Photogrammetric Systems and Operations Research

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ABSTRACT: *Operations Research is a useful tool in the engineering of systems containing a multiplicity of equipments. By providing quantitative measures of system performance, these techniques provide a concrete basis upon which equipment design and utilization can be optimized. This paper presents a specific example of operations research applied to the design of a photogrammetric support system.*

INTRODUCTION

DURING the initial design of photogrammetric equipment, a multitude of critical decisions are made. These decisions not only must take into account the many engineering problems of equipment design but also the equally important operational considerations of equipment performance. Operational considerations include those of output quality, cost, processing time, maintainability, reliability, etc., all of which form fundamental attributes of a system comprised of many equipments. Operations Research is a useful tool in such design deliberations as it provides a *quantitative* basis for evaluating the performance of a system before actual equipment construction is undertaken.

The following paragraphs discuss the application of Operations Research to a photogrammetric support system consisting of a Reformatting Device, Rectifier and Orthophotoscope. The details of the photogrammetric problems involved have been somewhat simplified in order to emphasize the Operations Research Techniques of decision making that are involved. The specific measures of performance used are cost and time and time per photo processed. The effects of changes in equipment design upon the system are quantitatively evaluated in terms of these performance criteria.

CONCEPTUALIZATION OF A SYSTEM

From the conceptual viewpoint, any particular system can be viewed in terms of the inputs; the possible system designs; and the outputs. In a photogrammetric system the inputs have certain specific attributes;quantity, timing, and quality predominating. Each of these attributes is associated with a probability distribution, which indicates the chance that any specific value of an attribute will occur in any particular input. The system itself can have several alternative configurations depending upon the design and arrangement of the individual components. Therefore, each system configuration can be considered to have an "efficiency" associated with each equipment design and arrangement. The outputs of the system as a result of these considerations also have the attributes of quantity, quality, plus attributes of time and cost per unit output.

The following development of system performance will be made in terms of quality, processing time, and cost rather than from the viewpoint of quantity and timing as these latter attributes have been considerably explored in the mathematics of waiting-line theory.

SYSTEM INPUTS

The inputs to the photogrammetric system under consideration are aerial photographs which have four quality characteristics or attributes:

Scale (s)
Tilt (t)
Relief displacement (r)
Resolution

While all these attributes are in some way mathematically interdependent, it will be obvious from an examination of the values specified for each attribute that the mathematical dependence is extremely limited. Experimental determination of the interdependence of the above attributes for the range of values specified in the following discussion allows treatment as statistically independent variables. Such a procedure sacrifices very little in accuracy while allowing concise mathematical presentation.

SCALE

The nominal scale of an aerial photograph is a function of focal-length and camera altitude although many additional factors must be considered in a truly rigorous analysis.[1] The physical restraints placed upon these factors which result from camera design, meterological encounters, as well as practical considerations of overlap allowances, determine the probability distribution of scale for a particular system. On this basis, the chance that a particular photograph entering the system will have a particular scale can be determined. While such an analysis of the chance occurrences of the four variables associated with the input photography is fundamental to the operations research analysis, the details of their determination have been abridged.

Scale is utilized as the fundamental input variable. An analysis of the specific scales which the system is required to process indicates that the probability distribution of scale is approximately normally distributed with a mean (μ) of 40,000 (scale number) and a standard deviation (σ) of 15,000.

The input scale distribution follows the form shown in Figure 1, where $f(s)$ represents the chance a given input photo will have a particular scale number, and:

$$f(s) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2}\left(\frac{s-u}{\sigma}\right)^2$$

Since no scale numbers less than 10,000 are inputs to the system, the .02275% of the distribution less than 10,000 is considered to be confined about the 10,000 scale number. This introduces no mathematical complications because the design variants are confined to the $\pm 1\sigma$ portion of the distribution, as will be seen later.

TILT

The ability of a camera mounting system to maintain verticality throughout the range of possible collection system attitudes will determine the tilts that can be expected in the resulting photography. Analysis of factors

[1] A. Katz, PHOTOGRAMMETRIC ENGINEERING, XIX, 63 (1952).

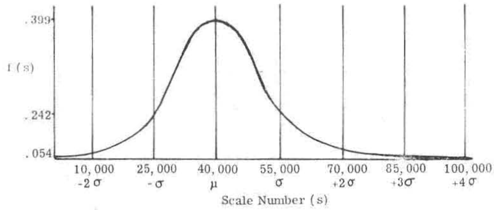

FIG. 1. The input photograph scale probability distribution—the chance that a particular incoming photo will have a particular scale.

associated with camera stability can be made on a statistical basis, thus allowing computation of the probable tilt values to be expected. The particular system under consideration here was analyzed in this manner which resulted in the expectation that the tilt of input photography would follow a Poisson distribution with a mean (μ) and a variance $(\sigma)^2$ of 2 degrees. This distribution is shown in Figure 2, where:

$$f(t) = \frac{e^{-u}u^{t}}{t!}$$

$f(t)$ represents the chance that a particular photo entering the system will have a particular value of tilt.

RELIEF DISPLACEMENT

The input variable of relief displacement is a complex function of camera altitude, object height, and radial image distance from nadir. Analysis of the camera station location, and ground contour interval at the expected camera altitudes indicate that relief displacement can be approximated by a Poisson distribution with a mean (μ) and variance $(\sigma)^2$ of 4 mm. (at the mean scale number of 40,000).

The input relief displacement distribution is shown in Figure 3, where:

$$f(r) = \frac{e^{-u}u^{r}}{r!}$$

This figure graphically indicates the probability that a particular photograph entering the system, when reproduced at a scale number of 40,000, will attain an image of a given value of relief displacement.

RESOLUTION

The output resolution of a series of photo-optical equipments which incorporate magnification can be maximized by performing the reproduction and scale change in the optimum sequence. The heuristic approximation for the resolution of a multi-element photo-optical system has been expressed as:[2]

[2] A. Katz, *J. Opt. Soc. Am.* 38, 604 (1948).

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

where:

R_T = total resolution
$R_1, \cdots, R_n$ = resolution of the individual elements in the system

For a system incorporating magnification, this equation can be rewritten in the following concise notation

$$\frac{1}{R_T} = x \sum_{i=1}^{m} \frac{1}{R_i} + \sum_{j=1}^{n} \frac{1}{R_j}$$

the limit of

$$x \sum \frac{1}{R_i} \text{ as } x \text{ approaches } \frac{1}{\infty} = \frac{1}{\text{Lens Resolution}}$$

where:

x = magnification
R_T = total resultant resolution
R_i = resolution of magnified elements of the system
R_j = resolution of the elements of the system not incorporating magnification

By maximizing this equation when magnification is greater than one (decrease in scale number), and when the magnification is less than one (increase in scale number), subject to the specified limit, the resulting resolution of the photo optical system will always be a maximum.

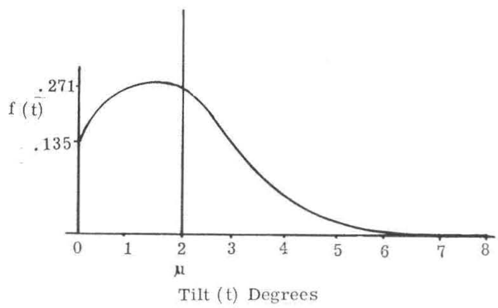

FIG. 2. The input photograph tilt probability distribution—the chance that a particular incoming photo will have a particular tilt.

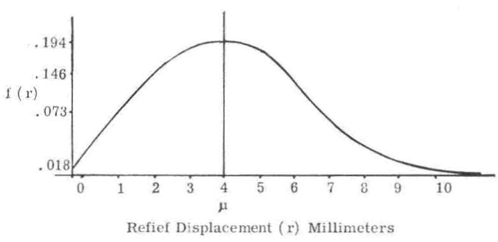

FIG. 3. The input photograph relief displacement probability distribution—the chance that a particular incoming photo will have imagery of a particular displacement due to relief.

As an example, consider the case where photography is to be reformatted and rectified to achieve a tilt-free photograph with a prescribed magnification at the isoline. What reproduction sequence will maximize the output resolution?

A specific case of the preceding equation for the total resolution of a two stage reproduction process can be written as:

$$\frac{1}{R_T} = \left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right) m_1 + \frac{1}{R_3} + \frac{1}{R_4}\right] m_2 + \frac{1}{R_5}$$

and

$$m_1 m_2 = M = 1$$

where:

R_T = total resultant resolution
R_1 = resolution of the original input
R_2 = lens resolution of the first equipment
R_3 = recording film resolution of the first equipment
R_4 = lens resolution of the second equipment
R_5 = recording film resolution of the second equipment
m_1 = magnification incorporated in first equipment
m_2 = magnification incorporated in the second equipment
M = total magnification performed

By combining the two expressions we can write:

$$\frac{1}{R_T} = M\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{m_1 R_3} + \frac{1}{m_1 R_4}\right] + \frac{1}{R_5}$$

From this equation it can be seen that regardless of the individual resolution values, the total resolution increases as m_1 increases. Therefore, when two photo-optical equipments are used in series the greatest magnification should be performed in the first equipment to maximize the resolution output of both equipments (subject only to the limiting resolution of the lens system involved). Therefore, by placing this constraint upon the operation of the system, the resolution will be maximized regardless of the scale of the inputs.

THE OUTPUT REQUIREMENTS

As in all systems, the quality of the output is dependent on the processing time permitted and the allowable costs. The output requirements for this particular photogrammetric system can be listed as:

(Q_1) = maximum output quality regardless of time or cost

(Q_2) = nominal output quality (to reduce cost from Q_1)

(Q_3) = minimum output quality (to minimize through-put time)

The probable occurrence of any one system output quality must be determined from experience. Here, experience dictates the use of the values (Q_1). 3, (Q_2).5, (Q_3). 2, for the probability that anyone of the three quality outputs will be required of the system at a particular time.

The specifications for the output levels of quality must be established. For this particular system they are tabulated in Table 1:

The System

The photogrammetric support system consists of 3 units of equipment.

(1) A Reformatting device with .4 to 4× magnification to accomplish scale change throughout the scale range of the input photography.

(2) A Rectifier to correct for photographic tilts which also incorporates a 1 to 2× magnification to accomplish a certain amount of isoline scale change.

(3) An Orthophotoscope to correct for relief displacement errors with no scale change capability.

Upon considering the three units of equipment that comprise the system, it is apparent that mathematically there are 17 states the system can occupy (including the zero state). This simply results from the expresssion:

$$\sum_{r=0}^{3} P_r^n + 1 = \sum_{r=0}^{3} \frac{n!}{(n-r)!} + 1 = \frac{4(3!)}{3! + 2! + 1! + 0!}$$

However, upon investigation of each of these possibilities, there are only ten states which have any meaning in the limitations of the "real world." These can be listed as follows:

S_0 = zero state (no operations performed)

S_1 = reformatting equipment

S_2 = rectifier

S_3 = orthophotoscope

S_{12} = reformat equipment followed by rectifier[3]

S_{13} = reformat equipment followed by orthophotoscope

S_{21} = rectifier followed by reformat equipment[3]

S_{23} = rectifier followed by orthophotoscope

S_{123} = reformat equipment followed by rectifier and orthophotoscope[3]

S_{213} = rectifier followed by reformat equipment and orthophotoscope[3]

Operational Analysis

Since all input variables are mutually independent for the range of values considered, the probable occurrence of any particular state of the system is determined by the probability that the variables of the scale, tilt and relief displacement for any particular photograph will or will not require reformatting, rectification or othophotoscoping. This is expressed mathematically as the product of the probabilities of the three input variables:

$$P(S_n) = P(s_n)P(t_n)P(r_n)$$

where

$P(S_n)$ = probability that a particular system state will occur

$P(s_n)$ = probability that the input variable

[3] The permutations S_{12}, S_{21} and S_{123}, S_{213} are required to maximize the resolution depending upon the magnification or minification of the input photography. If the input photography is being magnified (decreased in scale number), the maximum scale change is performed in the first equipment. Conversely when increasing scale number (magnification less than one), the maximum scale change is performed in the last equipment. Such a procedure maximizes the output resolution of the two equipments in series, and offers the simplicity in this analysis of eliminating resolution from the list of input variables as discussed previously.

Table 1

Specifications for Output Levels of Quality

Output Quality	Q_1	Q_2	Q_3
Scale Number	40,000±1,000	40,000±2,000	40,000±2,000
Tilt	Less than 2°	Less than 2°	Less than 4°
Relief Displacement	Not to exceed 4.5 mm.	Not to exceed 6 mm.	Any acceptable

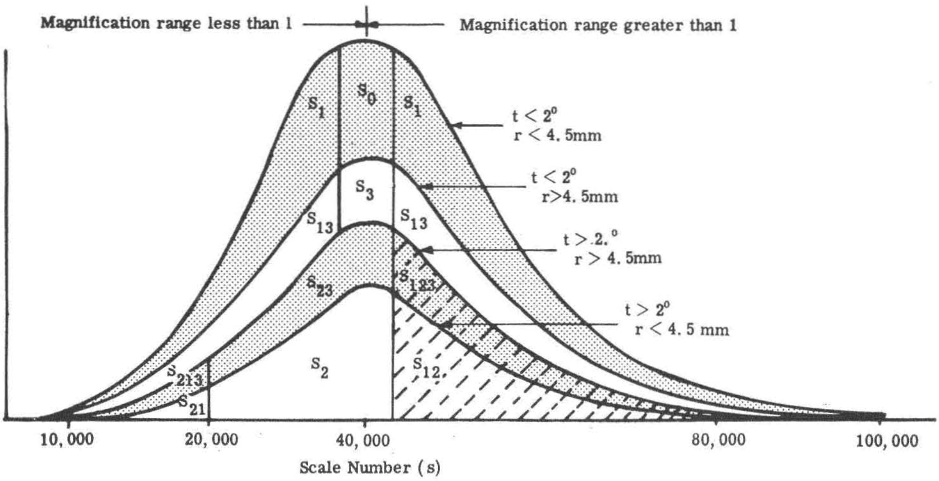

FIG. 4. Portioning of input variable among system states for maximum quality output—shaded area shows the effect of change in equipment design.

of scale will require only the operations included in the state S.

$P(t_n)$ = probability that the input variables of tilt will require only the operations included in the state S.

$P(r_n)$ = probability that the input variable of relief displacement will require only the operations included in the state S.

Thus, the probabilistic occurrence of any of the ten listed System States $P(S_n)$ can be calculated from the independent probabilities of scale (s), tilt (t), and relief displacement (r) for each of the system output quality levels of maximum quality (Q_1), nominal quality (Q_2) or minimum quality (Q_3).

The probabilistic distribution of any particular system state can be obtained in terms of any one of the input variables. Here it is convenient to use the input variable of scale since the other variables (tilt and relief displacement) are merely dichotomous events. By separating the scale continuum into all possible combinations of these "two valued" events, the probability that any system state will occur can be determined. Figures 4 and 5 graphically depict the probability that any one of the ten system states will occur as a function of scale under the specifications for maximum quality (Q_1) and nominal quality (Q_2). The random variable of scale has been partitioned into four normally distributed parts each of which represents an occurrence of the four possible combinations of the events

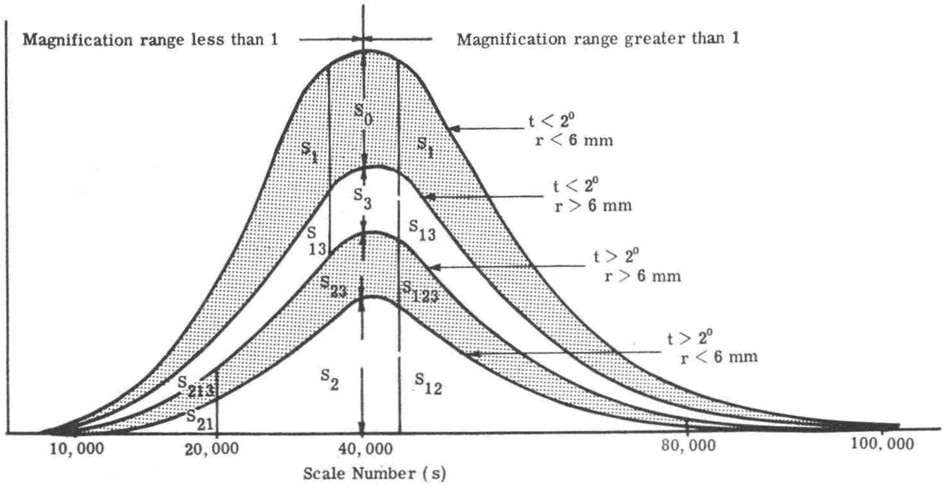

FIG. 5. Portioning of input variables among system states for nominal quality output.

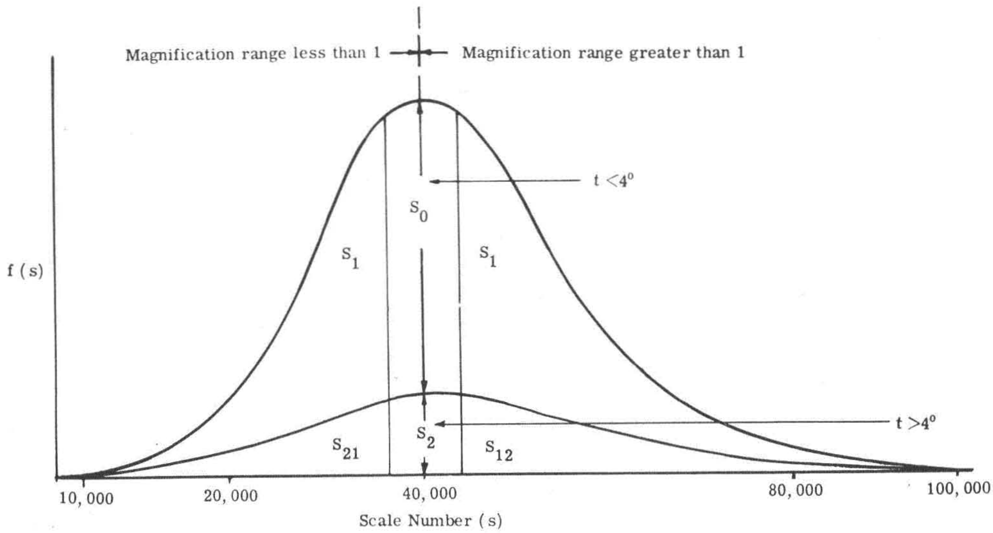

FIG. 6. Portioning of input variables among system states for minimum quality output.

tilt and relief displacement. Figure 6 graphically represents the probable distribution of the five possible system states that can occur under the specifications for minimum quality output (Q_3).

A value of time is associated with each equipment usage. Realistically such throughput times must reflect both the effects of increased handling time as the number of units increase and the decrease in processing time as the number of machine tasks are shared. In such a manner it is possible to assume a reasonable series of time relationships for the system as follows:

Time (Minutes)

$S_0=0$
$S_1=10$
$S_2=15$
$S_3=20$
S_{12} or $S_{21}=25$
$S_{13}=30$
$S_{23}=35$
S_{123} or $S_{213}=45$

In the same manner considerations of cost per unit processed based on initial equipment costs and maintenance considerations, (but not considering operator costs) may also be tabulated for use in this specific example of a photogrammetric support system.

Cost per photo processed

$S_0=0$
$S_1=\$2.00$
$S_2=\$4.00$
$S_3=\$3.00$
S_{12} or $S_{21}=\$6.00$
$S_{13}=\$5.00$
$S_{23}=\$7.00$
S_{123} or $S_{213}=\$9.00$

The reader must be aware that the accuracy of the following operations research analysis is directly dependent upon the accuracy of both time and cost assumptions. These values are the measures of performance that are used for system operation. Since the specific values of time and cost must be determined for each specific equipment design and system configuration, the utmost care must be made in their estimation.

THE DECISION MATRIX

The probabilities associated with any one system state can be used to formulate a "decision matrix." By placing the various system states with the time and cost associated with each state, in a row along the top of the matrix and placing the System output quality levels in a column to the left, the probability of each state for each quality level can be inserted. This probability associated with each state for each quality level is the area under the curves shown in Figures 4, 5, and 6. See Table 2.

Each cell in the matrix shows the probable sequence of operation which will be required for best quality, nominal quality and minimum quality. From this matrix, the average of all values of cost and time can be computed for each individual system output quality level, as well as for all three output quality levels by the probabilistic equation for Ex-

TABLE 2

	S_0	S_1	S_2	S_3	S_{12}	S_{13}	S_{21}	S_{23}	S_{123}	S_{213}
Time (T)	0	10	15	20	25	30	25	35	45	45
Cost (C)	0	2.00	4.00	3.00	6.00	5.00	6.00	7.00	9.00	9.00
$Q_1(.3)$	.0175	.3325	.14875	.0075	.16625	.1425	.035	.06375	.07125	.015
$Q_2(.5)$	.0475	.4275	.21375	.0025	.21375	.0225	.0475	.01125	.01125	.0025
$Q_3(.2)$	.08	.72	.02	—	.09	—	.09	—	—	—

pected Value:

$$EX = Px_1(X_1) + Px_2(X_2) + \cdots + Px_n(X_n)$$

Therefore the average cost per photograph processed for each quality level is:

$$\begin{aligned} EQ_1(C) &= .0175(0) + .3325(2.) + .14875(4) \\ &\quad + .0075(3.) + .16625(6.) + .1425(5.) \\ &\quad + .035(6.) + .06375(7.) + .07125(9.) \\ &\quad + .015(9.) \\ &= \$4.425 \end{aligned}$$

In a similar manner

$$EQ_2(C) = \$3.60$$
$$EQ_3(C) = \$2.60$$

The average time per photograph processed for each quality level is

$$\begin{aligned} EQ_1(T) &= .0175(0) + .3325(10) + .14875(15) \\ &\quad + .0075(20) + .16625(25) + .1425(30) \\ &\quad + .035(25) + .06375(35) + .07125(45) \\ &\quad + .015(45) \\ &= 21.025 \text{ minutes} \end{aligned}$$

and

$$EQ_2(T) = 15.75 \text{ minutes}$$
$$EQ_3(T) = 12 \text{ minutes}$$

The expected total system performance for all quality levels is computed again using the concept of Expected Value:

$$\begin{aligned} E(C) &= .3(4.425) + .5(3.60) + .2(2.60) \\ &= \$3.65 \\ E(T) &= .3(21.025) + .5(15.75) + .2(12) \\ &= 16.6 \text{ minutes} \end{aligned}$$

ANALYSIS OF THE PAYOFF MATRIX

By considering the operation of the system in terms of Expected Values, the cost of operation for each level of system performance can be *quantitatively* examined. For example, it can be determined from the payoff matrix that, by reducing the required output quality of the system from maximum to minimum, the cost per photograph processed is reduced from \$4.425 to \$2.60 while the processing time is decreased from 21 to 12 minutes.

Not only is it possible to compare various aspects of system performance, but what is more important from the Research and Development point of view, it is possible to quantitatively evaluate the operating performance of the system when changes in the equipment characteristics are introduced.

As an example, it is desired to evaluate a design change in Rectifier magnification in terms of the expected performance of the system. What will be the effect on the cost and time of system operation if a magnification capability of .5× is introduced into the Rectifier in addition to the current 2× magnification? (This will allow accommodation of scale change from 40,000 to 80,000 in addition to the present 20,000 to 40,000.)

By increasing the magnification it can be seen that the distribution of input variables among system states is changed. In case of the maximum quality output (Q_1), the probability distribution of input variables among the system states is changed as follows:

S_2 is increased to .28
S_{12} is decreased to .035
S_{23} is increased to .12
S_{123} is decreased to .015

The effect can be seen graphically as the hatched area in Figure 4.

Calculation of the expected performance of the system for the maximum quality output requirements, in the manner described previously, results in a new expected cost per photograph processed of:

$$\begin{aligned} EQ_1(C) &= .0175(0) + .3325(2.) + .28(4.) + .0075(3.) \\ &\quad + .035(6.) + .1425(5.) + .035(6.) \\ &\quad + .12(7.) + .015(9.) + .015(9.) \\ &= \$4.05 \end{aligned}$$

and an expected processing time of:

$$\begin{aligned} EQ_1(T) &= .0175(0) + .3325(10) + .28(15) \\ &\quad + .0075(20) + .035(25) + .1425(30) \end{aligned}$$

$$+ .035(25) + .12(35) + .015(45)$$
$$+ .015(45)$$
$$= 19.26 \text{ minutes}$$

Thus a quantitative comparison of the previous cost per photograph processed of $4.425 and through-put time of 21.025 minutes can be made by decision making personnel in order to determine the desirability of such an addition to Rectifier magnification. *In this manner it is possible to quantitatively evaluate equipment design in terms of system performance.*

CONCLUSION

This paper has presented a specific example of the utilization of operations research in the design of a specific photogrammetric support system. By the use of similar techniques, the expected performance of any system can be determined provided the distributions of input variables, possible system states and the specified output quality requirements are known or can be reasonably estimated. In this manner, not only may the initially expected system performance be determined, but also the effect of changes in equipment design on the system performance may be *quantitatively* evaluated. Thus, more precise measures of worth can be introduced into both system and equipment design decisions.

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*A Slit-Scan Electro Optical Rectifier**

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ABSTRACT: *A unique machine is described which has the capability of rectifying practically all types of oblique aerial photography. The rectifier takes advantage of optical projection for transforming the photographic information, and electronic computer controlled distortion for achieving dimensional restitution. By combining these two techniques a machine results which produces rectified photographs having very high resolution, excellent dimensional characteristics, high speed of operation, and flexibility in regard to the types of oblique photography which can be processed.*

Oblique frame, vertical panoramic, and oblique slit scan photography can be rectified. Oblique panoramic can be rectified by special adaptation. A large range of focal-lengths and oblique angles can be accommodated by the rectifier. A resolution figure of 80 lines-per-millimeter should be achieved in the rectified photograph with speed of operation being less than 15 minutes for 100 square inches of copy.

INTRODUCTION

THE requirement for improvements in equipment for rectifying oblique aerial photography has become of increasing importance in the past few years. This has come about because of the increased use of a variety of types of aerial photography and of the high resolution being obtained. In addition the requirement for reconnaissance and mapping has increased substantially.

Historically, rectification equipment has been limited to the optical projection type. With a few exceptions only oblique-frame type aerial photography can be processed by

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