*Analytical Aerotriangulation by the Direct Geodetic Restraint Methods**

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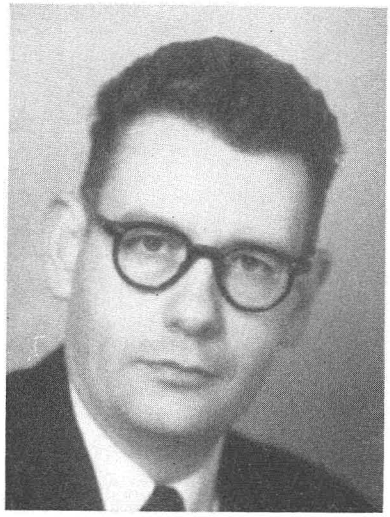
ABSTRACT: *The paper courses the history of the U. S. Geological Survey's research project on analytical aerotriangulation. The objectives of the program are discussed along with the general plan for attacking the problem. The main body of the text is devoted to describing the basic portion of the geometry of the Direct Geodetic Restraint Method of analytical aerotriangulation. This description is confined to terms familiar to the photogrammetrist and is given without recourse to mathematical formulas. A brief résumé of tests already conducted or in progress concludes the paper.*

HISTORY AND CURRENT STATUS

INSTRUMENTAL methods of photogrammetric control extension have long been used in the Geological Survey as an integral part of its map-production procedure. It has also been recognized for many years that an analytical solution to the control-extension problem holds certain theoretical advantages over instrumental methods. Developments in electronic computers caused the Geological Survey to activate a preliminary project in 1955 for the purpose of collecting and reviewing available information on analytical aerotriangulation and for selecting a procedure that could be adapted for electronic computation. It was decided in August 1957 that effort should be concentrated on a completely analytical method then being developed at Cornell University under a contract from the U. S. Army Engineer Research and Development Laboratories.[1,2]

Subsequent investigations demonstrated that the approach to the problem developed at Cornell required a certain amount of revision. The system as refined by the Geological Survey has been named "The Direct Geodetic Restraint Method." Tests of the method have been encouraging, but it must be emphasized that in the Geological Survey to date all work in analytical aerotriangulation has been exploratory in nature; it has not gone beyond the research stage.



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The Geological Survey is at present using a fixed-decimal-point, single-precision, Datatron 205 electronic computer program for these investigations. This program can accommodate the simultaneous adjustment of from one to nine single-camera photographs. In its present state, the program is not suitable for production use because development of the error theory and weighting system is not complete; also the program is inefficient and lacks many of the checks and peripheral components that would be required in practice. It is completely adequate, however, for the purposes of the research program.

[1] Dodge, Hugh F., "Geometry of Aerotriangulation" (a thesis), Cornell University, June, 1957.
[2] McNair, Prof. A. J., *et al.*, "A Solution of the General Analytical Aerotriangulation Problem," Cornell University, May 1958.

* Presented at the 25th Annual Meeting of the Society, Hotel Shoreham, Washington D. C., March, 10, 1959. Publication approved by the Director, U. S. Geological Survey.

Purposes of Research

The development of analytical aerotriangulation procedures in the Geological Survey is governed by two major objectives:

1. To provide a general solution to the aerotriangulation problem for both strips and blocks by a completely analytical means.
2. To develop knowledge of the basic laws and factors involved in aerotriangulation, with the expectation that such knowledge can be applied to other photogrammetric systems.

No attempt is being made to supplant human judgment with an electronic computer. On the contrary, the aim is to design a system that will relieve the photogrammetric operator of the tedious repetitive tasks that do not require judgment, and to provide him with a maximum amount of information so that his experience and intelligence may be applied more effectively. Since the operator can best utilize his experience and skill in situations that are familiar to him, the system is being designed so that the information supplied will not be foreign to current photogrammetric thinking. With the system so designed, the basic knowledge developed can be applied directly to other photogrammetric methods.

Plan of Attack

It was necessary to formulate a general plan of attack which would lead to the successful realization of the given objectives. This plan emphasizes theoretical development and analysis as a first step in every phase. This is followed by verification, first, in completely controlled tests involving fictitious photography and, second, in tests conducted under normal operating conditions. Such a procedure lends itself to the following sequence:

1. Development of a geometrical method of solution suitable for electronic computation and the preparation of an electronic computer program.
2. Verification of the geometrical development in a series of tests with fictitious photography.
3. Determination of the theoretical effects of the propagation of errors by means of a differential error analysis based on proven geometry.
4. Establishment of the validity of the error theory by controlled tests.
5. Construction of data-weighting computer routines.
6. Demonstration of the correctness of the weighting routines by electronic computer tests.
7. Appraisal of available physical components for the system (camera, film, glass plates, comparator) based upon the error theory and operational considerations.
8. Development of calibration procedures and computer routines for compensation of systematic errors in the physical components of the system.
9. Testing of the entire system under actual production conditions.

The first two items in this list—the development of a workable geometry and tests verifying it—have been completed for single-camera photography. The remainder of this paper will be devoted to an outline of the more important geometrical techniques used and a short résumé of tests completed or in progress.

Geometrical Basis

THE PHOTOGRAPH

The first item to be considered is the photograph itself. Figure 1 is a diagram of a "theoretical photographic system." The term "theoretical" is used in the sense that this photograph represents the simplest theoretical geometry, with the object rays all converging at a point O, the photograph lying in a perfect plane, and so on. Each photograph in a given bridge has its own set of coordinate axes with the origin at the perspective center. The z-axis is collinear with the camera axis; the x- and y-axes are parallel to lines joining the two sets of fiducial marks.

Coordinates of points on "theoretical photographs" are determined by making appropriate corrections to measurements ob-

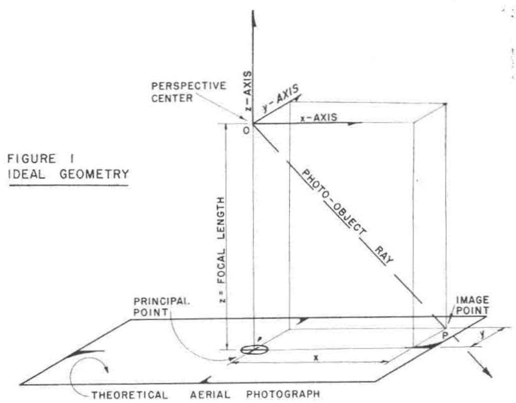

Fig. 1

tained from actual photographs. These corrections are based on camera-calibration data, film-distortion measurements and atmospheric-refraction information.

The "theoretical photographs" resulting from these corrections will not be perfect because residual errors in the photo-coordinate values will still exist after the application of the corrections. The theory is, however, that the resulting photographs may be represented by the simplest ideal geometry and will have greater fidelity than the actual photographs. These theoretical photographs are used in all subsequent calculations.

The coordinates of point P are shown in Figure 1 as x and y (as measured in a comparator and corrected as described above) and z, which is equal to minus the focal-length. These dimensions are used to find the direction cosines of each ray with respect to the photographic coordinate system. These direction cosines of all object rays are the digital form that the electronic computer uses to represent the bundle of rays associated with each photograph.

SATISFYING THE PHOTOGRAMMETRIC REQUIREMENTS OF THE BRIDGE

The question now becomes: How can the established photogrammetric requirements of the bridge be satisfied by using these digital representations of bundles of rays?

This is accomplished by executing the following general sequence.

1. Estimation of the position and orientation of the camera at each exposure station. These estimates comprise the following elements: latitude, longitude, altitude, x-tilt, y-tilt and camera heading.
2. Formation of an appropriate equation, based upon the initial estimates, for each photogrammetric requirement that is to be enforced. In other words, there will be one condition equation to remove parallax at a given point, one to force a fit to a bench mark, and so on.
3. Development of a set of normal equations from the condition equations and the simultaneous solution of these to determine the corrections necessary to the position and orientation of each bundle of rays so that the photogrammetric requirements will be closer to fulfillment.
4. Application of these corrections to produce a new position and orientation for the camera at each exposure station.

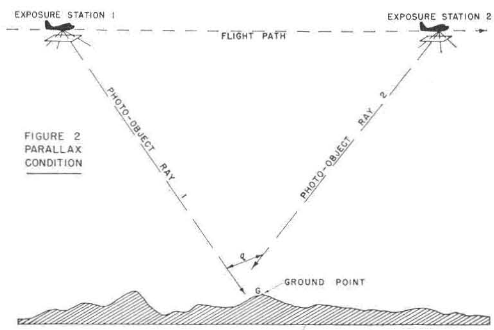

FIG. 2

5. Formation of a new set of condition equations based upon the new position and orientation of each bundle of rays.
6. Formation of new normal equations and their simultaneous solution.
7. Repetition of the cycling (or iterating) until the photogrammetric requirements are adequately satisfied.

While iterative procedures such as this are commonplace, the form of the condition equations used should be of interest.

REMOVAL OF PARALLAX

In taking up the photogrammetric requirements and the equations associated with each, one by one, a logical place to start is with the removal of parallax. Use is made here of the method of Dr. Paul Herget[3] whose work at Ohio State University forms the foundation for much of the work in this field at both Cornell University and the Geological Survey.

In Figure 2, two successive camera stations are represented by the two aircraft silhouettes with rays emanating from each. The long dashed lines represent the initially estimated locations of rays to a ground point whose true location is at G. When the bridge is absolutely oriented these two rays will intersect at G, the correct position in space, except for small errors. However, because the initial estimates of position and orientation for the bundles of rays are not exactly correct, these rays will, at first, neither intersect nor pass through the true position to the point.

Assuming that the elevation and horizontal position of the point G are unknown, the only condition that can be legitimately enforced is that the rays intersect. The correct positioning of this intersection is accomplished by

[3] Herget, Dr. Paul, "Interim Report on Ground Position Computation From Shoran-Controlled Photography," OSURF Technical Paper No. 179, January 1954.

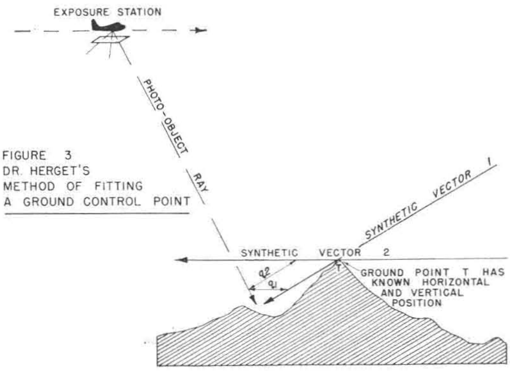

FIGURE 3
DR. HERGET'S
METHOD OF FITTING
A GROUND CONTROL POINT

FIG. 3

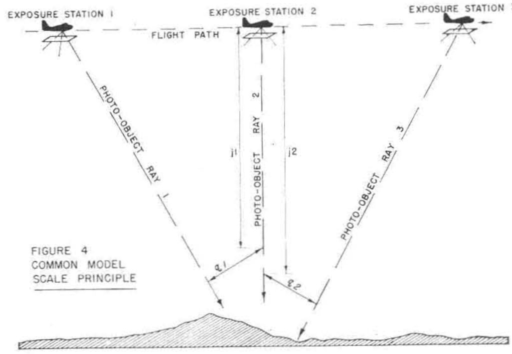

FIGURE 4
COMMON MODEL
SCALE PRINCIPLE

FIG. 4

the simultaneous enforcement of all conditions to be imposed on the photogrammetric bridge. To force these rays to intersect, or in other words, to remove the parallax, Dr. Herget developed an equation that reduces the shortest distance between the two rays (indicated by q) with each iteration until this distance becomes negligible. As already mentioned, the positions and orientations of the various bundles of rays in the bridge are adjusted with each iteration to achieve this.

An interesting sidelight is that Dr. Herget was able to use this same general form of equation to fit a ground control point that had a known elevation and horizontal position. Figure 3 shows such a ground-control point with two synthetic vectors (indicated by the solid lines) emanating from it. By forcing the dashed line, which represents an object ray, to intersect each of these two vectors, the ray was forced through the ground point itself. By thus forcing rays to intersect, Dr. Herget was able to resect the initial photograph of a strip and to add successive photographs in a cantilevering process.

However, resecting single photographs imposes far too severe ground-control requirements for practical topographic mapping in most areas of the world. The obvious need is for an operational analytical system broad enough to encompass those photogrammetric principles that will permit a general solution.

COMMON MODEL SCALE

The next photogrammetric principle to be considered is the requirement that all models of a strip have a common scale. Figure 4 illustrates three consecutive exposure stations with rays emanating from each that should intersect at a common point. The q terms again indicate the shortest distance between these rays. One of Dr. Herget's parallax equations is used to force ray 1 to intersect ray 2 and a second equation to force ray 3 to intersect ray 2. It is now necessary to use an equation that requires j_1 to equal j_2. This will bring the three rays together at a common point as the iterative procedure takes place. By enforcing such an equation for at least one point common to each three consecutive exposures of a strip, common model scale throughout the strip is assured.

RELATIONSHIP BETWEEN ADJACENT STRIPS

It is also necessary to enforce the proper relationship between adjacent strips in a block solution. In Figure 5 two flights are shown. The long rays all relate to the same ground point. The shortest distance between rays of the first flight is denoted by q_1 and for the second flight by q_2. The intersection of q_1 with ray 1 is indicated by D, and of q_2 with ray 12 by E. Three equations are used to force D and E to assume the same position in space. The first causes them to draw together in an X direction, the second in Y, and the third in Z. As the q dimensions go to zero in the iterative process, the effect is to bring all the rays through a common point. When two or more such ground points common to two adjacent

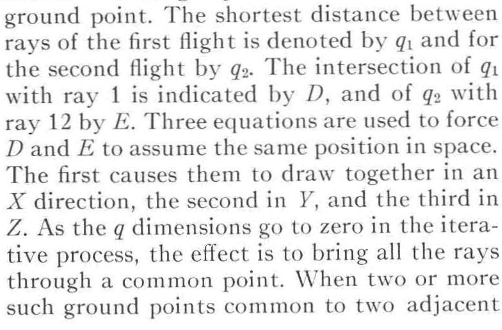

FIGURE 5
BLOCK ADJUSTMENT
PROCEDURE

FIG. 5

flights are used, the correct relationship will exist between the flights.

By enforcing these five equations (parallax, common model scale, and the three block-adjustment equations), wherever applicable, the internal integrity of a correctly planned photogrammetric bridge or block will be assured. All that remains for a completely general solution is a system for fitting all available ground control.

GEOCENTRIC COORDINATE SYSTEM

To understand the means of accommodating the various ground-control conditions, it is necessary to consider a second basic coordinate system. Figure 6 depicts the reference ellipsoid of the earth. Since the origin of the coordinate axes shown is at the center of the reference ellipsoid, the system is called a geocentric coordinate system. The positive Z-axis runs through the geographic North Pole and the positive X-axis is defined by the intersection of the plane of the meridian through Greenwich and the plane of the equator. The Y-axis is then chosen so that an orthogonal right-hand system will result.

Also shown in Figure 6 is a random meridian plane. All points in this plane have the same longitude, and any point with this longitude must lie in this plane.

The equation that is enforced for a point of known longitude in a photogrammetric bridge is one that causes the point to reach and remain on its meridian plane. A ground point or exposure station of known longitude is restrained in this way.

Similarly, if the latitude of a point is known, the point must be restricted to the surface of a known cone. Figure 7 shows this situation. It should be mentioned here that the use of this cone has certain limitations. When the latitude of the point is close to 0° or 90°, it is necessary to use the plane of the prime vertical to restrain the point. In this case, it is necessary also to keep the point on the correct meridian plane in order that the total effect will keep the point on the normal (N_2 in Figure 7).

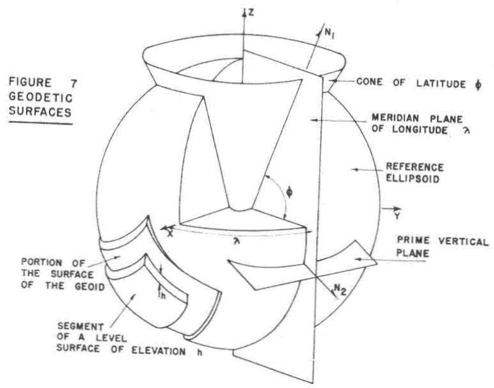

FIG. 7

To recapitulate on the matter of a horizontal station, assume a triangulation station of unknown elevation on two photographs of a strip. Three equations are written for this point: first, the parallax equation; second, an equation to force the intersection onto the meridian plane; and third, an equation to force the intersection onto the latitude cone (or prime vertical plane).

A similar situation exists for a point of known elevation. In Figure 7 a segment of a surface is delineated that represents all points of equal elevation. When the elevation of a ground point or exposure station is known, the specific surface that point is to reach and remain on is defined. A spherical surface is used locally to approximate this actual surface. The point of known elevation is restrained by an equation so that it reaches and remains on this spherical surface. Thus, in the case of a bench mark appearing on two photographs of a strip, two equations are enforced: first, the parallax equation and, second, the elevation restraint just described.

The utility of the geocentric coordinate system will be seen from the fact that these restraining surfaces are most easily defined mathematically in terms of the basic geodetic reference system.

Other equations and restraint methods are included in the Geological Surveys current computer program but the foregoing represents the basic portion of the Direct Geo-

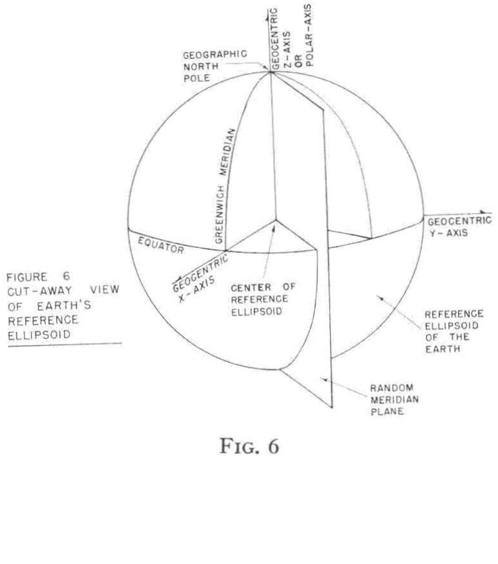

FIG. 6

detic Restraint Method and also indicates why the method was so named.

Results Obtained

The Direct Geodetic Restraint Method was first tested on a series of 26 problems. This initial series was made up from the fictitious data developed by Earl Church and given in the Harvard Computational Laboratory Report No. 25. Correct photographic-coordinates and ground-point positions were used in these tests, each of which was a minimum theoretical case. The tests ranged in complexity from the resection of a single photograph to the solution of a block of eight photographs. In addition to some normal cases, a number of bizarre problems were included, as well as a few that depended for solution upon exposure station restraints. The results of this series of tests are available at the U. S. Geological Survey's Washington office.

A second series of tests is now being conducted to determine the practical range within which the initial estimates must fall. A report on this series will soon be available.

A preliminary test using photography over the Arizona test area is also under way. The photographic coordinate data was measured by the U. S. Army Map Service on a Cambridge Stereocomparator. The results of this test will be available in the near future.

The results obtained with the Direct Geodetic Restraint Method to date have been satisfactory. It is contemplated that the present computer program will be used for additional testing, largely in relation to errors in the physical components of the system, the propagation of errors, and data weighting methods. It is expected that tests under production conditions will be possible in approximately one year.

In closing, I would like to acknowledge the important contributions to this work of the other two members of the U. S. Geological Survey's research team: Mr. Robert C. Eller and Mr. David Handwerker.

*An Evaluation of Aerial Photography for Detecting Southern Pine Beetle Damage**

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(*Abstract is on next page*)

Introduction

Forest insects are insidious killers of our Nation's timber. The most recent compilation of statistics (5) showed that they were responsible for killing seven times the sawtimber volume as that consumed by fire. In the South alone, during the past 5 years, the southern pine beetle (*Dendroctonus frontalis*, Zimm) killed more than 400 million board-feet of pine timber annually (3). How are we going to reduce these losses to tolerable levels? How can we best detect outbreaks while they are small and easy to control? The old army maxim, "Those that get there fustest with the mostest, win the battles!" applies equally well in the battle with the beetles. Because bark-beetle epidemics frequently cover large areas of timberland and fluctuate widely in amount and degree of damage, an aerial method to discover and locate outbreaks rapidly has more merit than the slow and costly ground methods used in the past.

Two aerial approaches for speeding up detection are possible. The first involves visual observation from the air by trained observers who plot suspected infestations on maps. The second involves taking special aerial photographs on which interpreters try to locate the infestations. The first method is inexpensive

* Presented at the Society's 25th Annual Meeting, Hotel Shoreham, Washington, D. C. March 8 to 11, 1959. This paper is a part of the Panel on The Application of Color Photography in Photo Interpretation.