*Relative Geometric Strength of Frame, Strip and Panoramic Cameras**

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ABSTRACT: *This paper contains an investigation into the stability of the interior orientation of the frame, panoramic and strip cameras. Mathematical expressions were formulated to relate the measured and calibrated data to ground points in terms of Universal Ground Coordinates. Using the General Law of Error Propagation, a theoretical error analysis was performed to determine the comparative accuracy between camera types when used for target location and charting.*

INTRODUCTION

THE primary objective of this paper is to determine the type of camera system with the strongest internal geometry which will subsequently secure the greatest possible amount of accurate interpretable information per pound of airborne equipment. These data are obtained under conditions where ground-control information is presumed unobtainable.

For the purpose of this paper the interior and exterior orientation parameters are defined as follows:

a) Interior orientation elements of the aerial camera consist of the focal length (f) and the photo-coordinates (x, y, frame camera and η, ϵ, panoramic camera).

b) Exterior elements of orientation are tilt, swing, azimuth of the principal plane, and position and elevation of the camera station.

In order to establish accurate ground position of photo-images without the use of geodetic-control points, it is essential to acquire all of the elements of interior and exterior orientation; some elements must be obtained on the film during the instant of exposure and the remainder from calibration reports. The problem of accurate target location is further complicated by the errors which are inherent in the design of each specialized camera, and the errors which occur in the chemical processing, correlation and data reduction of each photograph.

It is then, the primary intention of this paper to develop the relationships between photo-coordinates and ground-coordinates for the basic frame, strip and panoramic cameras in terms of the parameters discussed above and perform an error analysis on the cameras to determine the strength of their respective internal geometry. The method employed to formulate the coordinate equations for the frame, strip and panoramic cameras are

(1) to develop the frame coordinate equation which will be utilized as the basis for the other camera developments,
(2) to determine the photo-coordinate transformation equations which relate the frame photo-coordinates to the new geometry imposed by the panoramic and strip cameras, and

* Based on work done for Rome Air Development Center under Air Force Contract AF 30 (602)–2132.

(3) to substitute the panoramic and strip photo-coordinate equations into the basic frame camera-coordinate equations which will then generate the respective camera-coordinate equations.

Formulation of the General Frame Coordinate Equations

The basic equations which show the relationship between the photo-coordinates and the ground-coordinates for the frame camera must be developed before an error analysis can be performed. The formulation of these equations are based on three types of data:

(1) the object-space Cartesian coordinate-system for defining the position of point objects including the camera station (X, Y, H-h),

(2) the image-space coordinate system for defining the position of point images (x, y, f) and

(3) the relation existing between the coordinate system of object and image-space in terms of angular orientation (t, s, α), where

X, Y = Local Universal Ground-Coordinates with the origin at the camera nadir and the Y axis aligned with local north.
H-h = terrain clearance,
H = altitude above sea level of camera station,
h = elevation of object on terrain,
x, y = photo-coordinates,
f = focal-length,
s = swing,
t = tilt,
α = azimuth of the principal-plane.

If numerical values for t, s, and α and the image-space coordinates (x, y, f) of an image are assumed, the object ground-coordinates (X, Y) corresponding to the image-space coordinates may be calculated. The solution is based on expressing one coordinate system in terms of the other by use of functions of t, s, and α, as coefficients for the variables x and y. The Universal Ground Coordinate equations may be determined by means of successive rotations of the camera system through the angle of swing, tilt, and azimuth, and subsequent substitution into the standard equations for the vertical photograph.

To commence with the development of the frame coordinate equations the vertical photo-coordinates are first rotated through the swing angle (s) as indicated in Figure 1.

As determined from the geometry of the rotation, the new photo-coordinates embodying the swing angle are

$$x' = x \cos s - y \sin s$$

and

$$y' = y \cos s + x \sin s$$

Referring to Figure 2, the vertical photograph after being rotated through the swing angle is now tilted. As evident from the geometry, the photo-coordinates after undergoing tilt and swing are as follows:

$$x'' = x'$$

$$y'' = f \sin t + y' \cos t$$

and

$$-Z = f \cos t - y' \sin t$$

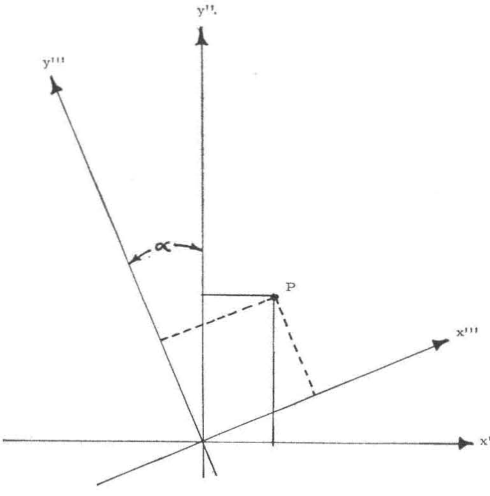

FIG. 1. Swing transformation.

where

Z is a negative quantity for it is measured from L to O. In keeping with the appropriate system of tilt, swing, and azimuth which fully describe the angular exterior orientation of the photograph, it is necessary to rotate the photo-coordinates through the azimuth angle (α) as indicated in Figure 3.

The new photo-coordinates embodying the tilt, swing and azimuth orientated systems are

$$x''' = y'' \sin \alpha + x'' \cos \alpha$$

and

$$y''' = y'' \cos \alpha - x'' \sin \alpha$$

These converted photo-coordinates are now reduced into terms of Local Universal Ground Coordinates by substituting, in turn, the developed photo-coordinate equations into equation (1) and (2) which are readily determined with reference to Figure 4, by application of the principles of similar triangles.

$$X = \frac{(H - h)}{Z} x''' \tag{1}$$

$$Y = \frac{(H - h)}{Z} y''' \tag{2}$$

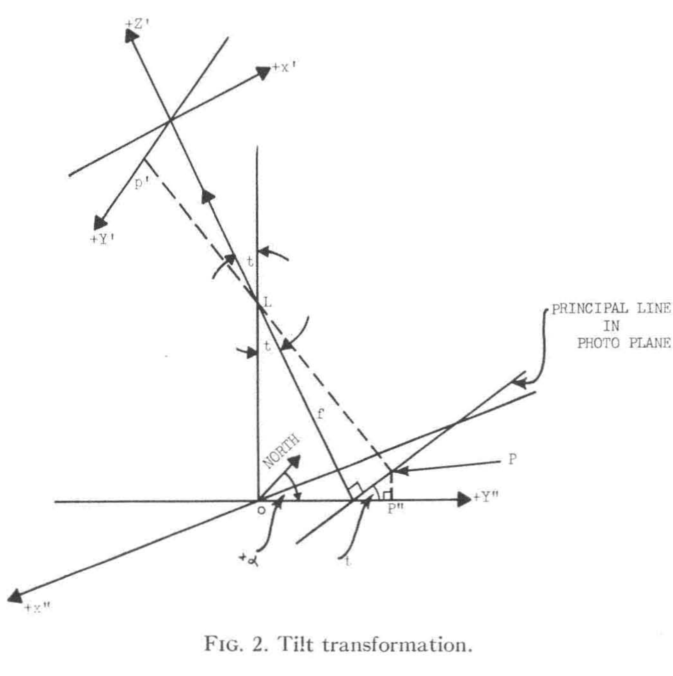

FIG. 2. Tilt transformation.

After substitution, the Local Universal Ground-Coordinate equations for the Frame Camera are:

$$X = (H - h)\left[\frac{(f \sin t + y \cos t \cos s + x \cos t \sin s) \sin \alpha + (x \cos s - y \sin s) \cos \alpha}{f \cos t - y \cos s \sin t - x \sin s \sin t}\right] \quad (3)$$

$$Y = (H - h)\left[\frac{(f \sin t + y \cos s \cos t + x \sin s \cos t) \cos \alpha - (x \cos s - y \sin s) \cos \alpha}{f \cos t - y \cos s \sin t - x \sin s \sin t}\right] \quad (4)$$

It must be reiterated that since the quantities measured from L downward are negative the magnitude of $(H\text{-}h)$ is negative. However, after substitution the negative signs drop out as equations (3) and (4) indicate. Assuming that all of the parameters are known, the above equations are sufficient to determine the ground coordinates of an object. However, the accuracy with which X and Y are determined depends of course, upon the accuracy of each of the parameters. An error analysis of equations (3) and (4) may be performed to establish the accuracy of calculating X and Y and results compared to similar analysis of other equations in order to ascertain relative orders of accuracy.

Formulation of General Panoramic Coordinate Equations

Similar to the frame camera, a prerequisite to the development of the pertinent error equation for panoramic photography is the development of equations and definition of the terms for transforming the general case of the panoramic photo-coordinates into a system of Local Universal Ground-Coordinates. The cylindrical platen rotating lens panoramic conforms to the geometry of the general transformation.

The development of the panoramic camera-coordinate equations are initiated with the determination of the photo-coordinate transformation equation. These equations (5) and (6) transform the plane rectangular coordinates measure on the

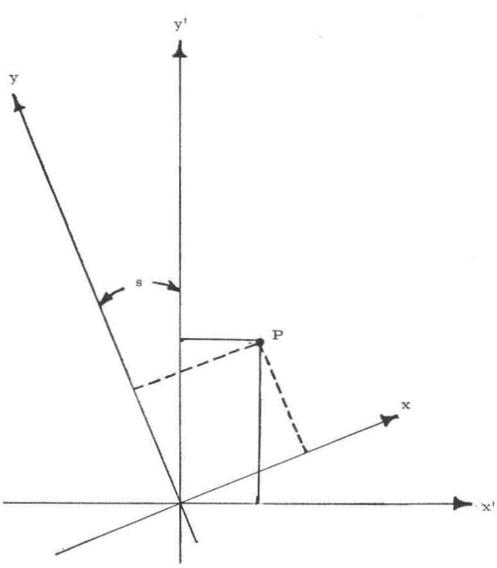

FIG. 3. Azimuth transformation.

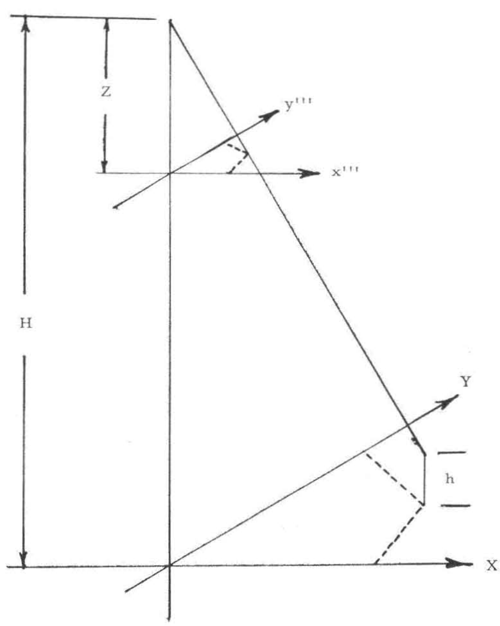

FIG. 4. Transformed photo coordinates.

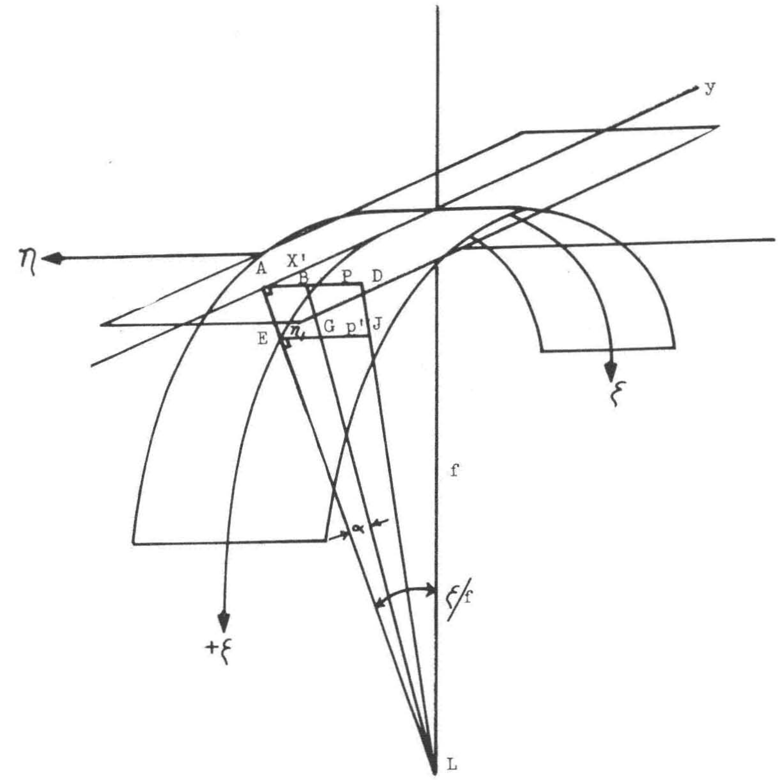

FIG. 5. Transformation from frame to panoramic coordinates.

panoramic photograph to their equivalent perspective position in a plane tangent to the panoramic cylinder at the line chosen as the center of mechanical scan.

Then as evident from Figure 5, triangle ABL is similar to triangle EGL.

It follows that

$$\frac{x}{\eta} = \frac{f \sec \epsilon/f}{f}$$

and

$$x = \eta \sec \epsilon/f \tag{5}$$

It is also evident that

$$y = f \tan \epsilon/f \tag{6}$$

where η-panoramic photo-coordinate axis, passing through the principal-point and parallel to the axis of the panoramic cylinder (negative in the direction of flight), ϵ-panoramic photo-coordinate axis normal to the η photo axis and on the surface of the panoramic negative (positive to the right of the flight direction) and f the calibrated focal-length measured along the normal to a plane tangent to the panoramic cylinder between the cylinder and the exit node of the lens. The center of the mechanical scan was arbitrarily chosen as one origin for the transformed photo-coordinate system. Airborne sensors providing angular data for eventual use in the data reduction process require a system of references with respect to the camera. The center of scan is chosen as one convenient reference for this purpose. In cases of oblique photography, another origin may be more convenient as a reference for external sensors.

The results of equations (5) and (6) are used directly in equations (3) and (4) for reducing the coordinates into panoramic Local Universal Ground Coordinates.

The panoramic coordinate equations relating the given data and transformed photo-coordinates considering tilt, swing and azimuth are as follows:

$$X = (H - h)\left[\frac{(f \sin t + f \tan \epsilon/f \cos t \cos s + \eta \sec \epsilon/f \cos t \sin s) \sin \alpha}{f \cos t - f \tan \epsilon/f \cos s \sin t - \eta \sec \epsilon/f \sin s \sin t} + \frac{(\eta \sec \epsilon/f \cos s - f \tan \epsilon/f \sin s) \cos \alpha}{f \cos t - f \tan \epsilon/f \cos s \sin t - \eta \sec \epsilon/f \sin s \sin t}\right] \quad (7)$$

$$Y = (H - h)\left[\frac{(f \sin t + f \tan \epsilon/f \cos s \cos t + \eta \sec \epsilon/f \sin s \cos t) \cos \alpha}{f \cos t - f \tan \epsilon/f \cos s \sin t - \eta \sec \epsilon/f \sin s \sin t} + \frac{(\eta \sec \epsilon/f \cos s - f \tan \epsilon/f \sin s) \sin \alpha}{f \cos t - f \tan \epsilon/f \cos s \sin t - \eta \sec \epsilon/f \sin s \sin t}\right] \quad (8)$$

The above equations differ from the frame-coordinate equations only in that they account for the new geometry imposed by the concept of the panoramic camera. As is evident from the above equations, to locate a ground point the tilt, swing and azimuth must be known for a given scan angle. As the optical axis rotates to image the most oblique rays the exterior orientation of the camera changes because of the motion of the vehicle. In contrast, the frame-camera equation, optimum in simplicity, needs only one set of t, s, and α to locate all points on the photograph.

Formulation of the Strip Camera Coordinate Equations

The inherent design concept of the strip-camera distinguishes itself from the basic frame and panoramic-cameras in that there exist the problems associated with moving film. The strip-camera is essentially an uncomplicated camera. Basically, it includes a lens, lens cone, variable exposure slit and motor drives that move the film past the exposure slit at a synchronous rate to V/H, the ratio of aircraft velocity to height above terrain. However, it is in this film movement where errors are generated which cause image displacements and deteriorate the accuracy of target location. It is probable that the film travels past the exposure slit in irregular non-uniform motions. This unpredictable movement can originate from many causes, some of which are varying roller radius, unprecise motor film drives and out-of-tolerance sprocket drives and holes.

The equations for transforming the coordinates measured on the strip type photography into equivalent coordinates of a Universal Coordinate System, must incorporate the image displacement caused by the inherent irregular film movement. A technique to mathematically determine the non-uniform and irregular motion of the film is to expose markers on the film as it passes the exposure slit. The markers are imaged on the film at a precise time rate. The time-reference mark device is mounted at the ends of the exposure slit rigidly fixed with respect to the lens, and the error associated with the time-flash device shall be considered negligible. Deviation of the markers in any direction from the predetermined positions will indicate irregular film movement. The transformation equations (equations 7 and 8) for the strip camera are developed as follows:

Referring to Figure 6, reference sets A and B are marks flashed on film as it passes over the exposure slit. Each set of marks is flashed on simultaneously at a constant time interval. Any twisting or nonuniform motion of the film will be shown by a displacement of the reference marks from the calibrated positions.

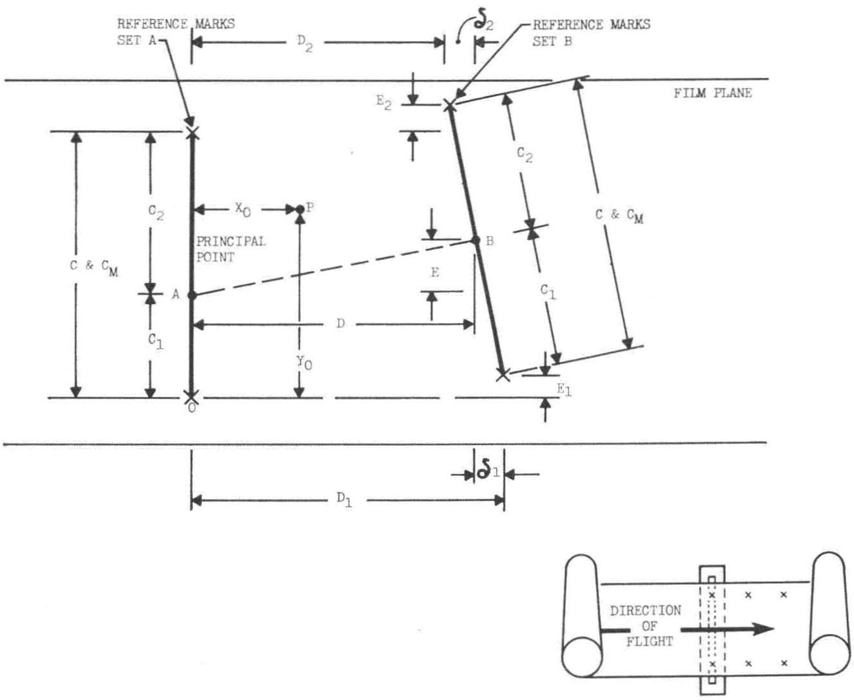

FIG. 6. Transformation from frame to strip coordinates.

C = calibrated distance between reference marks
C_m = measured distance between reference marks
C_1 & C_2 = calibrated distances from principal-point to reference marks
d_1 & d_2 = distance between adjacent reference marks
e_1 & e_2 = displacement of reference mark in the vertical direction
δ_1 & δ_2 = displacement of reference mark in the horizontal direction
a = principal point
b = new principal point due to movement of film
e = displacement of principal point
T = calibrated time interval between pulses A and B
T_a = real time that reference mark (Set A) is exposed (this time is given)
T_p = the instant of exposure of point p
x_0 & y_0 = measured distance from coordinate system which has O as origin
x & y = distances from coordinate system which has principal-point as origin

Starting the development we have

$$d = \frac{d_1 + d_2}{2} \qquad \text{(approx.)}$$

and

$$\delta_1 = \frac{d_1 - d_2}{2}$$

$$\delta_2 = \frac{d_2 - d_1}{2} \quad \text{or} \quad \delta_1 = -\delta_2$$

y is then

$$y = \frac{C}{C_m}\left(y_0 - e\frac{x_0}{d}\right) - C_1$$

where

$$e = \frac{e_1 + e_2}{2}$$

Then substituting for e and d we get:

$$y = \frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1 \tag{9}$$

For simplicity it is assumed that the longitudinal and transverse shrinkages are the same.

Now to develop the photo x coordinate for the strip camera we have

$$x = \frac{C}{C_m}\left[x_0 + \delta_1\left(\frac{x_0}{d}\right)\frac{\left(y_0 - \frac{C_m}{2}\right)}{\frac{C_m}{2}}\right]$$

Substituting for δ^1 and x_0 we get:

$$x = \frac{C}{C_m}\left[x_0 + \frac{(d_1 - d_2)}{2}\;\frac{2x_0}{(d_1 + d_2)}\left(\frac{2y_0}{C_m} - 1\right)\right]$$

and

$$x = \frac{C}{C_m}\left[x_0 + 2x_0\frac{(d_1 - d_2)}{(d_1 + d_2)}\left(\frac{y_0}{C_m} - 1/2\right)\right]$$

Substitute and simplifying:

$$x = \frac{x_0 C}{C_m}\left[1 + 2\frac{(d_1 - d_2)}{(d_1 + d_2)}\left(\frac{y_0}{C_m} - 1/2\right)\right] \tag{10}$$

Another fundamental distinction between strip-camera photography and frame type conventional photography for photogrammetric purposes lies in its instantaneous two-dimensional rather than three-dimensional "freezing" of the object space. Measurements made normal to the two dimensional perspective result effectively in measurements of time and not of image space. This basic characteristic of the strip camera requires that the x photo-coordinate measurement be used to derive the origin of the ground photo-coordinate system as a function of the direction and velocity or other vehicle positional parameters. An example of time to the position may be had in orbiting type vehicles when, after data "smoothing," the position of the vehicle may be predicted or interpolated as a function of orbital time. Assuming the strip-photo references are flashed with reference to orbital time, the value of T_p may be used to calculate an instantaneous origin of the ground coordinate system.

Substituting the transformation equations directly into equations (3) and (4) will generate the Universal Ground-Coordinate equation characteristic of the strip camera. Since the camera system is constantly moving there is an instantaneous nadir for every y. Therefore $x=0$ and the Universal Ground-Coordinate equations for the strip camera reduce to:

$$X = (h - h)\left\{\frac{f \sin t \sin \alpha + \left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\cos t \cos s \sin \alpha}{f \cos t - \left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\cos s \sin t}\right.$$
$$\left. - \frac{\left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\sin s \cos \alpha}{f \cos t\left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\cos s \sin t}\right\} \tag{11}$$

and

$$Y = (H - h)\left\{\frac{f \sin t \cos \alpha + \left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\cos s \sin t \cos \alpha}{f \cos t - \left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\cos s \sin t}\right.$$
$$\left. - \frac{\left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\sin s \sin \alpha}{f \cos t - \left[\frac{C}{C_m}\left[y_0 - x_0\frac{(e_1 + e_2)}{(d_1 + d_2)}\right] - C_1\right]\cos s \sin t}\right\} \tag{12}$$

Error Analysis

The frame, strip and panoramic equations previously developed permit a detailed error analysis which compares the target location accuracy and stability of the interior orientation of the respective cameras. The error analysis was accomplished by applying the General Law of Error Propagation to the Local Universal Ground-Coordinate equations.

The law of Error Propagation[1] asserts that for any given function of two or more independent variables, say $f(x, y)$, the total differential is given by

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

If there are substituted for the infinitesimal increments their finite equivalents, we obtain

$$\Delta f = \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y \tag{13}$$

which is a good approximation of the magnitudes if the increments are small, and the function is approximately linear at the point where the derivatives are evaluated. Since the errors are independent, it is true that

$$\sigma_0{}^2 = \Sigma C_i \sigma_i{}^2 \tag{14}$$

where $\sigma_0{}^2$ is the variance of the overall errors and σ_i refers to the component errors.

[1] Goode, Harry H., "Systems Engineering," McGraw-Hill Book Co., N. Y. 1957.

Then, combining equations (13) and (14) with the partial derivatives evaluated at the point being the C_i, to yield

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 (\sigma_x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\sigma_y)^2 \tag{15}$$

Using the above method, one could analyze several parameters of a complex function to determine how an error in each variable will affect the final result. This procedure also indicates the relative rate of change of the function with respect to each variable. This is a most important consideration for when systems are analyzed on the basis of operational experimentation it almost invariably appears that more effort could profitably have been spent on one phase than on another phase that ultimately contributed very little to the system error. Theoretical analysis permits determination of the comparative effects of the various numbers of the internal geometry of the respective cameras, and indicates which factors require the most consideration and design effort.

Utilizing equations (3, 4, 7, 8, 11 and 12) as the basis for the formulation of the error equations, the partial derivatives of each of the equations were taken with respect to each of the variables contained in the expressions. The determination of the partial derivatives is a straight-forward exercise and is not included herein. The squares of the partial derivatives were in turn multiplied by the individual square of the variance term. The squared product of the two were then summed and the square root obtained. For example, the error equations for the frame-camera are

$$M_X^2 = \left(\frac{\partial X}{\partial H} dH\right)^2 + \left(\frac{\partial X}{\partial h} dh\right)^2 + \left(\frac{\partial X}{\partial t} dt\right)^2 + \left(\frac{\partial X}{\partial s} ds\right)^2 + \left(\frac{\partial X}{\partial \alpha} d\alpha\right)^2 + \left(\frac{\partial X}{\partial x} dx\right)^2 + \left(\frac{\partial X}{\partial y} dy\right)^2 + \left(\frac{\partial X}{\partial f} df\right)^2 \tag{16}$$

and

$$M_Y^2 = \left(\frac{\partial Y}{\partial H} dH\right)^2 + \left(\frac{\partial Y}{\partial h} dh\right)^2 + \left(\frac{\partial Y}{\partial f} df\right)^2 + \left(\frac{\partial Y}{\partial x} dx\right)^2 + \left(\frac{\partial Y}{\partial y} dy\right)^2 + \left(\frac{\partial Y}{\partial s} ds\right)^2 + \left(\frac{\partial Y}{\partial t} dt\right)^2 + \left(\frac{\partial Y}{\partial \alpha} d\alpha\right)^2 \tag{17}$$

The error equations for the panoramic and strip cameras are similarly developed considering the new geometry. Thus, the numerical results of the error equations stemming from the application of the General Law of Error Propagation provides the theoretical inherent error which exists in compiling charts or in locating targets for each camera.

In order to establish a comparison of the geometric strength between the frame, strip and panoramic camera types it was necessary to assume reasonable numerical values for the interior and exterior orientation parameters. These values were substituted into the three sets of partial derivatives shown in the above equations. An additional procedure to assure an accurate comparison was to have the ray of the point imaged on the film plane leave the datum at a 45° angle for all three camera types. This technique enabled the photo-coordinates to be determined for the given set of conditions and subsequently substituted along with the other assumed parameters into the partial derivative equations.

The values of the individual error terms, dH, dt, ds, etc., used in the error analysis of the cameras are theoretically predicted errors and are as follows:

$$dH = 200 \text{ ft.}$$

$$dh = 20 \text{ ft.}$$

$$ds = 6 \text{ minutes} = .0017 \text{ radians}$$

$$dt = .5 \text{ minutes} = .00014 \text{ radians}$$

$$d\alpha = 6 \text{ minutes} = .0017 \text{ radians}$$

$$df = 25 \text{ microns} = 984.3 \times 10^{-6} \text{ inches}$$

$$\left.\begin{matrix} dx \\ dy \end{matrix}\right\} = \text{error in measuring photocoordinates} = .001 \text{ inches}$$

In choosing error terms it was necessary to keep the interior orientation errors relatively large to assure that their influence was apparent in the squared partial terms. The exterior orientation errors were kept small to assure that the effects of errors in interior orientation would be evident.

CONCLUSION

Table 1 contains a summary of the results obtained from the numerical evaluation of the error equations.

TABLE 1

RELATIVE STRENGTH OF CAMERA GEOMETRIES

Standard Errors Based on the Following Parameters

Tilt	3°
Swing	30°
Azimuth	0°
Altitude	101,000 Ft.
Focal Length	6″
Elevation	1,000 Ft.

Local Universal Ground Coordinates	*Frame Camera*	*Panoramic Camera*	*Strip Camera*
X-Coordinate	202 Ft.	220 Ft.	230 Ft.
Y-Coordinate	191 Ft.	221 Ft.	235 Ft.

The results in Table 1 were determined for the various camera types using the same conditions of observation and the same component error values. In an effort to make the effects on ground error as a function of camera types more apparent, the error values for factors of exterior orientation were kept small and equal for corresponding elements.

The error values assigned to elements of interior orientation were relatively large but correspondingly equal. In this manner, the relative accuracy of the camera types was made more apparent. It is important to note that the results indicated in Table 1 provide a comparison of the strength of the respective camera interior geometry and not a comparison of the camera under operational conditions. Therefore, the numerical values are insignificant except for the difference between them. For a more nearly operational comparison it will be necessary to evaluate the camera types using realistic error values rather than correspondingly equal values for errors in elements of interior orientation.

Table 1 then indicates that the theoretical strength of the solution, and subsequently the best target location capabilities, are greatest for the frame type, followed by the panoramic and the strip cameras respectively.

For the X-coordinate the percentage error in locating targets between frame and panoramic camera is 8.92%, between the frame and strip 13.87% and 4.73% between the panoramic and strip camera. For the Y-coordinate the percentage error between the frame and panoramic is 15.77%, between frame and strip 23.13% and 6.36% between the panoramic and strip.

There are a number of exterior orientation elements which have not been considered in the camera comparison; however, they are factors which affect the system error to the same extent regardless of camera type. Since one purpose of this investigation is an accuracy comparison between camera types, these added elements have not been considered.

*The Utilization of Constraints in Analytical Photogrammetry**

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(Abstract is on next page)

INTRODUCTION

WITH the introduction of high speed, large capacity digital computers, the possibility of solving elaborate problems in analytical photogrammetry, in particular the simultaneous adjustment of large blocks of aerial photographs in a control extension, has now become a reality. A number of techniques that adequately express the projective relations of photogrammetry have gained wide acceptance in recent years; however, most of these techniques have limited themselves to the case in which the camera and ground-point parameters are exactly known or completely unknown. It is the purpose of this paper to describe a number of constraints and their application to some of these techniques.

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Such constraints are obtained by enforcing the camera or points in the object space to conform to some functional or geometrical relationship or to lie within certain bounds as defined by weighting. The value of constraints lies in their ability (1) to utilize geometrical properties of the physical situation which may lead to a reduction in the number of unknown camera or ground-point parameters and, hence, to a reduction in the size of the normal equation matrix which must be inverted; and (2) to utilize these same geometrical properties and/or weighting fac-

* Presented at the Society's 27th Annual Meeting, The Shoreham Hotel, Washington, D. C., March 19–22, 1961.

† Subsequent to the Annual Meeting, the author became a member of the staff of Autometric Corporation, Alexandria, Va.