*Precision Photogrammetry a Tool of Geodesy**

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If one may consider photogrammetry to be a geodetic mensuration technique, as it is in Europe, then photogrammetry, by definition, is a tool of geodesy and one may wonder about the logic of the title of this paper. However, generally speaking, the application of photogrammetry for geodetic purposes has been limited to such problems as are conventionally associated with the terminology of surveying and topographic mapping.

In line with the subject of this session, "New Photogrammetric Systems" I do not intend to talk about the traditional use of photogrammetry. I should like to elaborate somewhat on the potential which photogrammetry, in my opinion, offers in connection with methods which are presently almost undisputed in the domain of classic geodetic triangulation.

It is easy to accept the expression: "Nothing succeeds like success." Now, it is exactly the lack of success by which photogrammetry is plagued when one hopes to establish precise geodetic control by the methods of strip and block triangulation. That is not saying that such data have not been successfully obtained for the purpose of the associated photogrammetry mapping procedure. But such control data definitely do not have the same properties as those obtained by conventional geodetic triangulation.

If photogrammetry intends to contribute to the problem of establishing geodetic control, it is necessary to develop an approach to the photogrammetric problem which in all of its individual steps equals in quality similar operations executed painstakingly during traditional geodetic triangulation work. The compatibility has to be assured in regard to the basic principles of an exact measuring method as well as to the problem of quality control by rigorous statistical means. To expect that such a method will emerge from the organic development of present-day photogrammetric restitution techniques, at the best, will delay unnecessarily the assistance which photogrammetry can give to the interest that was lately revived in three-dimensional geodetic triangulation.

The critical evaluation of any exact measuring system demands a rigorous mathematical model which serves as the basis of a least squares adjustment of the required excess observations. Furthermore, all individual steps encountered in the phase of the data acquisition, as well as in the process of the data evaluation, have to be investigated for the purpose of determining the metric quality of the individual components entering into a specific measuring system. Only by determining the internal accuracy of the individual instrumental components and environmental influences will it be possible to estimate in a significant way the unavoidable individual random errors.

The rigorous statistical treatment in connection with a clearly defined mathematical model will then enable making the study of the propagation of these random errors into the final result. Finally, the critical evaluation of actual full-scale experiments will provide the information by which the hypothesis of the measuring method can be validated or improved by the detection and isolation of unexpected bias errors.

An analysis of the method of geodetic triangulation will clearly demonstrate the close adherence of the geodetic triangulation practice to the aforementioned principles.

Let us now discuss some of the consequences which one has to accept if one wants to develop the photogrammetric method as a tool of geodetic control triangulation.

Photogrammetry has—with justified pride—always proved to be a method independent of the need of any surface activity within the area to be surveyed. The resulting consequences are the problems of pass-point identification and transfer. One may consider stereoscopic viewing as an adequate tool for identification and furthermore, one can hopefully anticipate the development of image correlation techniques to provide an automatized solution to the problem of pass-point transfer. Nevertheless, the fact must be taken

* Paper presented at the Semi-annual Convention of the American Society of Photogrammetry, New York, October 1961.

into account that even the most accurately identified, and accordingly accurately determined, point represents for geodetic control purposes an entirely useless triple of coordinates, as long as it is not possible to correlate these coordinates with a specific permanently existing physical feature in nature. As a matter of fact, this identification must be possible and well within the limits of the mean errors of the correspondingly computed coordinates. In short, it will be necessary to develop an effective and economical method for establishing suitable markers, thus reducing, for the sake of precision, the target identification problem during the process of evaluating the photogrammetric record, and at the same time enabling the coordination of the computed triangulation results with a specific location in the field.

Obviously, the camera is the most important instrumental component in any photogrammetric system. The over-all quality of modern cameras has been proven to be satisfactory, provided the photographs are evaluated by conventional restitution techniques. However, the question arises: "Can such cameras be considered adequate for the purpose under discussion?" Results of preliminary calibration tests indicate a cautious "no." Figure 1 shows the radial distortion components of a modern lens, indicating clearly an elliptical distortion pattern. For the purpose of point triangulation, a strictly numerical evaluation method must be considered. Therefore, such a pattern can be simulated by simple analytical terms during the reduction of the corresponding comparator measurements. Considerably more difficult is the correct compensation for the tangential distortion components as shown in Figure 2. No simple analytical expression can adequately simulate the complex topography. However, over limited areas—no more than one centimeter wide—changes of as much as 10–12 microns are shown in the tangential distortion components. Such amounts cannot be ignored if one considers the necessity for maintaining, for our purposes, the over-all accuracy of the coordinate measurements to about ± 3 microns. Modern photogrammetric lenses offer satisfactory resolution and light distribution characteristics for recording point sources or high contrast targets, considering available types of emulsion. Similarly, the corresponding theoretical radial distortion patterns are acceptable even for highest precision requirements. The individual physical samples of a certain lens type, however, seem to suffer from the lack of a sufficiently accurate manufacturing and assembly control. The use of the photogrammetric method for triangulating the space positions of in-

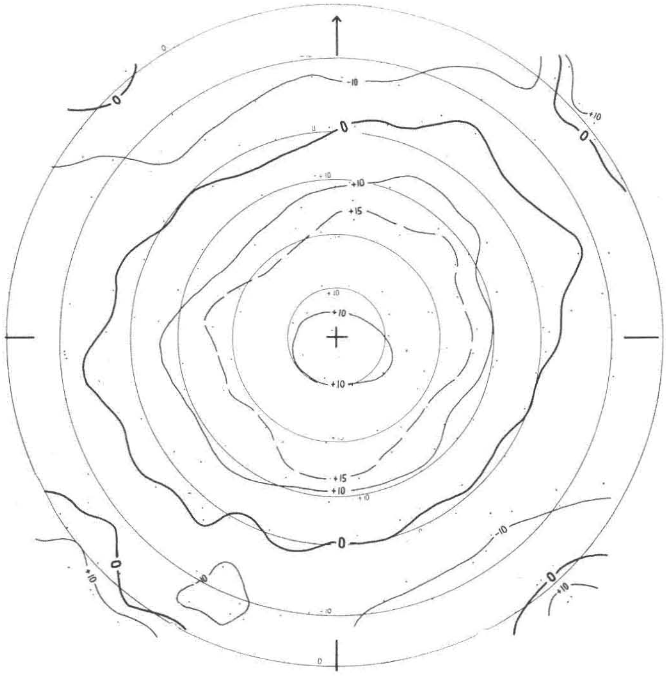

FIG. 1. Radial distortion components of a modern lens. Astrotar BC-4 No. 265.

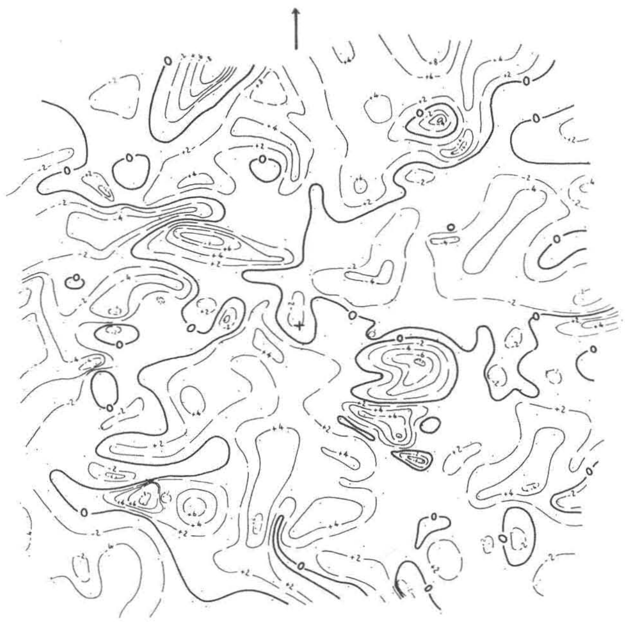

FIG. 2. Tangential distortion components. Astrotar BC-4 No. 265.

dividual flash points in connection with the geodetic satellite program, can be successful only if lenses can be made available which, above all, are manufactured to tolerances guaranteeing the geometric fidelity of the bundle of light rays, not only in an over-all average sense, but with respect to any one of the rays within the angle of view.

The significance of the dimensional stability and flatness of the emulsion carrier is obvious. For the problems under consideration, no real disadvantage of operation can be mentioned which is not outweighed by the gains from the use of sufficiently thick and flat glass plates.

The sources of concern just mentioned, namely the lens and the emulsion carrier, are only two of the critical components which influence the metric quality of the photogrammetric record. The most careful laboratory calibration is useless if the dimensional stability of the camera body is not assured within the wide range of environmental conditions encountered in practice. All these problems point to the necessity of investigating and evaluating with respect to precision and stability, the individual components of the recording camera which is the basic tool of photogrammetry. The significance of such a program becomes obvious if one recalls the effort which has been made in the investigation of all of the components which comprise a theodolite in terms of internal precision, stability and over-all operational accuracy. Because the photogrammetric determination of directions in space can not, like the corresponding theodolite measurements, be executed by a null-method which eliminates any remaining bias errors, the dependence of precision photogrammetry on component research can not be overemphasized.

The effort initiated by Coast and Geodetic Survey and apparently supported by numerous agencies and private concerns to establish a highly precise and well-targeted photogrammetric test area, is a significant and absolutely necessary step for the development and calibration of precise photogrammetric triangulation systems.

The importance of high-quality raw data becomes even more evident when there is considered as a third area, the data evaluation problem. It is in this phase that the most drastic reorientation is necessary. The task of data reduction must not be seen as an alchemic activity where one can regain by mathematical manipulation what has been lost during the preliminary phase of data acquisition. All that the best data evalution process can do is to come up with the best answer, the accuracy of which is predetermined by the laws of propagation of the errors inherent in the basic data, in accordance with the particular geometry of the specific case of triangulation.

The avoidance of numerical computations

has become an outstanding characteristic of the photogrammetric evaluation technique. Thus computations of the photogrammetric triangulations are performed with optical-mechanical analogue computers—the wide variety of photogrammetric restitution equipment—which are simulators so far as they are built according to the so called rigorous principle. The most accurate instruments of this type are capable of operating with an internal accuracy of 1 part in 40,000, thus being just accurate enough to do justice to high quality photogrammetric records.

This high degree of precision has been obtained by sacrificing essential flexibility, thus limiting the absolute accuracy of the triangulation result by the inability to cope with the various perturbations affecting the photogrammetric triangulation process, predominantly refraction and asymmetric lens distortion. Likewise, the definition of the condition of relative orientation as the intersection of five pairs of associated rays is correct only for bundles generated according to the geometrical concept of central perspective. In reality, the light rays are, due to refraction, lines curved in space, and if during the process of restitution these lines are replaced by their tangents at the center of projection, the condition of intersection is obviously no longer a valid criterion for a correct relative orientation.

The real reason for rejecting the analogue approach to the problem of precision photogrammetric control point triangulation, however, is more basic than is indicated by the aforementioned limitations.

It is a fact that photogrammetry, as a measuring technique was developed from a predominantly geodetic background. Therefore, it is somewhat difficult to understand the almost stubborn devotion with which the photogrammetric community has pursued the problem of triangulating control data on first-order plotters using exclusively the method of the "Folgebildanschluss."

Decades ago, theoretical studies concerned with the propagation of random errors in geodetic triangulation systems explained why a triangulation chain, despite the geometrical strength of its individual figures, is not an instrument for carrying either direction or scale with any appreciable degree of accuracy over extended distances.[1] In other words, angular triangulations suffer from unfavorable error propagation. In geodesy the correct conclusions were drawn: any extended triangulation chain, despite the rigorous least squares adjustment, must be supported by (a) independent astronomical azimuth observations in order to control the unavoidable bending of the chain and by (b) independent base line measurements for the purpose of reducing the intolerable accumulating scale error.

Photogrammetry, in the early stage, may have been influenced by systematic errors to such an extent that to investigate the propagation of random errors was felt unnecessary. But only if one assumes that the bias errors in our basic observations are sufficiently small—a situation which holds in modern precision photogrammetry—can the triangulation of worthwhile control data be envisaged. Nevertheless, photogrammetry has not shown any signs of being affected by the findings of its colleagues in the field of geodetic triangulation. The method of the "Folgebildanschluss" is an incorrect least squares adjustment comparable to a solution where each individual figure in a geodetic triangulation chain is adjusted independently, instead of allowing the unrestricted flow of the random errors by an adjustment of the triangulation system as a whole. In addition, the relative orientation which is the basis for the principle of photogrammetric triangulation is, geometrically speaking, not a strong configuration. To make things worse, the use of only six points in each step of triangulation, where five constitute the unique solution, is next to no adjustment at all. If this chain of weak steps is being considered, in the light of the aforementioned basic limitation of any triangulation based solely on angular measurements, then it should be clear that genuine photogrammetric aerial triangulations will not and cannot work.

Based on studies of error propagation in geodetic triangulations Prof. Gotthardtt[2] showed, almost 20 years ago, that the photogrammetric triangulation method was suffering from conditions which could, if separated from the complex geometry, be explained by the pseudo-systematic character of the double summation curves of randomly distributed errors.

The practice of photogrammetry, however, did not interpret this result by acknowledging its dependence on additional direc-

[1] Compare "Die Übertragung von Richtungen in weite Fernen" by Prof. Dr. Ing. A. Berroth, Aachen, 1938.

[2] "Der Einfluss unregelmässiger Fehler auf Luftbildtriangulationen" *Z. f. Vermessungswesen*, 1944.

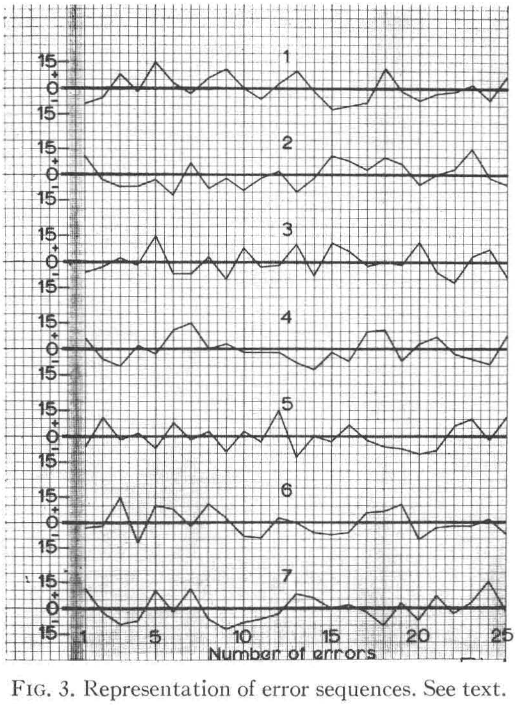

FIG. 3. Representation of error sequences. See text.

tional and scaling parameters, but misused the pseudo-systematic character of the double summation curves by trying to improve the triangulation results by smoothing techniques using empirically conceived high order functions.

Let me demonstrate two approaches which in my opinion show the uselessness of these attempts.

Accepting the resemblance of the error propagation in aerial triangulation to the double summation curves of random errors, a group of 25 errors[3] which approximate a normal distribution were selected and arranged by a method of chance in seven different sequences. Figure 3 shows the corresponding graphical presentation of these error sequences.

For each of the seven error sequences, the double summation curves were computed and plotted as shown in Figure 4. The chosen population of 25 errors is obviously a small sample and similarly the number of seven different arrangements will render only a limited amount of information. Nevertheless, this limited investigation demonstrates (a) that, undoubtedly, each of the double summation curves displays a certain systematic trend, but (b) that it is not possible to express a specific curve by a pre-selected function. For example, despite the equality of accumulated errors at the end of the sequence, the curves No. 3 and No. 5 follow entirely different courses; and the curves No. 2 and No. 7, although coinciding almost entirely in the beginning, later show widely diverging patterns.

The arrival of electronic computers revived the interest in the numerical evaluation of photogrammetric records. Thus, for the first time, it became possible to replace the incorrect principle of the Folgebildanschluss by a rigorous least squares adjustment, the treatment of the entire configuration of a photogrammetric triangulation as a whole. At the same time, the numerical approach to the general photogrammetric problem made possible incorporating in a rigorous statistical manner all independent control data. The outstanding features of a rigorous least squares adjustment could thus be assured for the evaluation of complex photogrammetric triangulations. Obviously, such a result represents the most probable answer, thus by definition making unnecessary and senseless trying any further improvement by mathematical means. In addition, the corresponding expressions for the mean errors of the triangulated points are available, thus permitting deriving, by strictly numerical analysis, the laws of error propagation free from any hypothetical assumptions, aside from the

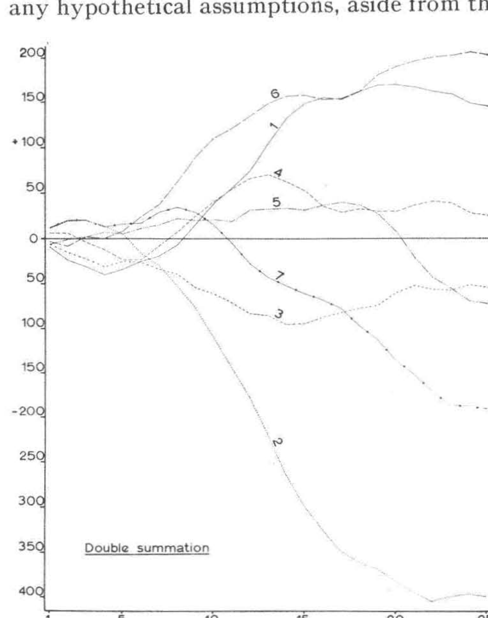

FIG. 4. Double summation curves for the seven error sequences. See Fig. 3 and text.

[3] Taken from *Photogrammetrie* by Prof. Dr. Finsterwalder, page 239.

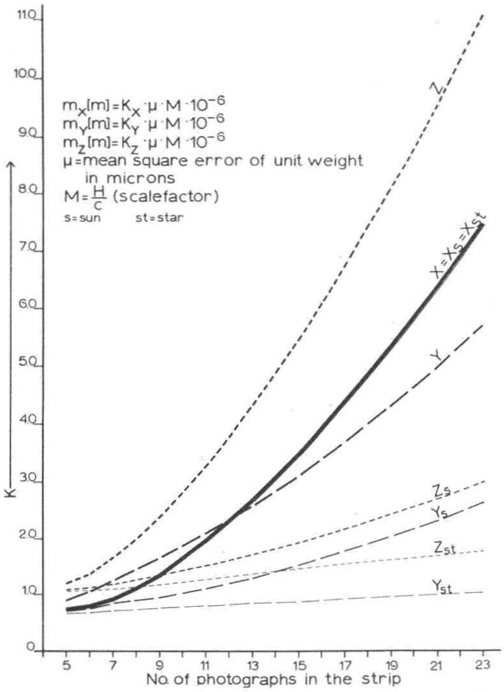

FIG. 5. Maximum mean errors for XYZ coordinates at center of strip as a function of various strip lengths. See text.

obvious necessity of accepting the validity of the least squares principle.

As an example, Figure 5 shows for a strip of vertical photography with control at the beginning and end, and flown with $\frac{2}{3}$ overlap, the maximum mean errors for the XYZ coordinates at the center of the strip as a function of various strip lengths. These curves demonstrate the aforementioned unfavorable accumulation of errors. The conclusion which must be drawn from this result is that the maximum length an unaided photogrammetric strip can have is only the length of two adjacent non-overlapping photographs. This obviously is only a sophisticated way of expressing the fact that the method of photogrammetric strip and block triangulation is geometrically too weak to be useful for the purpose of triangulating precise geodetic control data in extended triangulation schemes.

Relief from this situation can only be obtained, as mentioned earlier, by providing additional independent directional and scale information. The curves denoted by $X_s Y_s Z_s$ and $X_{st} Y_{st} Z_{st}$ show the beneficial influence of additional sun and star photography, respectively. The bending of the strip is drastically reduced as evidenced by the improvement of the Y and Z coordinates. Scale as expressed by the mean error of the X coordinate, however, remains unchanged.

Figure 6 shows the result, assuming an independent measurement of the length of an individual ray in each bundle, as might be obtained by APR (Altitude Precision Radar) type measurements. With such a method, the scale along the strip is brought under control as is shown by the improved accuracy of the X coordinates. However, the bending of the strip is not helped as indicated by the only slightly changed Y and Z curves.

Finally, Figure 7 presents the situation with both additional directional and scale information introduced. The corresponding laws of error propagation are now improved for all three coordinates to such an extent that the problem of precise control extension seems within reach.

As basic and important as this information is, in order to appreciate the fundamental requirements for precise extended photogrammetric triangulation, such data are still not sufficient to solve the problem of the optimum photogrammetric system. Certainly, it is essential to learn about the influence of various geometrical patterns resulting from the use of cameras with various angles of views, arranged according to the principles of vertical

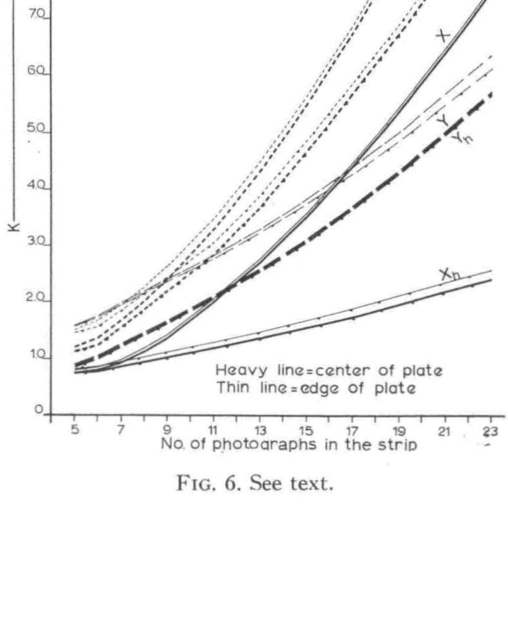

FIG. 6. See text.

and/or convergent photography thus covering a wide range of base height ratios. Equally important is the problem of finding the optimum locations of given control data, as well as the specific precision with which auxiliary data have to be obtained in order to lead to a result optimized in terms of accuracy and economy. Again, numerical analysis, making use of a somewhat modified approach to the least squares problem, makes possible studying these problems in all desirable detail.

The following two examples are chosen to illustrate these possibilities. It is customary to distinguish between cantilever extension having all of the control in the first model, and bridging where the given control is arranged in the first and last models of the strip. Obviously, it is possible to imagine various other control point distributions. Figure 8 shows the accuracy of the Z-coordinate along a strip composed of nine models. The indices 1 to 9 refer to the model in which the third and the fourth control points are situated keeping the first two control points fixed in the first model. The pattern shows clearly that the curve indexed by "7" belongs to the optimum point arrangement. Here the maximum error has its smallest value and, in addition, the area under the curve, which is a fair indication of the overall Z-accuracy of the strip, is far smaller than the corresponding area associated with curve 9 which represents the conventional arrangement of control points in the process of bridging.

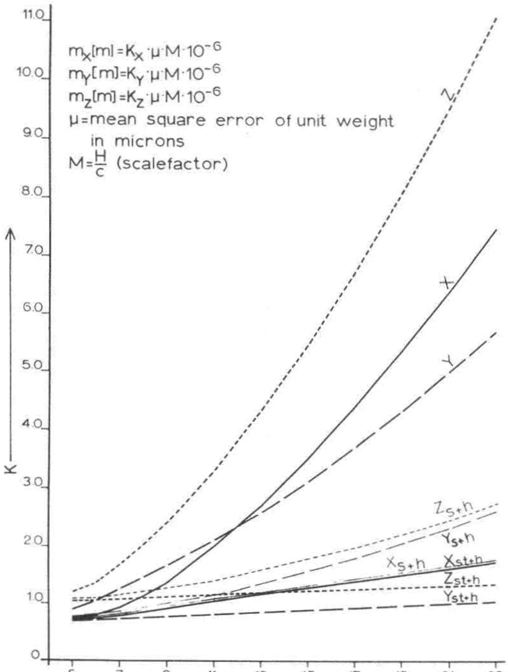

FIG. 7. See text.

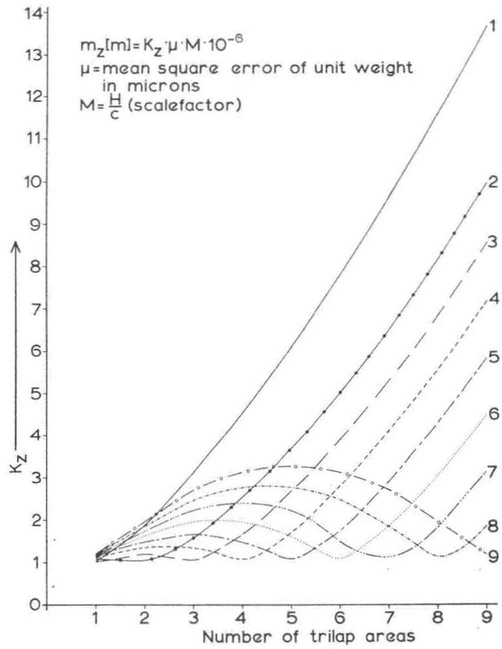

FIG. 8. Accuracy of Z coordinate along a strip composed of nine models.

The other example portrays the following situation. Two airplanes are employed to take photographs in such a way that the photographs are taken simultaneously in pairs along the strip. Between each pair of exposures the spatial distance is assumed to be measured electronically. The basic idea is obviously to introduce independent scale information, similar to the above mentioned APR method. Our problem is to study the influence of this additional auxiliary information assuming the accuracy of these length measurements to change over a wide range.

Figure 9 shows the quite interesting result. The abscissa gives the mean error in X (scale) as a function of the number of photographs in the strip. The ordinate is denoted in terms of the accuracy (weight) of the additional length measurements.

The result indicates that for short strips, let us say to $n=10$, there is hardly any gain by measuring additional scale information. For long strips e.g. $n=50$, a decisive improvement is noticed on comparing the values obtained when the weight equals zero, meaning

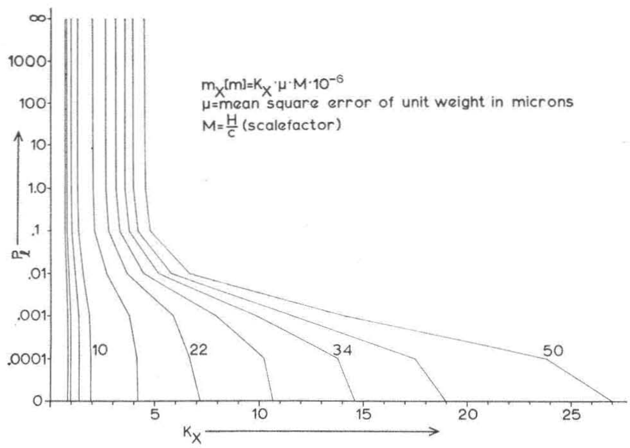

FIG. 9. Abscissa—mean error in X (scale) as a function of number of photographs in the strip. Ordinate—terms of accuracy (weight) of the additional length measurement.

that the auxilary quantity is not measured at all, with the values obtained when the weight equals ∞, indicating a flawless measurement for the additional lengths. The really significant information, however, is (a) that inaccurate length measurements do not contribute much; (b) that an area exists which is greatly benefited by a slight increase in accuracy of the corresponding length measurements, and finally (c) that there is a very definite optimum level for the accuracy of the length information. Any further improvement of the length measurements does not lead to any additional gain in the accuracy of the triangulation result.

It is hardly necessary to stress the importance of this type of information when it comes to optimizing the planning and instrumentation of any specific measuring system.

The examples and the explanations have had to be short in order to fit the allotted time of this presentation. I hope they have been elaborate enough to support the following concluding remarks:

Photogrammetry has, at least in my opinion, the potential of assisting in the establishment of precise geodetic control data. In order to meet the resulting accuracy requirement, photogrammetry has to make an effort to develop from an engineering tool—a term used with highest regard—to an exacting mensuration technique. Numerical analysis, possible by electronic computing, allows one to investigate any anticipated triangulation system in order to optimize the necessary instruments as well as the geometry of the triangulation, for satisfying certain *a priori* established accuracy requirements. Thus the performance requirements obtained for the corresponding measuring instruments will enable the photogrammetric engineer to confront the industry with unambiguous demands for essential measuring equipment. From results already available, it is clearly indicated that a precision triangulation method will depend on the availability of hybrid systems, combining angular and length measuring devices, under optimum conditions which can be determined by corresponding numerical investigations.

Again, making use of the potential of numerical evaluation methods, the individual components of such instrumentation have to be investigated. If they have been found to be consistent with the pre-established accuracy requirements, the execution of precise photogrammetric triangulations will then in essence become a problem of logistics and organization. Thus, it will be possible to guarantee a certain amount of information in accordance with a planned amount of effort.

Those who feel such an approach too extreme have to realize that photogrammetry has already committed itself in connection with certain space projects, especially in connection with the geodetic satellite, to the support of complex triangulation projects, the results of which will only be acceptable if they can withstand the most critical scrutiny as dictated by the principles governing exact measurements.