to

\* PHOTOGRAMMETRIC ENGINEERING, Vol. XXIV, No. 3, June, 1958.