Levelling the Stereo Model

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ABSTRACT: *A simplified arithmetic routine is described for determining the amounts that a stereoscopic model needs to be tipped and tilted for levelling, based on readings at control points in the four corners. The routine also displays first the amount of model warp which the operator may wish to correct before proceeding any further.*

EXAMPLE

SUPPOSE that the errors in feet in the four corners of a stereoscopic model are represented as

$$\begin{array}{ll} a = +3.6 & 0 \\ b = -0.4 & c = -0.8 \end{array}$$

Then the correction necessary for omega-warp after leveling will be

$$\begin{array}{ll} -0.8 & +0.8 \\ +0.8 & -0.8 \end{array}$$

where -0.8 is obtained by formula

$$\begin{aligned} \omega &= 1/4(b - a - c) \\ &= 1/4(-0.4 - 3.6 + 0.8) = -0.8 \end{aligned}$$

which will be explained presently. If this is too great, it can be corrected before proceeding any further with the computation. Then the amount the near side of the model needs to be raised is

$$x = 1/2(a - b - c) = 1/2(3.6 + 0.4 + 0.8) = 2.4$$

and the amount the left side needs to be raised (actually lowered) is

$$\begin{aligned} y &= 1/2(c - a - b) \\ &= 1/2(-0.8 - 3.6 + 0.4) = -2.0 \end{aligned}$$

EXPLANATION

The operator of a stereoscopic plotting instrument, after completing a procedure called relative orientation, achieves an "absolute" orientation which consists of levelling and scaling the model preparatory for drawing contour lines. These remarks pertain only to the levelling phase.

In levelling a model, an operator observes terrain elevations at the four corners and compares them to known correct ground values, after which he adjusts the attitude of the model to make subsequent observations agree with the known elevations. It is convenient to think of a model as having three degrees of freedom corresponding to appropriate adjusting screws on the instrument:

1. A raising or lowering of the front side of the model relative to the far side;
2. A raising or lowering of the left side of the model relative to the right side;
3. A small x-tilt motion of one projector to correct for one of the four corners not lying in the same plane as the other three.

Finally, a change in datum index is required because the three motions ordinarily leave the model uniformly too high or too low.

The operator can always assign a zero error to one of the four corners by setting his vertical reading index at that point. Therefore in this discussion it is assumed that the error in the upper-right corner is set to zero. If it is not, the same effect can be achieved in either of two other ways:

1. A constant equal to the error in the upper-right corner can be subtracted from each of the four errors, or
2. The tabular system of notation can be rotated cyclicly in all four terms of Equation 2.

Then Equations 4, 5, and 6 apply immediately to indicate the corrections to be accomplished in each of the three manners.

DERIVATION

The discrepancies in elevations in the four corners of the model are shown symbolically as a table of the four numbers, with zero arbitrarily located in the upper right:

$$\begin{array}{ll} a & 0 \\ b & c. \end{array}$$

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The x-correction is similarly indicated as

$$\begin{matrix} 0 & 0 \\ x & x \end{matrix}$$

and the y-correction as

$$\begin{matrix} y & 0 \\ y & 0 \end{matrix}$$

and the omega-warp correction as

$$\begin{matrix} \omega & -\omega \\ -\omega & \omega. \end{matrix}$$

It is desired that the sum of the errors, the three corrections, and the datum change should leave a flat model:

$$\text{error} + \begin{matrix}x\text{-tilt}\\ \text{correction}\\ \text{of model}\end{matrix} + \begin{matrix}y\text{-tilt}\\ \text{correction}\\ \text{of model}\end{matrix} + \begin{matrix}\text{omega}\\ \text{correction}\end{matrix} + \begin{matrix}\text{datum}\\ \text{change}\end{matrix} = \begin{matrix}\text{zero}\\ \text{error.}\end{matrix} \quad (1)$$

If this equation is expressed in symbolic tabular (matrix) notation

$$\begin{bmatrix} a & 0 \\ b & c \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ y & 0 \end{bmatrix} + \begin{bmatrix} \omega & -\omega \\ -\omega & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega \\ \omega & \omega \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (2)$$

This equation implies four simultaneous linear equations in which x, y, ω are unknowns:

$$\begin{aligned} a + y + 2\omega &= 0 \\ b + x + y &= 0 \\ c + x + 2\omega &= 0. \end{aligned} \quad (3)$$

The solution of these equations is

$$x = 1/2(a - b - c) \quad (4)$$

$$y = 1/2(c - a - b) \quad (5)$$

$$\omega = 1/4(b - c - a) \quad (6)$$

which completes the derivation.

If all four terms of Equation 2 are rotated cyclicly the same direction and amount, Equation 4, 5, 6 continue to apply. For example,

$$\begin{bmatrix} b & a \\ c & 0 \end{bmatrix} + \begin{bmatrix} x & 0 \\ x & 0 \end{bmatrix} + \begin{bmatrix} y & y \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -\omega & \omega \\ \omega & -\omega \end{bmatrix} + \begin{bmatrix} \omega & \omega \\ \omega & \omega \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

in which now the sense of x and y is interchanged.

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Although the author has noted others (particularly Dr. B. Hallert and Ing. H. Trager) making equivalent mental calculations, he has not been able to find an expression of these principles in the literature, especially American literature.

Application of Aerial Photographs and Regression Technique for Surveying Caspian Forests of Iran

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(*Abstract is on next page*)

Introduction

IRAN with its 20 million people is starving for forest products while just a few miles from Tehran, the capital city, are virgin hardwood forests. These forests are located throughout the north slopes of the rugged Elburz Mountains facing the Caspian Sea. The major objective is to place these forests under intensive management and to increase the supply of forest products on a sustained yield basis as soon as possible. A secondary objective is to produce maps at 1:50,000 scale for this forest area.