*The Problem of Exterior Orientation in Photogrammetry**†

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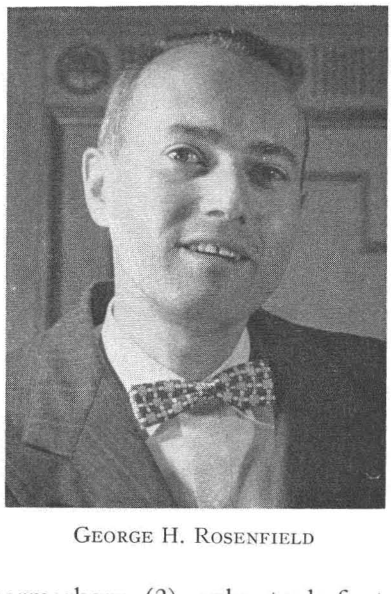
ABSTRACT: *Increased interest in Analytical Photogrammetry has focused attention upon a chaotic situation which complicates research and communication between photogrammetrists. The designation of the orientation parameters in a different manner by almost every investigator has resulted in the need for a universal non-ambiguous system which depicts the actual physical orientation of the photograph. This paper presents a discussion of the basic concepts involved in the various systems in use, and recommends the adoption of a standard system within the basic framework of the Stockholm resolution of July 1956, concerning the sign convention to be used in Photogrammetry.*

I. INTRODUCTION

A CHAOTIC situation exists within the science of Photogrammetry which complicates research and communication between photogrammetrists. When the theoretician studies a technical paper, he finds the work unnecessarily laborious until he has first become familiar with the coordinate system, definitions, and rotations employed by the writer. When the skilled instrument operator sits down to a new stereoplotting instrument he finds the orientation procedure unnecessarily difficult until he becomes familiar with the positive direction of the translational and rotational motions. Thus it is that almost every individual has devised his own unique system of designating the exterior orientation of the photograph.



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In order to alleviate these complications, the VIII International Congress for Photogrammetry at Stockholm in July 1956, accepted a resolution on the sign convention to be used in photogrammetry. The resolution asked that instrument manufacturers, authors of papers, and teachers, adopt *a right-handed rectangular coordinate system with its origin at the left-hand air station and Z-axis downward. Further, the signs of the base components are to be the same as those of the coordinates of the right-hand air station in the above system.* (1)

Background for this resolution regresses to Schermerhorn (2), who took first formal note of the chaos existing in the signs on spatial plotting instruments. He then requested Jerie (3) to make an investigation into the various systems in use by the instrument manufacturers to designate the rotational and translational elements of exterior orientation. Jerie also tabulated and listed the systems used for the *y*-parallax equations and for the formulae of numerical orientation.

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Past application of photogrammetry to strictly stereoscopic restitution methods was not concerned with the choice of the positive directions of the base components and manufacturers could choose them in any manner which best suited their particular instrument. The advent of the electronic computer has taken some of the emphasis away from purely instrumental methods, and interest in numerical (or analytical) methods has increased. The choice of the positive directions becomes more important in the analytical approach; and as a result, uniformity of designation would be advantageous. The spatial system to be used to designate the coordinates of models, the camera movements, and the camera rotations, is a matter for agreement. Such was the problem as presented by Schermerhorn. He then suggested that since many countries designate their geodetic coordinates in a right-turning system that it is obvious that a right-turning system should be applied to the photogrammetric coordinate systems.

The resolution adopted at Stockholm was based upon the recommendations of Schermerhorn. The right-handed system adopted has the Z-axis downwards and the

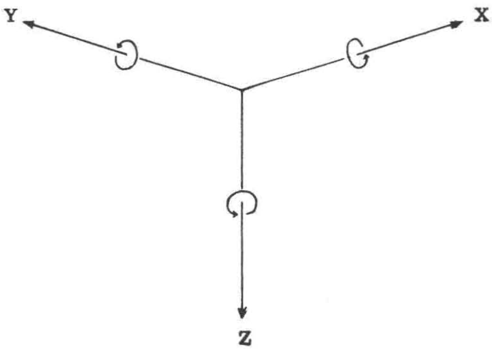


FIG. 1.

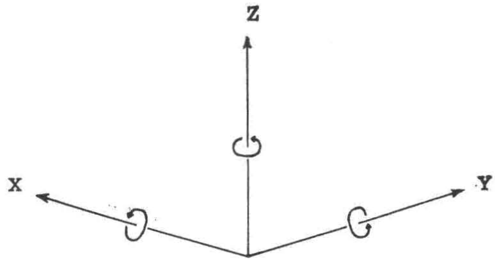


FIG. 2.

X-axis to the right; the position of the Y-axis is then predetermined. The Stockholm system is equivalent to the geodetic coordinate system of many European countries: X North, Y East, and Z downward.

The determination of the positive direction of the rotations is such that if we face along the positive direction of any axis (e.g. the X-axis) and turn through a right angle clockwise about this axis, a line originally in the positive direction of the axis and coming next in the cyclic order of the letters (e.g. the Y-axis) will be carried into coincidence with the remaining axis (e.g. the Z-axis). Thus a rotation is to be taken as positive when it is clockwise about the positive direction of the respective axes (1). The arrows in Figure 1 indicate the positive directions of the axes and rotations.

In the United States the geodetic coordinate system upon which surveying and mapping is based regards the Z-axis as upward, the X-axis to the East, and the Y-axis to the North. This is also a right-handed system. Again in this country, the survey and navigation angles are measured clockwise for the positive direction. Thus the basis for the recommendation of Schermerhorn and of the Stockholm resolution can be adopted in this country by assuming a photogrammetric system based upon our geodetic system. Figure 2 illustrates the positive directions of the axes and rotations of such a system.

It is readily seen that the two systems are the same in abstract space and that the formula for any rotation about a given axis in either system would therefore be the same. These rotation formulae can be readily expressed in matrix form. Thus the matrix for an independent rotation through the angle θ about the X-axis is:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \tag{1.1}$$

about the Y-axis is:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \tag{1.2}$$

and about the Z-axis is:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \tag{1.3}$$

in which the non-primed axes represent the original position of the axes and the primed axes represent the rotated (or new) position of the axes.

If the non-primed axes represent the survey system, and the primed axes represent the camera system, then the individual rotation matrices are in the form given by equations (1.1), (1.2), and (1.3).

If, on the other hand, the non-primed axes represent the camera system, and the primed axes represent the survey system, then the individual rotation matrices take the inverse form. Since for orthogonal axes the inverse of the rotation matrix is equal to its transpose, then the matrix for the independent rotation through the angle θ about the X'-axis is:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}, \tag{1.4}$$

about the Y'-axis is:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}, \tag{1.5}$$

and about the Z'-axis is:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}. \tag{1.6}$$

The Stockholm resolution does not specify which set of axes (the non-primed or the primed) is to represent the survey system and which set is to represent the camera system. Therefore, the decision for which to use appears to be a matter for choice.

There is another item of practicality which must be considered, but which is not specified by the Stockholm resolution. Consider the ordered rotation of a rectangular coordinate system about its respective axes. A primary rotation is given to the system about the chosen primary axis; the other two axes now take on auxiliary positions. The secondary rotation can now no longer take place about the original position of the secondary axis (since it has been rotated) but must take place about the secondary auxiliary axis or about the secondary axis as rotated through the primary angle. The tertiary rotation must then take place about the third axis as rotated through the primary and secondary angles.

In illustration of the above remarks, let us consider first a rotation about the X-axis:

$$\begin{pmatrix} X \\ Y' \\ Z' \end{pmatrix} = R_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \tag{1.7}$$

then about the new position of the Y axis, or about Y':

$$\begin{pmatrix} X' \\ Y' \\ Z'' \end{pmatrix} = R_2 \begin{pmatrix} X \\ Y' \\ Z' \end{pmatrix}, \tag{1.8}$$

and lastly about the new position of the Z axis, or about Z'':

$$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix} = R_3 \begin{pmatrix} X' \\ Y' \\ Z'' \end{pmatrix}, \tag{1.9}$$

in which the non-primed axes represent only the original position of the axes, and the various primed axes represent the auxiliary and final positions of the axes. The R quantities represent the individual rotation matrices. This particular development is independent of which is the survey and which the camera system.

In line with the above approach, the relationship of the X'', Y'', Z'' axes can be expressed in terms of the X, Y, Z axes by sequential substitution of the respective auxiliary expressions. The form for this relationship is:

$$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix} = R_3 R_2 R_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{1.10}$$

The inverse relationship can also be developed, that of expressing the X, Y, Z axes in terms of the X'', Y'', Z'' axes by inversion of each of the separate rotation matrices which will then necessitate sequential substitution in reverse order. The form for this relationship is:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_1^T R_2^T R_3^T \begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix}, \tag{1.11}$$

where the superscript T is used to denote transposition.

[The inversion of the expression (1.10) to form (1.11) follows the theorems of matrix algebra: 1) The product of two or more orthogonal matrices is an orthogonal matrix; thus the product of the three orthogonal rotation matrices R_1, R_2, R_3, is an orthogonal matrix. 2) If a matrix is orthogonal, the product of itself by its transpose is equal to the unit matrix, and the inverse of the matrix is equal to its transpose; thus if A is an orthogonal matrix, $AA^T = I$ and $A^{-1} = A^T$. 3) If a matrix is transposed the order of the matrices forming the product must be reversed and each individual matrix forming the product must be transposed; thus if $R_3R_2R_1 = A$, then $A^T = R_1^T R_2^T R_3^T$.]

II. The Orientation Matrix

The orientation of one rectangular coordinate system with respect to another can be considered as the ordered arrangement of the direction cosines between the axes of one system with each of the axes of the other. For photogrammetric operations let the image-space coordinate system represent the one system (the photographic system) with the axes denoted by x, y, z; while the object-space coordinate system represents the other, denoted by X, Y, Z. It is not necessary that the object-space coordinate system represent the primary geodetic survey system; it can represent some local survey system, or even a purely photogrammetric system wherein the X-axis represents only the approximate direction of the flight line.

The ordered arrangement can then be expressed in matrix form as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \tag{2.1}$$

in which A, B, C, $\cdots$ F, represent the respective direction cosines between the axes of the two systems. A, B, C denote the direction cosines between the image-space x-axis and the object-space X, Y, Z axes respectively; similarly A', B', C' denote those between the image-space y-axis and the object-space X, Y, Z axes respectively; and D, E, F denote those between the image-space z-axis and the object-space X, Y, Z axes respectively.

This relationship can also be expressed in its inverted form. Since the axes being considered are orthogonal, the inverse of the orientation matrix is equal to its transpose, and:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} A & A' & D \\ B & B' & E \\ C & C' & F \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \tag{2.2}$$

The general form for this type matrix is well known. One variation of it appears in the Manual of Photogrammetry (4) on page 368. The problem arises when one attempts to designate the trigonometric expressions for the direction cosines representing a particular camera orientation. Since the angles between the axes cannot be directly measured, their direction cosines must be computed from the known measurable parameters of orientation. It is the designation of these parameters of orientation together with the elements of the orientation matrix which we are presenting for standardization.

The elements of the matrix referred to in the Manual of Photogrammetry is that developed by Professor Earl Church of Syracuse University and is well known to the American photogrammetrist (5). The Church system is based on the so-called angles of tilt, swing, and azimuth of the principal plane, and are related to the Eulerian Angles of mathematics (6). The major disadvantage of this system is that it breaks down for the ideal aerial photograph, that of zero tilt. In this case the principal plane disappears and the angles of swing and azimuth become undefined. Since most analytical methods in photogrammetry are based on iterative methods using initial approximations, the Church system is found lacking because the obvious approximation, the truly vertical photograph, cannot be used.

[The relationship between the Church and Eulerian Angles is given in Appendix A to this paper. The appendix also presents a development of the Church orientation matrix by the method of matrix operators. It is presented with this paper in order to make the development readily available to the photogrammetric public. This derivation is shorter and easier to present than the development in the original Church bulletin, and it is also believed to be easier to follow and to understand. It is suggested that reading of Appendix A be postponed until after reading of the main text.]

There is another basic concept for designating the angles of rotation in photogrammetry; this is the Von Gruber (7) concept of rotation about each of the axes of the coordinate system wherein the three rotations take on a primary, secondary, and tertiary order. These angles are usually designated as omega, phi, kappa. This is the basic system adopted by the instrument manufacturers due to its adaptability to the mechanical motions and adjustment screws of the stereoscopic plotting instruments. This system readily lends itself to analytical computations since the initial approximation can be taken as the ideal photograph with all three angles as zero. We have already commented on the chaos existing with the use of these designations. There are almost as many different definitions for the angles and elements for the orientation matrix as there are instrument manufacturers and the theoreticians.

It must be pointed out that there exists, for whatever system of designation is used, a particular rotation which when carried through a particular angle causes the other two rotations to become indeterminate and thus make undefinable the absolute orientation of the photograph. Thus, the Church system becomes undefined for a truly vertical photograph, where the tilt angle is zero. So it is with any of the variations of the Von Gruber system. If the system is established for a vertical photograph it becomes undefined when tilted through 90 degrees to become a horizontal photograph. The condition also applies to the reverse situation. It therefore becomes necessary to select the particular angular designations which best fit the particular orientation of the photography (vertical or horizontal) and to define the angles separately for each particular system.

Let us accept the basic concept of the Stockholm resolution: that we adopt a right-handed image-space coordinate system following from the right-handed geodetic coordinate system, and also a right-handed object-space coordinate system; and that we take the positive direction of the rotations as clockwise about the positive direction of the axes. We cannot adopt their resolution in toto since the American geodetic system is different from that European system which was the basis for their resolution. We shall therefore take the positive Z-axis upward.

Again, since the ordered rotation about the three axes must proceed in sequence, it is not possible to perform each rotation independently about the original position of its respective axis, but the rotations must be presented utilizing the concept of secondary and tertiary auxiliary axes.

Thus, in order to develop the elements of the orientation matrix, it is not sufficient to define only the positive direction of the axes and rotations; it is also necessary to specify the order of the rotations about the respective axes as primary, secondary, and tertiary. Here we are given a choice, even so far as the Stockholm resolution is concerned. It has been customary to consider the primary rotation as about either the X- or the Y-axis, with the secondary rotation about the alternate axis. The tertiary rotation has always been considered as about the Z-axis.

When once the coordinate system and positive direction of the rotations have been defined, and the ordering of the rotations decided upon, the development of the elements of the orientation matrix becomes unique and indisputable.

1. AERIAL PHOTOGRAMMETRY

In considering the development of the orientation matrix for aerial photogrammetry, the photograph should be considered as a diapositive* with the x- and y-axes

* Diapositive is here defined per the MANUAL OF PHOTOGRAMMETRY: A positive photographic print on a transparent medium.

as perpendicular lines on the photograph defined by the images of the fiducial marks in the camera shown on the photograph. The positive x-axis is that which most nearly coincides with the direction of the flightline. The y- and z-axes are chosen to make a right-handed coordinate system as described above. Thus the image-space x, y, z coordinates of the nodal point are 0, 0, $-f$ (where f is the focal length of the lens). The origin of the object-space coordinate system must then be considered as translated through space to coincide with the principal point of the photograph.

The diapositive is given primary consideration in this treatment since the aerial photogrammetrist works mostly with either a positive print or a glass plate diapositive. The film-base aerial negative is rarely used for photogrammetric measurement. The development for use with the aerial negative is given following that for the diapositive.

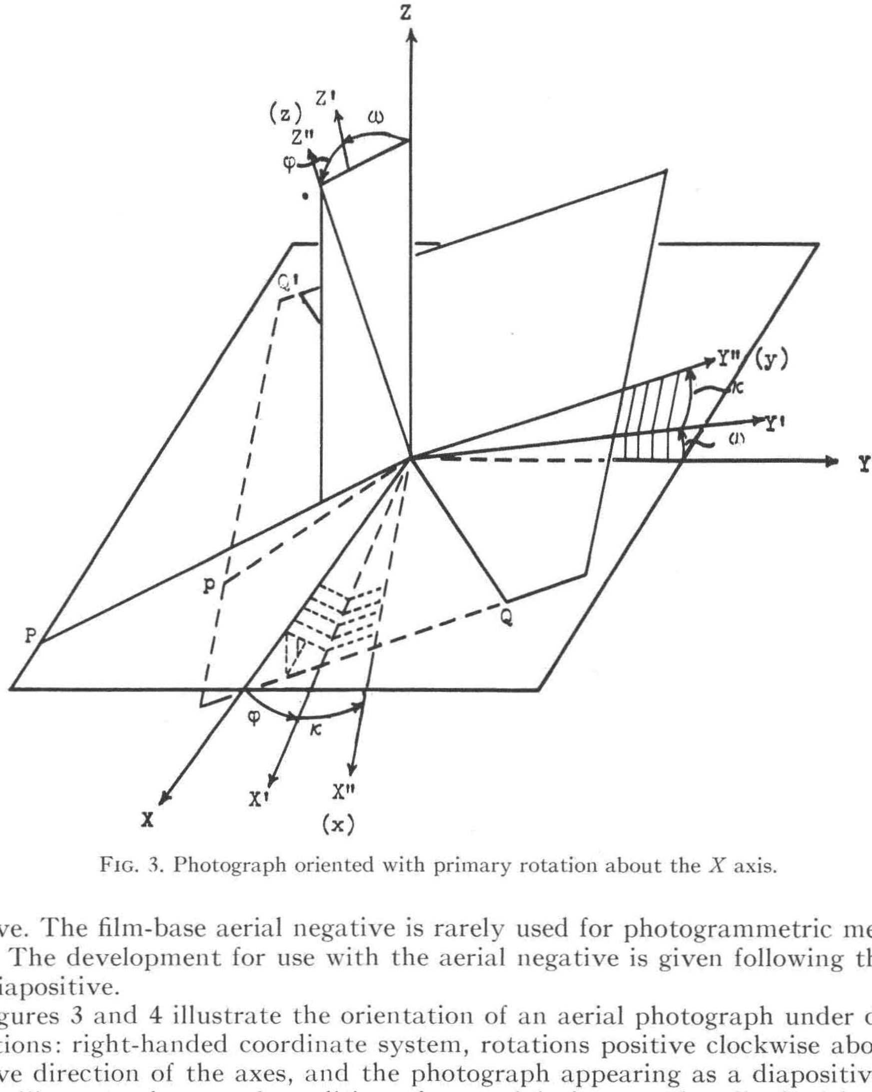


FIG. 3. Photograph oriented with primary rotation about the X axis.

Figures 3 and 4 illustrate the orientation of an aerial photograph under defined conditions: right-handed coordinate system, rotations positive clockwise about the positive direction of the axes, and the photograph appearing as a diapositive. The figures illustrate the actual condition of an aerial photograph—tilted with respect to the object-space coordinate system.

The positive rotation about the X-axis corresponds to the position of the aircraft in which the left wing is raised. The positive rotation about the Y-axis corresponds to the nose down position. Figure 3 illustrates the condition for primary rotation

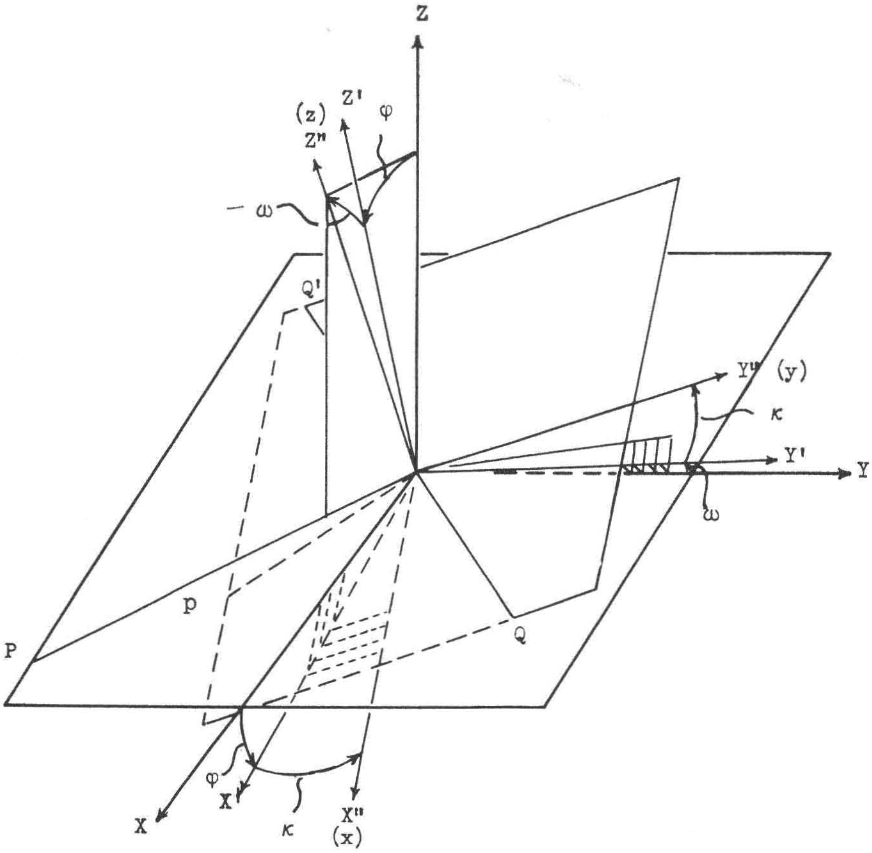


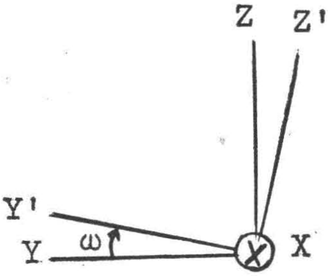
FIG. 4. Photograph oriented with primary rotation about the Y axis.

about the X-axis, while Figure 4 represents the condition for primary rotation about the Y-axis. The following items in the figures are identified:

- X, Y, Z: the object-space coordinate axes
- X', Y', Z': a system of auxiliary axes
- X'', Y'', Z'': the image-space coordinate axes (x, y, z)
- QQ': the line of intersection between the plane of the photograph and the XY-plane
- P: an axis in the XY-plane perpendicular to QQ'
- p: an axis in the xy-plane perpendicular to QQ'
- ω: the omega rotation angle
- ϕ: the phi rotation angle
- κ: the kappa rotation angle

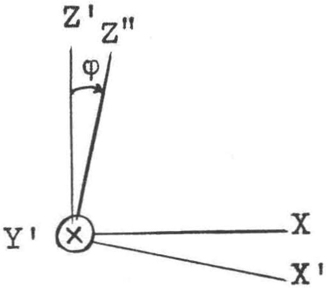
From the illustrations (Figures 3 and 4) the individual rotation diagrams and matrices can be developed. The final orientation matrix can then be produced from these individual sub-matrices.

Let us first consider the condition of primary rotation about the X-axis. The diagram and matrix for the primary rotation through the angle omega about the X-axis is:



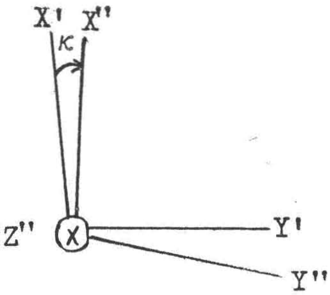
$$\begin{pmatrix} X \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \qquad (2.3)$$

for the secondary rotation through the angle phi about the new position of the Y-axis is:



$$\begin{pmatrix} X' \\ Y' \\ Z'' \end{pmatrix} = \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} X \\ Y' \\ Z' \end{pmatrix}, \qquad (2.4)$$

and for the tertiary rotation through the angle kappa about the new position of the Z-axis is:



$$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix} = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z'' \end{pmatrix}. \qquad (2.5$$

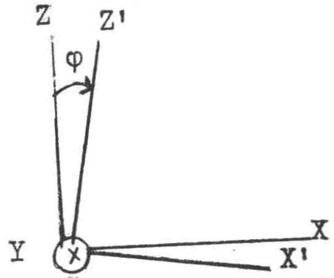
Sequential substitution into the above expressions and substitution of x, y, z for X'', Y'', Z'' leads to:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \qquad (2.6)$$

and matrix multiplication of the elements in expression (2.6) gives the final orientation matrix:

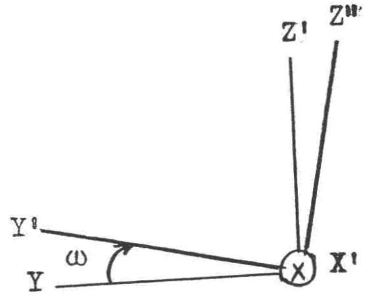
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \qquad (2.7)$$

Now let us consider the other condition of primary rotation about the Y-axis. The diagram and matrix for a primary rotation through the angle phi about the Y-axis is:



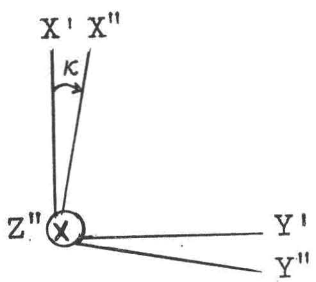
$$\begin{pmatrix} X' \\ Y \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \qquad (2.8)$$

for the secondary rotation through the angle omega about the new position of the X-axis is:



$$\begin{pmatrix} X' \\ Y' \\ Z'' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} X' \\ Y \\ Z' \end{pmatrix}, \qquad (2.9$$

and for the tertiary rotation through the angle kappa about the new position of the Z-axis is:



$$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix} \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z'' \end{pmatrix}. \qquad (2.10)$$

Sequential substitution into the above expression and substitution of x, y, z for X'', Y'', Z'' leads to:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \qquad (2.11)$$

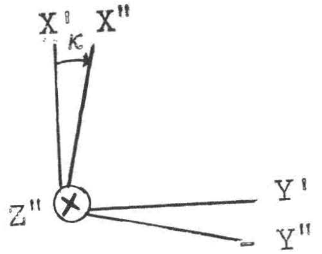
and matrix multiplication of the elements in expression (2.11) gives the final orientation matrix:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\phi\cos\kappa + \sin\phi\sin\omega\sin\kappa & \cos\omega\sin\kappa & -\sin\phi\cos\kappa + \cos\phi\sin\omega\sin\kappa \\ -\cos\phi\sin\kappa + \sin\phi\sin\omega\cos\kappa & \cos\omega\cos\kappa & \sin\phi\sin\kappa + \cos\phi\sin\omega\cos\kappa \\ \sin\phi\cos\omega & -\sin\omega & \cos\phi\cos\omega \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \qquad (2.12)$$

It should now be expressly stated that for a photograph of a given orientation, while the numerical values for the respective elements of each orientation matrix (equations 2.7 and 2.12) would be the same, the magnitude of the omega and phi angles in each case would be different.

In certain instances where the measurements are to be made on the emulsion side of the original negative (as with glass plate negatives) the positive direction of the y-axis should be reversed. The x-axis still represents the axis which most nearly coincides with the flight line, considering that the negative is rotated 180 degrees from its original position in the camera. The image-space x, y, z coordinates of the nodal point for this case are 0, 0, f.

Referring back to expressions (2.5) and (2.10) the kappa rotation diagram and matrix would then appear as:



$$\begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ \sin\kappa & -\cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z'' \end{bmatrix}. \qquad (2.13)$$

The orientation matrix for primary rotation about the X-axis would then be:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ \cos\phi\sin\kappa & -\cos\omega\cos\kappa + \sin\omega\sin\phi\sin\kappa & -\sin\omega\cos\kappa - \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \qquad (2,14)$$

and the orientation matrix for primary rotation about the Y-axis would then be:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi\cos\kappa + \sin\phi\sin\omega\sin\kappa & \cos\omega\sin\kappa & -\sin\phi\cos\kappa + \cos\phi\sin\omega\sin\kappa \\ \cos\phi\sin\kappa - \sin\phi\sin\omega\cos\kappa & -\cos\omega\cos\kappa & -\sin\phi\sin\kappa - \cos\phi\sin\omega\cos\kappa \\ \sin\phi\cos\omega & -\sin\omega & \cos\phi\cos\omega \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}. \qquad (2.15)$$

The above matrix expressions have been developed for the condition wherein the image-space coordinate system is expressed in terms of the object-space coordinate system. If the inverse case is desired, that of the object-space coordinate system in terms of the image-space coordinate system, then the final orientation matrix must be inverted. The matrix can be inverted directly by transposition of its elements since for an orthogonal matrix the inverse is equal to its transpose. The inverse form can also be developed by inversion of each of the individual rotational sub-matrices which would necessitate inverting the order of matrix multiplication (in order to meet the substitution requirements). The Stockholm resolution does not specify which ordering is to be followed.

Almost all photogrammetric methods developed in the past have been based on the latter system of relationship, that of expressing the object-space coordinate system in terms of the image-space coordinate system. It is only since the recent work of Schmid (8) that the former system, that of expressing the image-space coordinate system in terms of the object-space coordinate system, has been utilized. The use of this former method simplifies least squares adjustment and analytical computation since the observation measurements are separated and handled independently.

2. TERRESTRIAL PHOTOGRAMMETRY

Most modern day terrestrial photogrammetry is concerned with high precision measurements, and as a result much of the basic data is obtained from the emulsion side of the negative itself. In many instances the precision cameras use glass plate negatives. In this situation the z-axis is taken as the optical axis of the lens with positive direction from the plane of the negative to the lens. The y-axis is designated by the fiducial mark which most nearly represents the vertical position, considering that the negative is rotated 180° from its original position in the camera. The x-axis is chosen to complete the right-handed coordinate system. The image-space x, y, z coordinates of the nodal point for the terrestrial negative are 0, 0, f. The object-space coordinate system is considered as before, a right-handed system with the Z-axis upward. Thus we are in keeping with the Stockholm resolution concerning right-handed coordinate systems.

It is now necessary to modify slightly the Stockholm resolution concerning the positive direction of the rotations. Terrestrial photogrammetry is analogous to geodetic surveying with respect to the direction of the optical axis, which can be compared to the line of sight in surveying. Since the convention for measuring terrestrial angles has been long established, it is not desirable to change at the present time. Therefore, the elements of orientation for the terrestrial camera can be expressed as the angles of azimuth, elevation and roll. Azimuth is the horizontal angle measured positive clockwise from North; elevation is the vertical angle measured positive up from the horizontal; and roll is the rotation in the negative plane about the optical axis. It is to be noted that both elevation and roll are in keeping with the Stockholm resolution concerning the direction of positive rotation. The primary rotation is azimuth, counterclockwise about the positive direction of the Z-axis. The secondary rotation is elevation, clockwise about the new position of the X-axis; the tertiary rotation is roll, clockwise about the new position of the Y-axis. Thus to illustrate the orientation of the terrestrial photograph, the origin of the object-space coordinate system must be considered as translated through space to coincide with the principal point of the negative.

[At first it would appear that the above ordering of the rotations violates the ordering as established by the Stockholm resolution. This is not the case, provided that the following is considered. Early in photogrammetry the concept was established that the survey axes must be expressed to coincide with, or be related to, the camera axes. This concept holds readily for aerial applications as evidenced by the previous section of this paper. The concept can be applied to terrestrial photogrammetry if the Y and Z axes are considered interchanged. Then the Y-survey axis will coincide with the y-camera axis, and Z-survey axis with the z-camera axis. The analogy can be made to a universal stereo plotting instrument wherein it is necessary to interchange the Y and Z axes to convert from aerial to terrestrial photography. The ordering of the rotations is then as follows: primary rotation about the Y-axis (corresponds to primary phi of the aerial application), secondary rotation about the new position of the X-axis, and tertiary rotation about the new position of the Z-axis.]

[One problem which must be recognized with the terrestrial system of orientation: if the photograph is tilted through the angle omega = 90° (to a zenith position) then the orientation angles take on the same relationship to the Eulerian angles as do the Church angles, and the angles of azimuth and roll become undefined. This condition rarely exists in normal terrestrial photogrammetry. The relationships for the zenith photograph are simple and well known.]

Figure 5 illustrates the orientation of the terrestrial negative under the conditions of a right-handed coordinate system, with the angles measured as defined above.

The following items in the figure are identified:

X, Y, Z: the object-space coordinate axes
X', Y', Z': a system of auxiliary axes
X'', Y'', Z'': the image-space coordinate axes (x, y, z)

QQ': the line of intersection between the plane of the photograph and the XY plane
α: the azimuth rotation angle
ω: the elevation rotation angle
κ: the roll rotation angle

From the illustration (Figure 5) the individual rotation diagrams and matrices can be developed. The final orientation matrix can then be produced from these indi-

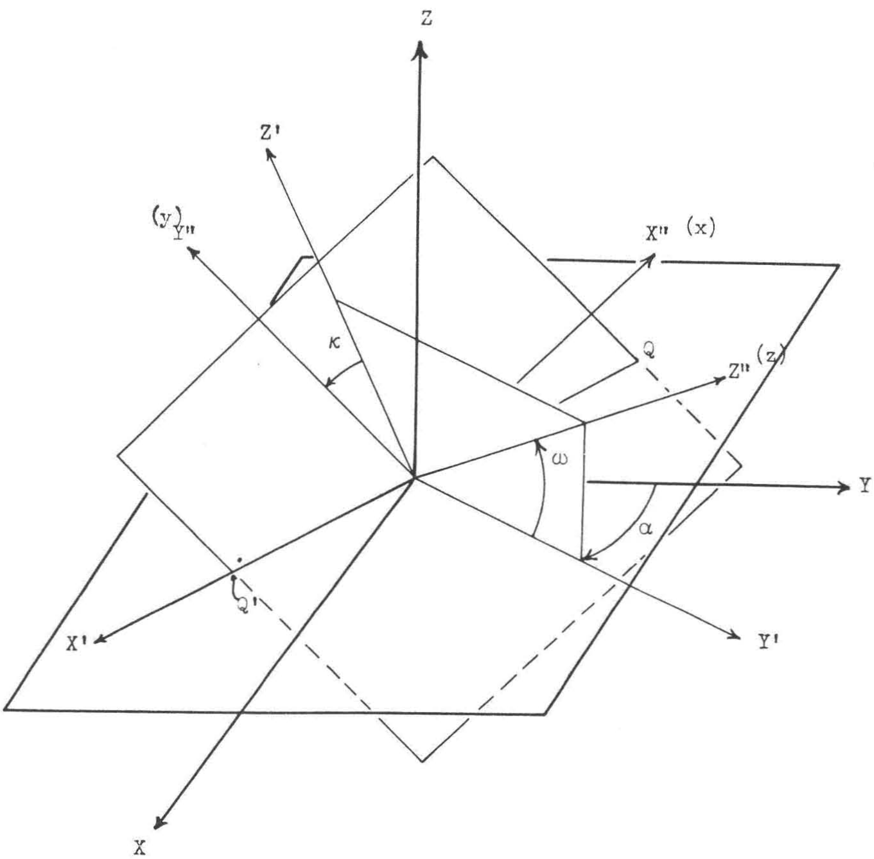
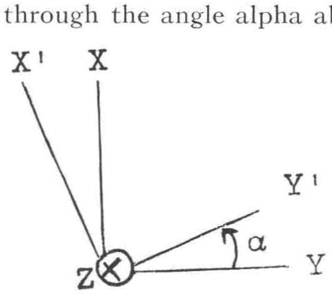


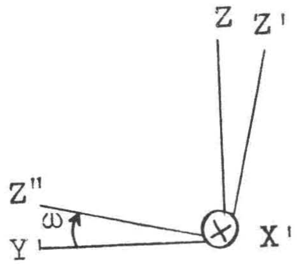
FIG. 5. Orientation of the terrestrial negative.

vidual sub-matrices. The diagram and matrix for the primary azimuth rotation through the angle alpha about the Z-axis is:



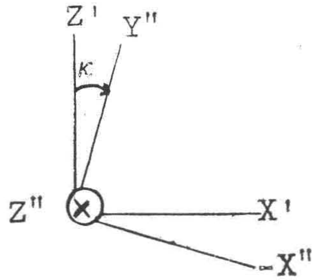
$$\begin{pmatrix} X' \\ Y' \\ Z \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \qquad (2.16)$$

for the secondary elevation rotation through the angle omega about the new position of the X-axis is:



$$\begin{pmatrix} X' \\ Z'' \\ Z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sin\omega & \cos\omega \\ 0 & \cos\omega & \sin\omega \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z \end{pmatrix}, \tag{2.17}$$

and for the tertiary roll rotation through the angle kappa about the new position of the Y-axis is:



$$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix} = \begin{pmatrix} -\cos\kappa & \sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X' \\ Z' \\ Z'' \end{pmatrix}. \tag{2.18}$$

Sequential substitution into the above expression and substitution of x, y, z for X'', Y'', Z'' leads to:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\cos\kappa & \sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sin\omega & \cos\omega \\ 0 & \cos\omega & \sin\omega \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \tag{2.19}$$

and matrix multiplication of the elements in expression (2.19) gives the final orientation matrix:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\cos\alpha\cos\kappa - \sin\alpha\sin\omega\sin\kappa & \sin\alpha\cos\kappa - \cos\alpha\sin\omega\sin\kappa & \cos\omega\sin\kappa \\ \cos\alpha\sin\kappa - \sin\alpha\sin\omega\cos\kappa & -\sin\alpha\sin\kappa - \cos\alpha\sin\omega\cos\kappa & \cos\omega\cos\kappa \\ \sin\alpha\cos\omega & \cos\alpha\cos\omega & \sin\omega \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{2.20}$$

Again it should be pointed out that this orientation matrix expresses the relationship of the image-space coordinate system in terms of the object-space coordinate system. If desired the inverse relationship, that of the object-space coordinate system in terms of the image-space coordinate system, can be obtained by inversion of the orientation matrix; or it can be developed by inversion of the individual rotation sub-matrices and matrix multiplication in reverse order.

III. Conclusion and Recommendations

It has been shown that the Stockholm resolution of July 1956, concerning coordinate systems and rotations, is not entirely compatible with United States surveying and mapping practice. But it has also been shown that the basic concepts of the resolution; that of using right-handed coordinate systems and taking the rotations as positive when clockwise about the positive direction of the respective axes can be applied to aerial photogrammetry in this country; and that only a slight modification of the concept will allow its application to terrestrial photogrammetry.

It has also been shown that when the object-space and image-space coordinate systems have been specified, together with the positive direction and ordering of the rotations, as well as which set of axes is to be referred to the other, that the orientation matrix can be developed uniquely and indisputably by a series of substitutions and matrix multiplications of the rotation operators.

It is recommended that the American Society of Photogrammetry adopt the following resolution as being the best possible compromise to the original Stockholm resolution; and that the concept of the orientation matrix as developed herein be utilized in photogrammetric education, research, and operations, and in all contributions to Photogrammetric Engineering.

> Be it resolved to adopt a right-handed rectangular coordinate system for both image-space and object-space. Further, the signs of the air-base components are to be the same as those of the air-base coordinates in the object-space system. Further, that the positive direction of the rotations be taken as clockwise about the positive direction of the respective axes. Further, that the above designations apply to aerial and terrestrial photogrammetry, except that the azimuth angle of terrestrial photogrammetry be taken as positive clockwise from North.

Several alternative orientation matrices dependent upon varying conditions have been presented in this paper. *It is therefore recommended that the Society standardize upon only two orientation matrices; one for aerial photogrammetry and one for terrestrial.* It is suggested: 1) that the orientation matrix for aerial photogrammetry be selected as that given in expression (2.7) as developed for the aerial diapositive with primary rotation about the X-axis; and 2) that the orientation matrix for terrestrial photogrammetry be selected as that given in expression (2.20) as developed for the terrestrial negative.

The comments and suggestions from members of the Society, favorable or otherwise, are cordially requested.

Acknowledgment

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Appendix A

The Church System

The development of the Church system of exterior orientation is presented in detail in reference (5). The method of development utilizes the concepts of analytical space geometry, is involved in procedure, and difficult to visualize. In this paper, the Church orientation matrix is developed by a series of matrix multiplications utilizing rotations through the respective Church angles. [Since the recent independent development of this procedure by the author, it has been learned that a similar approach was developed by Mr. R. D. Esten of the Engineer Research and Development Laboratories, Fort Belvoir, Virginia, as an appendix to his unpublished thesis at Syracuse University in 1947–48.]

The Church angles are defined as follows:

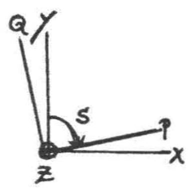
Tilt, t—the angle between a horizontal plane and the focal plane at the instant of exposure.

Swing, s—the direction of the tilt on the photograph itself. It is conventionally defined as the clockwise angle in the plane of the photograph, from the positive y-axis in the photographic system to the principal line.

Azimuth of the Principal Plane, α_{vo}—the survey direction of the principal plane measured in the same manner as the azimuth of any vertical plane in ordinary terrestrial surveying.

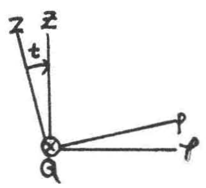
The principal plane of any photograph is the vertical plane containing the camera axis.

s rotation:



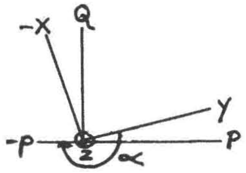
$$\begin{Bmatrix} P \\ Q \\ z \end{Bmatrix} = \begin{Bmatrix} \sin s & \cos s & 0 \\ -\cos s & \sin s & 0 \\ 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

t rotation:



$$\begin{Bmatrix} P \\ Q \\ Z \end{Bmatrix} = \begin{Bmatrix} \cos t & 0 & \sin t \\ 0 & 1 & 0 \\ -\sin t & 0 & \cos t \end{Bmatrix} \begin{Bmatrix} P \\ Q \\ z \end{Bmatrix}$$

α rotation:



$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} P \\ Q \\ Z \end{Bmatrix}$$

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} \cos t & 0 & \sin t \\ 0 & 1 & 0 \\ -\sin t & 0 & \cos t \end{Bmatrix} \begin{Bmatrix} \sin s & \cos s & 0 \\ -\cos s & \sin s & 0 \\ 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} -\cos s \cos\alpha - \sin s \cos t \sin\alpha & +\sin s \cos\alpha - \cos s \cos t \sin\alpha & -\sin\alpha \sin t \\ +\cos s \sin\alpha - \sin s \cos t \cos\alpha & -\sin s \sin\alpha - \cos s \cos t \cos\alpha & -\cos\alpha \sin t \\ -\sin s \sin t & -\cos s \sin t & +\cos t \end{Bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

FIG. 6. Rotation through the Church Angles (t, s, α).

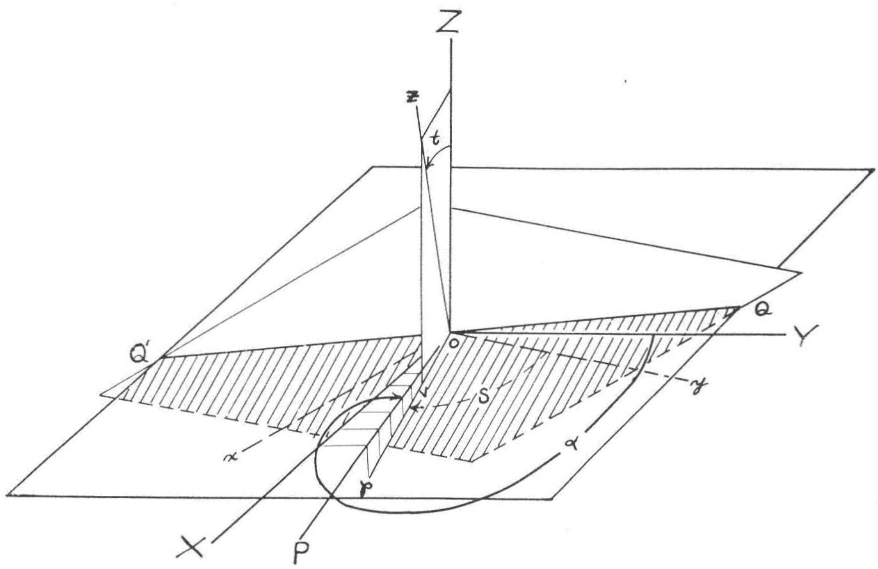


FIG. 7. Photograph oriented by Church System (t, s, α).

The principal line on any photograph is the line of intersection of the focal plane with the vertical plane containing the camera axis.

In considering this development by rotations, the origin of the Survey axes must first be considered as translated through space to the point of origin of the photographic axes (the principal point of the photograph) as indicated in Figure 7. The various angles of rotation are then determined as indicated in the diagram. This illustration is in keeping with Church's definitions: x, y, and z are the photographic axes; X, Y, and Z the Survey axes; o—the principal point; v—the nadir point; op—the principal line; oP—the projection of the principal line on the XY-plane; QQ'—

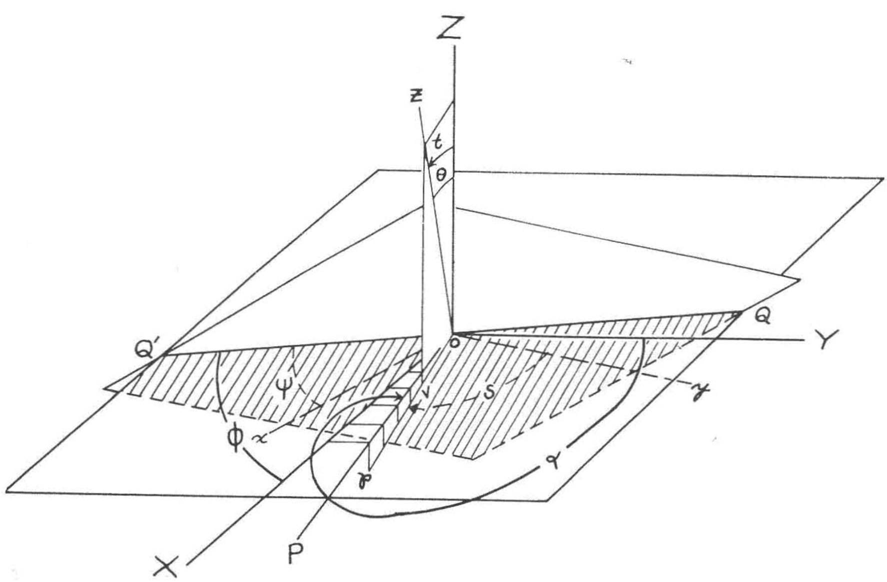


FIG. 8. Relation between Church angles and Euler angles.

the line of intersection between the photographic and Survey planes. The positive Y-axis points toward the North.

The rotation operations and matrix multiplications then take place as indicated in Figure 6, and lead to the Church orientation matrix.

As stated earlier, the Church system is based directly upon the standard Eulerian angles. Euler's angles are three angles chosen to fix the directions of a new set of rectangular space coordinate axes (photo) with reference to an old set (Survey). They are the angles between the old and new Z-axis, θ, (corresponds to t), the angle between the new x-axis and the intersection of the new xy-plane with the old xy-plane, ψ, (corresponds to s), and the angle between this intersection and the old x-axis, ϕ' (corresponds to $\alpha-180°$ (6) Figure 8 illustrates this relationship.

A major disadvantage of the Church system is that it breaks down for the ideal aerial photograph, the true vertical, the primary purpose for which it was developed. When there is no tilt, the two planes are either parallel or coincident, and there is no line of intersection (or principal line); thus the angles of swing and azimuth are undefined.

Use of Photogrammetry in Architecture and Other Civil Engineering Construction

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INTRODUCTION

ARCHITECTURAL Photogrammetry has developed into a field of considerable technical and professional importance, during its use since World War II. Sweden, France, The Netherlands and Switzerland have found it advisable to record their historical Architecture by Photogrammetry. In the U.S.A. developments in this field have progressed at Ohio State University's School of Architecture and Landscape Architecture, combined with the Institute of Photogrammetry, Geodesy, and Cartography. Professor Bertil Hallert, Director of the Photogrammetric Institute of the Royal Swedish Institute of Technology at Stockholm, published a paper on the extensive work done there in determining accuracy and the theory of error, in architectural and other terrestrial photogrammetry.

Photographic surveying of the historic buildings has been carried out systematically in Belgium during the years 1955/56 and a publication of "Photographic Surveying of Historic buildings in Belgium" was issued by the director of the "Service de Topographie et de Photogrammetrie du Ministerè des Travaux Publics," Ing. F.J.G. Cattelain & Ing. P. Vermeir.

A demonstration of the practical uses of the method of stereophotogrammetry as applied to Architecture/Archaeology is given below. It has also proved its worth in city planning, mining development and other civil engineering work, where the projects are investigated and studied from elevation plans.

This paper is based on a project in architectural photogrammetry at the Swiss Federal Institute of Technology, in which the WILD phototheodolite and WILD autograph A-5 were used. However this procedure does not need specific instruments but can be carried out with any standard existing instruments. Presented in this paper are some simple concepts of architectural photogrammetry, which may serve as an aid in explaining the relation of the subject to other branches of civil engineering.

The project under review involved the preparation of elevation plans at a scale of 1:100 of 50 ancient buildings within the old city of Lucerne, Switzerland. Although modern buildings are being constructed in this part of the town, too, the town-planning office aims at retaining as much as is possible, the character of the "old city." The elevation plans of the buildings to be demolished and of the neighboring houses will be of great