The x-correction is similarly indicated as

$$\begin{matrix} 0 & 0 \\ x & x \end{matrix}$$

and the y-correction as

$$\begin{matrix} y & 0 \\ y & 0 \end{matrix}$$

and the omega-warp correction as

$$\begin{matrix} \omega & -\omega \\ -\omega & \omega. \end{matrix}$$

It is desired that the sum of the errors, the three corrections, and the datum change should leave a flat model:

$$\text{error} + \begin{matrix}x\text{-tilt}\\ \text{correction}\\ \text{of model}\end{matrix} + \begin{matrix}y\text{-tilt}\\ \text{correction}\\ \text{of model}\end{matrix} + \begin{matrix}\text{omega}\\ \text{correction}\end{matrix} + \begin{matrix}\text{datum}\\ \text{change}\end{matrix} = \begin{matrix}\text{zero}\\ \text{error.}\end{matrix} \quad (1)$$

If this equation is expressed in symbolic tabular (matrix) notation

$$\begin{bmatrix} a & 0 \\ b & c \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ y & 0 \end{bmatrix} + \begin{bmatrix} \omega & -\omega \\ -\omega & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega \\ \omega & \omega \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (2)$$

This equation implies four simultaneous linear equations in which x, y, ω are unknowns:

$$\begin{aligned} a + y + 2\omega &= 0 \\ b + x + y &= 0 \\ c + x + 2\omega &= 0. \end{aligned} \quad (3)$$

The solution of these equations is

$$x = 1/2(a - b - c) \quad (4)$$

$$y = 1/2(c - a - b) \quad (5)$$

$$\omega = 1/4(b - c - a) \quad (6)$$

which completes the derivation.

If all four terms of Equation 2 are rotated cyclicly the same direction and amount, Equation 4, 5, 6 continue to apply. For example,

$$\begin{bmatrix} b & a \\ c & 0 \end{bmatrix} + \begin{bmatrix} x & 0 \\ x & 0 \end{bmatrix} + \begin{bmatrix} y & y \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -\omega & \omega \\ \omega & -\omega \end{bmatrix} + \begin{bmatrix} \omega & \omega \\ \omega & \omega \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

in which now the sense of x and y is interchanged.

ACKNOWLEDGMENT

Although the author has noted others (particularly Dr. B. Hallert and Ing. H. Trager) making equivalent mental calculations, he has not been able to find an expression of these principles in the literature, especially American literature.

Application of Aerial Photographs and Regression Technique for Surveying Caspian Forests of Iran

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(*Abstract is on next page*)

INTRODUCTION

IRAN with its 20 million people is starving for forest products while just a few miles from Tehran, the capital city, are virgin hardwood forests. These forests are located throughout the north slopes of the rugged Elburz Mountains facing the Caspian Sea. The major objective is to place these forests under intensive management and to increase the supply of forest products on a sustained yield basis as soon as possible. A secondary objective is to produce maps at 1:50,000 scale for this forest area.

The Plan Organization of Iran (the agency responsible for financing the economic developments of Iran using oil revenues) recognized this forestry problem and allotted funds to the Iranian Forest Service to conduct the Caspian Forest Survey. The Iranian Forest Service enlisted five U. S. Forest Technicians to develop, train, organize and furnish advice to this program encompassing 3,420,000 hectares comprising the Caspian Forest Region. This program was started in 1958. Completion is expected in about two years for the overall program. The results of applying aerial photographs and regression techniques together are given in this report for a portion of the area recently completed.

This program was administered by the Iranian Forest Service in Tehran, Iran, and the author is indeed grateful to that Service for the privilege of using their data.

ABSTRACT: *A Forest Survey project of the Caspian Forests was started in 1958. United States technicians were selected to help in the operations of this project covering 3,420,000 hectares. Forest area, volumes and growth were determined using new applications of photo interpretation and ground techniques. A new scheme for determining photo-scale was used for the first time on a large project. Photo-plot measurements, ground-plot volumes and ground-plot growth were compiled using a multiple regression. Sampling errors and the source of sampling error were shown and presented in a compact package. This report is limited to 98,000 hectares where the survey was completed. The methods used proved satisfactory and are recommended for other Forest Survey projects. Material presented in this paper was drawn from the Program Instructions of the Caspian Forest Survey which is a project of the Iranian Forest Service, Tehran, Iran.*

The report is limited to 98,000 hectares of land known as the Tavalesh Forest located about 18 miles west of Rasht, Iran (Figure 1). All the details for conducting such a program cannot be covered but those pertaining to new developments will be emphasized—especially those pertaining to the use of aerial photographs.

This Forest Survey used a design discussed in detail in "PHOTOGRAMMETRIC ENGINEERING," (Volume XXVI, No. 3, page 441). In applying this design to the Tavalesh Forest three intensities of sampling are used. These included 2,831 photo-points, 2,308 photo-plots and 54 ground-plots. Briefly, the photo-points are classified for forest and non-forest land, the photo-plots are the locations for making stand measurements on aerial photos, and the ground-plots are locations for collecting the ground forest data. The photo-points were the basis for estimating forest area. The photo-plots and ground-plots were the basis for the construction of a regression of photo-measurements to ground values. This regression is an important key to measure gains made through photo interpretation.

AERIAL PHOTOGRAPHS AND MAPS

Aerial photographs of four types and maps at scale of 1:250,000 were available for this program. These are discussed separately.

Vertical aerial photographs at 1:50,000 scale completely cover the project area. The photos were delineated to show four forest types (beech, oak, mixed hardwoods, and cypress); three density classes, (poor, medium and well stocked); and four stand-height classes corresponding with seedling and sapling trees, pole trees, young sawtimber trees and old sawtimber trees. These aerial photographs were also used for field delineation of map features and for field completion of geographic names, road classifications, trails etc. These photographs also served as map substitutes in the absence of detailed maps. The present 1:250,000 maps were not satisfactory for field planning and guidance. But the 1:50,000 scale photos were very useful for this purpose and were part of the field parties' equipment.

The coverage by 1:25,000 oblique panchromatic aerial photography was incomplete. This was only used to assist in the delineation, planning and field operations. In addition it served to supplement the coverage of 1:50,000 scale photos.

The 1:10,000 scale vertical panchromatic aerial photography completely covered the Tavalesh Forest. On the Tavalesh area these photos were used instead of the 1:50,000 scale photos for the forest delineations. Also these photos were used for classifying and measuring the photo-plots which consisted of a 0.1 hectare area. The three tallest trees

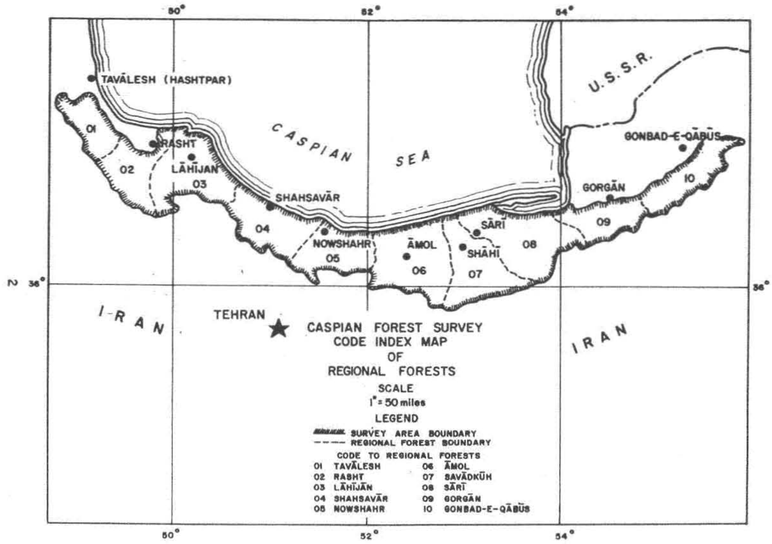

FIG. 1. Code index map of regional forests.

were measured here, and the crown density was estimated for the dominant and codominant trees.

Tree heights were determined with the help of a parallax wedge. The principles used in applying the wedge were not new, but the method of determining scale of the photo at the plot was unique and was used for the first time on such a project. A further comment on this is desirable. Since the 1:250,000 scale maps were of little value for determining photo-plot scales it was necessary to develop another technique. The technique developed applies radial line triangulation, the basis for map compilation. All that was needed from this triangulation network was the location of photo-centers to a known scale. This gave the ground distance (B) between photo-centers. By dividing this ground distance by the parallax at the photo-plot, the photo-scale for the plot was determined. This is the formula:

$PSR = B/P$ where:

PSR = Reciprocal of photo-scale in representative fraction.

B = Ground distance in meters between photo-centers.

P = Parallax in meters at photo-plot.

Having solved for photo-scale, the measurement of tree heights, crown diameters, slopes and distance became routine. Crown density was estimated using a crown density guide. The photo-plots were marked with ink and were located so as to get a complete systematic sample of the area. The mark consisted of 4 straight lines, all heading towards the plot-center but stopping short so that the plot area (.1 hectare) was not marked. Photos were viewed stereoscopically using a pocket stereosope and data were placed on a photo interpretation record form.

Strip photography at a scale of 1:5,000 on panchromatic film was taken at about 20 mile intervals across the project area. These were used for photo-plot measurements and for the selection of ground-plots in those areas outside the Tavalesh Forest.

The only maps available for the entire project were 1:250,000 scale; these were useful for planning purposes but were of very little value to the field crews. The scale was too small. The aerial photographs were more useful for this purpose as pointed out earlier. For forest management purposes these maps were also of very little value.

PROCEDURES FOR FOREST AREA ESTIMATE

The 1:250,000 maps were used to determine the project area. The project boundaries were marked on these maps. Then a dot-grid was laid over the maps and a dot-count was made of those dots in or out of the project. A ratio of project dots inside the project to total dots was next obtained. The area of the map was determined from the U. S. Geological Survey standard quadrangle area tables. The area in the project equaled the portion of the dots in the project area times the map area. This information was ob-

TABLE 1

CASPIAN FOREST SURVEY AREA

Summary of Gross Area, by Map Quads

1. *Map QUAD No.*	2. *Total area QUAD (Hectares)*	3. *Total DOTS in QUAD*	4. *Dots in Project Area*	5. *Ratio project dots to total dots (4÷3)*	6. *Caspian forest Survey area (2×5) (Hectares)*
NJ 39-9	1,472,166	5,854	1,373	.2345	345,223
39-10	1,472,166	5,854	1,223	.2089	307,223
39-13	1,491,218	5,950	224	.0376	56,070
39-14	1,491,218	5,950	1,498	.2518	375,489
39-15	1,491,218	5,950	2,754	.4629	690,285
39-16	1,491,218	5,950	4,197	.7054	1,051,905
NJ 40-9	1,720,194	6,830	305	.0447	76,893
40-10	1,472,166	5,854	388	.0663	97,605
40-13	1,491,218	5,950	1,574	.2645	394,427
40-14	1,491,218	5,950	21	.0035	5,219
1-39-E	1,006,847	3,933	78	.0198	19,936
Total					3,420,487

tained for each map sheet covering the project, as shown in Table 1.

By this procedure it was determined that 3,420,000 hectares of gross land area were in the Caspian Forest Survey Project and 98,000 hectares of gross land area were in the Tavalesh area.

The forest area within the project area was estimated by using aerial photographs. For this purpose the 1:50,000 aerial photographs were used for all the project except the Tavalesh area; for this the 1:10,000 scale aerial photographs were used. The boundary line of the project was marked on the aerial photographs. Then, on every alternate print a dot grid (16 dots per inch) was laid over the photo. The dots actually observed and recorded are only those within the net area of the photo so as to prevent duplication or gaps in covering the project area. Each dot was studied to determine whether it fell on a forest or non-forest area. A careful record was made by each photo of the number of dots falling on forest or on non-forest in the project area. For this purpose 338,580 points were classified. Only 2,831 of these points were classified on the Tavalesh area and 2,308 of these were forest. The portion of forest area was obtained by dividing 2,308 by 2,831 which is .8151. The area in forest was .8151 times 98,000 hectares or 79,880 hectares. Other information was noted for each dot falling in Tavalesh forest such as Forest Region, watershed, political division, ownership, and commercial forest. This information was used for breakdown area estimates of the forest land (see Table 2).

PROCEDURES FOR VOLUME ESTIMATES

The following formula is the basis for computing the average volume-per-hectare:

$\overline{V}_t = P_f(a + b_1\overline{X}_1 + b_2\overline{X}_2)$ where:

V_t = mean volume-per-hectare of gross land area.

P_f = portion of forest land to gross land area, interpreted from photo-points.

a = constant, computed by method of least squares.

b_1 = regression coefficient for heights, computed by method of least squares.

b_2 = regression coefficient for crown-density, computed by method of least squares.

$\overline{X}_1$ = average total height of 3 tallest trees measured on all forest photo-plots.

$\overline{X}_2$ = average crown-density measured on all forest photo-plots.

The regression constant and coefficients were computed from the data collected from 54 ground-plots and the corresponding photo-plots, using a method of least squares. For each ground-plot the field data were reduced to a net volume-per-hectare by employing usual procedures used in forest survey compilation. The ground-plots were mechanically selected as every 43rd photo-plot. The data produced the following regression formula:

$$\overline{V}_t = .8151(21.8297 + 3.6506\overline{X}_1 + 1.3565\overline{X}_2)$$

TABLE 2

COMMERCIAL FOREST LAND AREA BY MAJOR FOREST TYPES

Tavalesh Forest

Forest Type	*Hectares*	*% of Total*
Beech Forest	21,489	26.9
Oak Forest	19,891	24.9
Mixed Forest	38,423	48.1
Cypress	80	0.1
Total All Types	79,883	100.0

The 2,308 photo-plots furnished the average stand height ($\overline{X}_1=18.7$) and the average density ($\overline{X}_2=63.0$). Applying these average values to the formula, the average volume-per-hectare of gross land area ($\overline{V}_t$) equals 143.0952 cubic meters. The product of the average volume-per-hectare times the gross area (98,000 hectares) gave a total cubic meter volume of 14,023,300.

This total volume was distributed by types or other classes using volume portions obtained from ground plots as shown in Table 3.

The variance of the mean volume-per-hectare of gross land area included the contribution independent of the regression, contributions from photo-measurements of height and crown-density, contribution from forest area, and the contribution from regression coefficients (see Table 4). The variance of the mean volume-per-hectare of gross land area was 173.3. The standard error was 13.17 or 9.2% of the mean volume.

TABLE 3

NET VOLUME OF GROWING STOCK ON COMMERCIAL FOREST LAND BY STAND VOLUME CLASSES

Tavalesh Forest

Stand Volume Class Cubic Meters per Hectare	*Cubic Meters*	*% of Total Volume*
0–49	252,000	1.8
50–99	1,447,000	10.3
100–149	729,000	5.2
150–199	1,528,000	10.9
200–299	5,778,000	41.2
300–399	1,541,000	11.0
400–499	1,900,000	13.5
500–749	848,000	6.1
750 or more	None	None
Total	14,023,000	100.0

PROCEDURES FOR ANNUAL GROWTH ESTIMATES

The estimate of total net growth followed the procedures used for volume. A regression was used similar to volume except that growth-per-hectare replaced volume-per-hectare and only crown-density was used for photo-measurements since tests showed that height was not significant for growth. Data for the regression were based on the same plots as were used for volume. Growth was determined from measurements of annual rings which were reduced to growth-per-hectare ($\overline{G}_t$) by the usual forest survey procedures. The growth regression formula was as follows:

$$\overline{G}_t = .8151(.8997 + .331\overline{X}_2)$$

By applying the average crown-density ($\overline{X}_2=63.0$) to this formula, average net cubic meter growth-per-hectare was found to be 2.4331. The product of growth-per-hectare times gross area was the total cubic meter net-annual-growth; this equals 238,443.

As for volume the distribution of growth is made in proportion to the ground-plot growth as illustrated in Table 5. The annual mortality was deducted from the computed net annual growth.

The variance of the mean growth-per-hectare of gross land area included the same sources as those for volume. This variance was .0588. The standard error was .2421 or 9.9% of the mean value.

TABLE 4

DISTRIBUTION OF VOLUME VARIANCE BASED ON CUBIC METERS

Source	*Population Variance*	*No. of Samples*	*Variance of Mean*
Independent of regression	9,122	54	168.9
Photo measurements	1,635	2,380	0.7
Forest area	4,647	2,831	1.6
Regression coefficients	2.1	—	2.1
Total	15,406		173.3

$$\text{Standard error} = \sqrt{173.3} = 13.17$$

$$\%\ \text{standard error} = \frac{\text{standard error}}{\text{mean}} = \frac{13.17}{143.09}(100) = 9.2$$

TABLE 5

NET ANNUAL GROWTH AND MORTALITY OF LIVE SAWTIMBER AND GROWING STOCK ON COMMERCIAL FOREST LAND

Tavelesh Forest

Item	Sawtimber		Growing Stock	
	Cubic Meters	%	Cubic Meters	%
Total net annual growth	*202,936*	100.0	*238,442*	100.0
Sound trees	180,012	88.7	207,247	86.9
Sound cull trees	12,935	6.4	20,328	8.6
Rotten cull trees	*9,989*	4.9	*10,813*	4.5
Annual Mortality	*26,119*		*26,119*	
Adjusted net annual growth	176,817		212,323	

FOREST MAPS

The only maps available (250,000 scale) are not satisfactory for forest management purposes. As a minimum maps at 1:50,000 scale are needed. In order to fulfill this need a program was designed to furnish these maps. The Iranian Forest Service assumed responsibility for the delineation and field completion part of the program, while the compilation and printing were to be contracted to private firms or other government agencies.

The delineation consisted of two parts—forest delineation and map features delineation. The forest delineations included land use, forest-type, stand-density and stand-height classes. The 1:10,000 scale photos were used for this work on the Tavalesh area and 1:50,000 photos were used for the remaining area. The map features delineation followed the general procedures established by the U. S. Army Map Service and adapted by the Geographic Department of the Iran Imperial Army. As this part of the program has not been completed for the Tavalesh area, it cannot be reported on at this time.

TABLE 6

DISTRIBUTION OF GROWTH VARIANCE BASED ON CUBIC METERS

Source	Population Variance	No. of Samples	Variance of Mean
Independent of regression	3.1360	54	.0581
Photo measurement	.3946	2,380	.0002
Forest area	1.3429	2,831	.0005
Regression coefficients	0.0000	—	—
Total	4.8735		.0588

$$\text{Standard error} = \sqrt{.0588} = .24$$

$$\%\ \text{standard error} = \frac{.24}{2.43}\ 100 = 9.9$$

DISCUSSION

Gains made by the use of aerial photographs and regression techniques are expressed in the number of ground-plots required when these techniques are not used. If these techniques were not used 92 ground-plots would be needed to furnish the same sampling error of volume estimate and 87 ground-plots for the same sampling error of growth estimate in the Tavalesh area. Cost comparisons should be made if reliable cost data are available. But at this time such data are not available, so this evaluation cannot be shown. Regardless of cost the same accuracy of forest area would be practically impossible without aerial photographs. Also without aerial photographs the costs of the mapping program would be increased. The technqiues used in this project performed according to expectancy. As the photo interpretation improves the population variance independent of the regression can be expected to get smaller thus further improving the efficiency of the program.