On Certain Instrumental Errors in the A7

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THE Wild A7 Autograph which is one of the most important instruments of present-day photogrammetry reconstructs the central projection mechanically. The instrument is not capable of reconstructing the geometric model which it aims to restitute, with mathematical precision, because of minute inaccuracies in the machining and assembly. This leads to small deviations in the geometry of the resulting projection, as compared with the theoretical one. These deviations define the quality of the instrument and the precision of the measurements made with it. The latter is usually given by the standard error of the coordinates of a point as measured in the instrument.

Photogrammetrical measurements, like all other measurements, must comply with specifications of precision in accordance with the purposes for which they are required. For this reason it must be assured that the instrument used for making measurements will not cause systematic errors which exceed the required precision standards. The essential condition for this is optimal adjustment of the instrument.

Besides the unavoidable instrumental errors (e.g. errors of the gimbal axis) additional errors may occur due to the lack of adjustment; this causes systematic errors, influencing the measurement. In order to avoid a loss in the quality of the measurement, it is necessary to take these systematic errors into account, i.e. we have to be able to detect them and to correct the results of the measurement if needed. The correction may be made mechanically—adjusting the instrument, and/or analytically.

The paper is a contribution to solving the above problem by showing how to test for the presence of certain systematic errors, evaluating their form and analysing their influence.

1.1 SYSTEMATIC PROJECTION ERRORS IN THE AUTOGRAPH

The central projection projects one space into another by means of a bundle of rays which pass through one point—the projection centre. In the A7 Autograph the bundle of rays is represented by the space rod, which swings around a fixed point in the instrument—the mechanical projection centre. The upper end of the rod is connected with an objective whose optical axis is everywhere perpendicular to the negative plane of the instrument. (This objective constitutes the first optical part in the observation system.) The point of intersection of the optical axis with the negative plane is the point which is to be projected to the projection space, in which the photogrammetric model is formed in the instrument.

The objective slides along a swinging girder. The combined movements, the linear movement of the objective, and the swinging movement of the girder enable the objective to cover the whole of the negative. The principle explained above and employed in the Autograph is known as the principle of the orthogonal observation. If this is not satisfied then the instrument is out of adjustment and systematic errors arise. The resulting error is usually resolved into two components, one in the direction of the swinging girder—the longitudinal inclination, *dl*, the other in the direction perpendicular to the swinging girder—the transversal inclination, *dt*.

The directions *dl, dt* form an orthogonal system whose relation to the coordinate system of the negative plane depends on the coordinates of the point on which the space rod is directed.

The components *dt, dl* may be interpreted as shifts *dx, dy* of the projected points, in directions parallel to the coordinate axis of the instrument. This interpretation is in fact a linear transformation, which we write as:

$$dx = a_{11}dt + a_{12}dl$$
$$dy = a_{21}dt + a_{22}dl \tag{1}$$

The coefficients of the transformation equations are trigonometrical functions of the variable angle between the two coordinate systems, and may be derived as follows:

P—point of intersection with the negative of the perpendicular to the negative passing through the upper end of the space rod.
P_1—point of intersection with the negative of the inclined optical axis of the objective fixed to the upper end of the space rod.
$\overline{PP_1}$—the resulting error.

From Figure 1 we derive:

$$PP_1 \cos\beta = dt$$
$$PP_1 \sin\beta = dl$$
$$PP_1 \cos(\beta+\alpha) = dx = dt\cos\alpha - dl\sin\alpha$$
$$PP_1 \sin(\beta+\alpha) = dy = dt\sin\alpha + dl\cos\alpha$$
$$\cos\alpha = \frac{g-y}{\sqrt{x^2+(g-y)^2}} = \frac{g-y}{s}$$
$$\sin\alpha = \frac{x}{\sqrt{x^2+(g-y)^2}} = \frac{x}{s} \tag{2}$$

Substituting (2) into (1) we finally obtain:

$$dx = \frac{g-y}{s}dt - \frac{x}{y}dl$$
$$dy = \frac{x}{s}dt + \frac{g-y}{s}dl \tag{3}$$

The expressions (2) are useful as they enable us to correct the measured coordinates for the inclinations dl, dt.

1.2 DETERMINATION OF THE dl AND dt

The inclinations are to be determined analytically. Testing instruments are usually made by measuring coordinates of points projected from the negative to the projection plane. The errors in the coordinates of the projected points do not depend on the lack of adjustment alone; they are also a function of small errors in the elements of orientation; only an analytical approach enables us to distinguish between these factors.

The relationship between the coordinates of a point projected by a central projection from one plane to another, while the planes are parallel and the angle between the coordinate systems which cover the planes is equal to zero, is given by the expression:

$$X = \frac{Z}{f}x$$
$$Y = \frac{Z}{f}y \tag{4}$$

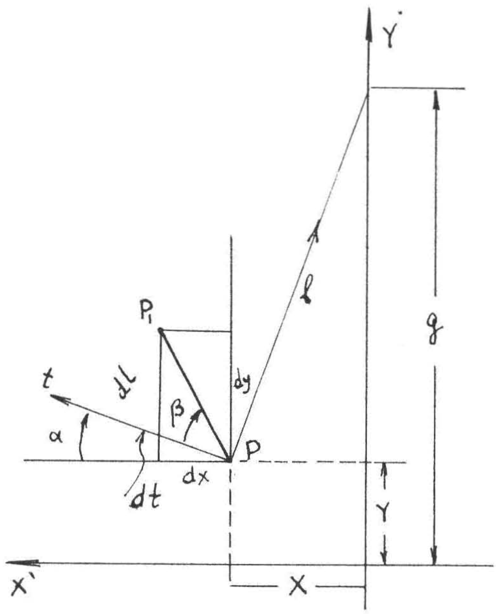

FIG. 1. The negative plane of the A7.

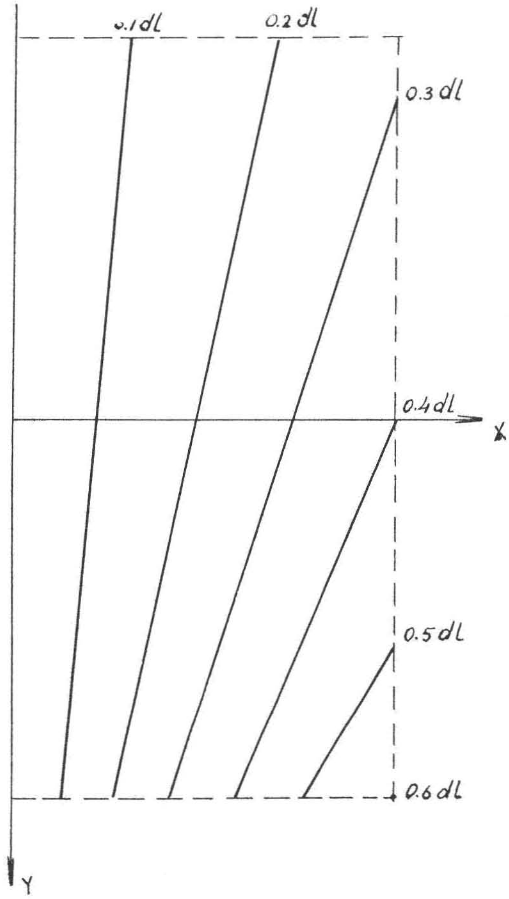

FIG. 2. The X-Parallax distribution in the projection plane of the A7 due to the inclination dl.

But this is a theoretical case only. In practice there is no possibility to achieve parallelism between the planes in a geometrical sense. Therefore the coordinates in the projection plans X, Y, will not satisfy the relations (4) perfectly.

The discrepancies dX and dY as functions of changes in the elements of orientation, and the inclination dt, dl, are given in the coordinate system of the A_7 by the following formulae:

$$dX = -db_x - \frac{X}{Z}db_z - Yd\kappa - \left(1 + \frac{X^2}{Z^2}\right)Zd\phi + \frac{XY}{Z^2}Zd\omega + m\frac{g - y'}{s}dt$$
$$- m\frac{X'}{s}dl$$

$$dY = -db_y - \frac{Y}{Z}db_z + Xd\kappa - \frac{XY}{Z^2}Zd\phi + \left(1 + \frac{Y^2}{Z^2}\right)Zd\omega$$
$$+ m\frac{X'}{s}dt + m\frac{g - y'}{s}dl \qquad (5)$$

where the dX and dY are defined as corrections i.e. given value minus measured value.

The measurements are carried out in the projection plane, while the coefficients of the transformation are derived in the negative plane; therefore, they have to be multiplied by the factor $m=Z/f$.

Each point measured in the projection plane gives rise to two equations of type (5). Since eight unknowns have to be evaluated we have to measure at least four points. In order to get some information about the accidental errors of the measurement, and to obtain a check on the results, it is necessary to measure more than the four essential points. Thus then arises a need for adjustment which is here carried out according to the method of least squares. Solving a system of equations we obtain the values of the dt and dl. In order to simplify the solution of a system containing eight unknowns we chose symmetrically situated points.

The coordinates of the points in the projection plane are:

	x	y
1	$-a$	$-a$
2	$+a$	$-a$
3	$-a$	$+a$
4	$+a$	$+a$
5	0	0

(6)

Substituting the coordinates from (6) into (5) we get a system of observation equations from which the following normal equations are derived:

db_x	db_y	db_z	$d\kappa$	$Zd\phi$	$Zd\omega$	dt	dl	$-l$
5	0	0	0	$5+\frac{4a^2}{Z^2}$	0	$-K_1$	0	$[dX]$
	5	0	0	0	$-\left(5+\frac{4a^2}{Z^2}\right)$	0	$-K_2$	$[dY]$
		$8\frac{a^2}{Z^2}$	0	0	0	0	$\frac{a}{Z}K_2$	$\frac{a}{Z}N_1$
			$8a^2$	0	0	aK_2	0	aN_2
				$5+\frac{8a^2}{Z^2}+\frac{8a^4}{Z^4}$	0	$-\left(K_1+\frac{a^2}{Z^2}K_3\right)$	0	$\left(1+\frac{a^2}{Z^2}\right)[dX]+\frac{a^2}{Z^2}N_3$
					$5+\frac{8a^2}{Z^2}+\frac{8a^4}{Z^4}$	0	$K_1+\frac{a^2}{Z^2}K_4$	$-\left(1+\frac{a^2}{Z^2}\right)[dY]-\frac{a^2}{Z^2}N_4$
						K_5	0	M_1
							K_6	M_2
								$[dX^2]+[dY^2]$

(7)

The solution of the system (7) gives:

$$dl=\frac{-M_2-\frac{K_1}{5}[dY]+\frac{K_2}{8}N_1-\frac{5}{24}(K_4-0.8K_1)(0.2[dY]+N_4)}{K_5-\frac{K_1^2}{3}-\frac{K_2^2}{8}-\frac{5}{24}K_4^2+\frac{1}{3}K_1K_4}$$

$$dt = \frac{-M_1 - \frac{K_1}{5}[dX] + \frac{K_2}{8}N_2 - \frac{5}{24}(K_3 - 0.8K_1)(0.2[dX] + N_3]}{K_5 - \frac{K_1{}^2}{3} - \frac{K_2{}^2}{8} - \frac{5}{24}K_3{}^2 + \frac{1}{3}K_1K_3}$$

$$Zd\omega = \frac{5}{24}\frac{Z^2}{a^2}(0.2[dY] + N_4) - \frac{5}{24}\frac{Z^2}{a^2}(K_4 - 0.8K_1)dl$$

$$Zd\phi = -\frac{5}{24}\frac{Z^2}{a^2}(0.2[dX] + N_3) + \frac{5}{24}\frac{Z^2}{a^2}(K_3 - 0.8K_1)dt$$

$$d\kappa = -\frac{N_2}{8a} - \frac{K_2}{8a}dt$$

$$db_z = -\frac{ZN_1}{8a} - \frac{2K_2}{8a}dl$$

$$db_y = -0.2[dY] - \frac{5Z^2 + 4a^2}{24a^2}(0.2[dY] + N_4) + \frac{5Z^2 + 4a^2}{24a^2}(K_4 - 0.8K_1)dl - 0.2K_1dl$$

$$db_x = -0.2[dX] - \frac{5Z^2 + 4a^2}{24a^2}(0.2[dX] + N_3) + \frac{5Z^2 + 4a^2}{24a^2}(K_3 - 0.8K_1)dt - 0.2K_1dt$$

$$K_1 = 2m\left(\frac{g + a}{s_1} + \frac{g - a}{s_2}\right) + 1$$

$$K_2 = 2m\left(\frac{g + a}{s_1} - \frac{g - a}{s_1}\right) + 2m\left(\frac{a}{s_1} + \frac{a}{s_2}\right)$$

$$K_3 = 2m\left(\frac{g}{s_1} + \frac{g}{s_2}\right)$$

$$K_4 = 2m\left(\frac{g + 2a}{s_1} + \frac{g - 2a}{s_2}\right)$$

$$K_5 = 4m^2 + 1$$

$$M_1 = m\frac{a}{s_1}(dY_1 - dY_2) + m\frac{a}{s_2}(dY_3 - dY_4) - m\frac{g + a}{s_1}(dX_1 + dX_2)$$
$$- m\frac{g - a}{s_2}(dX_3 + dX_4)$$

$$M_2 = -m\frac{g + a}{s_1}(dY_1 + dY_2) - m\frac{g - a}{s_2}(dY_3 + dY_4) - m\frac{a}{s_1}(dX_1 - dX_2)$$
$$- m\frac{a}{s_2}(dX_3 - dX_4)$$

$$s_1 = \sqrt{a^2 + (g + a)^2}$$

$$s_2 = \sqrt{a^2 + (g - a)^2}$$

$$N_1 = -dY_1 - dY_2 + dY_3 + dY_4 - dX_1 + dX_2 - dX_3 + dX_4$$

$$N_2 = + dY_1 - dY_2 + dY_3 - dY_4 - dX_1 - dX_2 + dX_3 + dX_4$$

$$N_3 = dY_1 - dY_2 - dY_3 + dY_4$$

$$N_4 = dX_1 - dX_2 - dX_3 + dX_4 \tag{8}$$

The expressions (8) enable us to determine the correlation between the elements of orientation and the inclinations dt, dl. In case of $dl = dt = 0$, we obtain the known expressions for the elements of orientation of the projector (Hallert, 1954–55).

The coefficients $K_1 \cdots K_5$ may be calculated in advance for different values of a. Choosing $a = 100$ mm. $m = Z/f = 2$ and $g = 230$ mm. we obtain:

$$\begin{aligned} K_1 &= 8.00 \\ K_2 &= 4.26 \\ K_3 &= 8.28 \\ K_4 &= 5.72 \\ K_5 &= 17.00 \end{aligned} \tag{9}$$

With the values according to (9), after some simple computations we derive linear equations from (8) for determinations of the dl and dt:

$$\begin{aligned} dl &= 0.10(dx_1 - dx_2) + 0.30(dx_3 - dx_4) - 0.10(dy_1 + dy_2) + 0.30(dy_3 + dy_4) \\ dt &= 0.12(dx_1 + dx_2) + 0.49(dx_3 + dx_4) + 0.28(dy_1 - dy_2) - 0.90(dy_3 - dy_4) \end{aligned} \tag{10}$$

The formulas (10) are useful because according to them it is possible to test the projector of the instrument and to determine the dt and dl from linear functions of the directly measured discrepancies dx and dy.

1.3. SOME ADDITIONAL REMARKS

(*A*) Instruments are generally tested by measuring the projected coordinates of given points. These points can be provided in two ways:

a) by placing a precision grid on the plateholder of the projector,

b) by marking points directly on the plateholder.

The second method is preferred, because it permits testing the instrument without interrupting measurements which may be in progress. The coordinates of the marks on the plateholder have to be supplied by the producer.

(*B*) In order to obtain the values of the dt and dl with high precision, it is advisable to measure more than the five points suggested above. But increasing the number of points complicates the test to some extent; therefore, in order to reduce the influence of the accidental errors on the determination of the inclinations, without complicating the adjustment formulas (10), it is necessary to measure the coordinates of the points several times, for instance three times, and to substitute the mean value of the results into the equations (10).

(*C*) A suitable combination of points can be chosen as a standard procedure for testing instruments. For this combination, as is shown above, we have to derive equations from which dt and dl can be easily computed.

Since there is no possibility of adjusting the instrument perfectly, residual errors of adjustment remain, and the instrument is to be regarded as adjusted, if these errors do not exceed certain allowable values. These will be derived further.

II. THE INFLUENCE OF dt AND dl ON THE RESTITUTION OF A SINGLE MODEL

As explained above, the instrumental errors dt and dl disturb the mechanical central projection causing erroneous coordinates of the projected points. As a consequence additional x and y parallaxes arise, which being compensated by the elements of the relative orientation, lead to model deformations.

2.1. THE FIRST STAGE OF THE RESTITUTION

The first stage of the restitution of a photogrammetric model is the relative orientation. Its geometrical sense is intersection of five pairs of corresponding rays. In the Autograph it is achieved by removing y-parallaxes in five or more points. (Usually six.)

The y-parallax is defined as: $P_y = y' - y''$, where y' is the ordinate of the point projected from the left projector, and y'' from the right one. In an orientated model, free from deformations, the $P_y - s$ must be equal to zero. It is clear that if the plotting instrument is not adjusted, the condition $P_y = 0$ will not be fulfilled.

Suppose that a grid model is placed in the instrument. Since parallaxes are coordinate-differences we can assume, without loss of generality, that only one projector of the plotting instrument, the left one, is out of adjustment. The y-parallax appearing at a point will then be equal to the increment in the ordinate of the point:

$$P_y = dy' = a_{21}dt + a_{22}dl$$

The model coordinates of the usual orientation-points are:

	x	y
1	0	0
2	b	0
3	0	$-d$
4	b	$-d$
5	0	$+d$
6	b	$+d$

Substituting these values into the formula for the y-parallax, we obtain the $P_y - s$ in the points $1 \cdots 6$ as functions of the dt and dl.

$$\begin{aligned} P_{y1} &= dl \\ P_{y2} &= a_{21_2}dt + a_{22_2}dl \\ P_{y3} &= dl \\ P_{y4} &= a_{21_4}dt + a_{22_4}dl \\ P_{y5} &= dl \\ P_{y6} &= a_{21_6}dt + a_{22_6}dl \end{aligned} \tag{11}$$

The y-parallaxes according to (11) are substituted into the expressions for the elements of orientation. Thus giving the resulting errors in these:

a. For independent pairs of pictures

$$\begin{aligned} d\omega'' &= \frac{h}{4d^2}\left\{(2a_{21_2} - a_{21_4} - a_{21_6})dt + (2a_{22_2} - a_{22_4} - a_{22_6})dl\right\} \\ d\kappa' &= \frac{1}{3b}\left\{(a_{21_2} + a_{21_4} + a_{21_6})dt + (a_{22_2} + a_{22_4} + a_{22_6})dl\right\} \\ d\kappa'' &= \frac{1}{3b}\left\{3dl \pm Fd\omega''\right\} \end{aligned} \tag{12}$$

$$d\phi' = \frac{h}{2bd}\left\{(a_{21_4} - a_{21_6})dt + (a_{22_4} - a_{22_6})dl\right\}$$

$$d\phi'' = 0$$

b. For dependent pairs of pictures

$$
\begin{aligned}
d\omega'' &= \frac{h}{4d^2}\left\{(2a_{21_2} - a_{21_4} - a_{21_6}) + (2a_{22_2} - a_{22_4} - a_{22_6})dl\right\} \\
d\kappa'' &= \frac{1}{3b}\left\{-(a_{21_2} + a_{21_4} + a_{21_6})dt + (3 - a_{22_2} - a_{22_4} - a_{22_6})dl\right\} \\
d\phi'' &= \frac{h}{2bd}\left\{(-a_{21_4} + a_{21_6})dt + (-a_{22_4} + a_{22_6})dl\right\} \qquad (13) \\
db_z'' &= -\frac{h}{2d^2}\left\{(a_{21_4} - a_{21_6})dt + (a_{22_4} - a_{22_6})dl\right\} \\
db_y'' &= \tfrac{1}{3}\left\{(a_{21_2} + a_{21_4} + a_{21_6})dt + (a_{22_2} + a_{22_4} + a_{22_6})dl \pm F d\omega''\right\}
\end{aligned}
$$

The coefficients α_{iK} depend on the dimensions of the model. For a wide-angle model we may suppose $b=d=0.6h$. Assuming $b=100$ mm in the negative plane, we obtain for the $\alpha_{iK}-s$, the following values:

$$
\begin{aligned}
a_{21_2} &= 0.40 & a_{22_2} &= 0.92 \\
a_{21_4} &= 0.29 & a_{22_4} &= 0.96 \qquad (14) \\
a_{21_6} &= 0.61 & a_{22_6} &= 0.79
\end{aligned}
$$

From (12), (13) and (14) the elements of orientation are determined. A single model is generally orientated as an independent pair of pictures. According to (12) we obtain the following elements:

$$
\begin{aligned}
d\omega'' &= -\ 86dt + 77dl \\
d\kappa' &= 716dt + 1995dl \\
d\kappa'' &= -\ 177dt + 2222dl \qquad (15) \\
d\phi' &= -\ 550dt + 292dl \\
d\phi'' &= 0
\end{aligned}
$$

The angles $d\omega''$, $d\phi''$ etc. are found directly in seconds, if dt and dl are given in millimeters.

From (15) we see that the elements $d\kappa'$, $d\kappa''$ are affected the most seriously, which results in a swing of the restituted model in its plane.

2.2 ESTIMATION OF ALLOWABLE VALUES

The relation (12) provides a basis for estimating the maximum allowable values for dt and dl, and to determine a criterion for adjustment of the Autograph.

The instrument can be regarded as adjusted if the model deformations caused by lack of mathematically exact adjustment do not exceed the model deformations resulting from the mean square errors of the elements of relative orientation, i.e. if the systematic errors of the instrument do not exceed the accidental errors of the relative orientation procedure. This condition is expressed mathematically as:

$$
\begin{aligned}
d\omega'' &\leq \mu\sqrt{Q_{\omega\omega}} \\
d\phi'' &\leq \mu\sqrt{Q_{\phi\phi}} \\
d\phi' &\leq \mu\sqrt{Q_{\phi\phi}} \\
d\kappa'' &\leq \mu\sqrt{Q_{\kappa\kappa}} \\
d\kappa' &\leq \mu\sqrt{Q_{\kappa\kappa}}
\end{aligned} \tag{16}
$$

The weight numbers Q are according to Hallert (1944):

$$
Q_{\omega\omega} = \frac{3h^2}{4d^4}
$$

$$
Q_{\phi\phi} = \frac{h^2}{2b^2d^2}
$$

$$
Q_{\kappa\kappa} = \frac{9h^4 + 8d^4 + 12d^2h^2}{12b^2d^4}
$$

Assuming a wide-angle model with the above mentioned dimensions, and accepting the mean square error of parallax measurement to be 0.007 mm, we obtain:

$$
\begin{aligned}
m_\omega &= \pm\ 21'' \\
m_\phi &= \pm\ 17'' \\
m_\kappa &= \pm\ 42''
\end{aligned} \tag{17}
$$

Equating the model deformations $d\omega$, $d\phi$ etc. to the mean square errors m_ω, $m\phi$ etc. we derive from (15) and (17) a system of equations whose solution gives:

$$
dl \cong 0.020 \text{ mm.}
$$

$$
dt \cong -\ 0.009 \text{ mm.}
$$

It is obvious that the allowable errors of adjustment should be smaller than those found according to (18). An order of size of 5 microns in the negative plane seems to be reasonable.

2.3 THE INFLUENCE OF THE dt AND dl ON THE HEIGHT MEASUREMENT

Lack of adjustment affects the heights in the model in two ways, i.e., by deforming the relative orientation, and by causing x-parallaxes.

In order to simplify the analysis, we assume that only one projector is out of adjustment, namely the left one.

As a consequence errors are made in the relative orientation, which are given by (15), and horizontal parallaxes occur, which we write according to (3) as:

$$
P_x = dx' = \frac{g - y}{\sqrt{x^2 + (g - y)^2}}\, dt - \frac{x}{\sqrt{x^2 + (g - y)^2}}\, dl \tag{18}
$$

2.3.1 We deal with the deformation of the relative orientation first. The x-parallax is defined as $P_x = x' - x''$. In the coordinate system of the *A*7 the dp_x is given by the familiar differential formula:

$$
dp_x = Y(d\kappa'' - d\kappa') - \left(1 + \frac{X^2}{Z^2}\right) z d\phi' + \left(1 + \frac{(x - b)^2}{Z^2}\right) z d\phi'' + \frac{(x - b)\,Y}{Z}\, d\omega'' \tag{19}
$$

The relation between small changes in the x-parallax and height differencies is: $dh=(H/b)dp$.

Since $d\phi''=0$ we obtain from (19) the following expression for the error in the measured height of any point in the model (prior to levelling):

$$dh = AX^2 + BXY + CY + DX + E$$

where:

$$\begin{aligned} A &= -\frac{H}{bf}\,d\phi' \\ B &= -\frac{H}{bf}\,d\omega'' \\ C &= +\frac{H}{b}\left[d(\kappa'' - \kappa') + \frac{b}{f}\,d\omega''\right] \\ D &= 0 \\ E &= -\frac{H}{b}\,fd\phi' \end{aligned} \tag{20}$$

Absolute orientation will compensate the linear and part of the quadratic terms. Though this deforms the absolute orientation, it is unimportant, since the aim of measuring a single model is to obtain correct coordinates X, Y, Z, and not correct elements of outer orientation. It may be important when measuring strips. For the sake of simplicity we then will consider that only the linear terms have been compensated by the absolute orientation, and will proceed to investigate the remaining errors caused by quadratic terms only. We then obtain the following height errors:

$$dh = AX^2 + BXY = -\frac{H}{bf}\,(d\phi' X^2 + d\omega'' XY)$$

According to (15) we obtain:

$$dh = -\frac{H}{bf}\left[(550dt - 292dl)X^2 + (-86dt + 77dl)XY\right] \tag{21}$$

From (21) it is possible to deduce a mean value for the height-error in the model. For this purpose it is customary to integrate the expression (21) and to divide the result by the area of the model:

$$\begin{aligned} M_h &= -\frac{H}{bf}\,\frac{1}{bd}\int_{x=0}^{x=b}\int_{y=0}^{y=d}\left[(550dt - 292dl)X^2 + (-86dt + 77dl)XY\right]dxdy \\ &= -\frac{H}{f}\cdot b(162dt - 78dl) \end{aligned} \tag{22}$$

For a wide-angle model we assume $b/f=0.65$. Thus the (22) may be written as:

$$M_h = \frac{H}{\zeta''}\,(51dl - 105dt) \tag{23}$$

From (23) it is obvious that the errors in the relative orientation caused by lack of adjustment do not influence the height measurements.

For instance, assuming $dt-0.02$ mm. and dl 0.02 mm. (which are from practical experience extreme values) and assuming H 5,000 meters, we obtain the mean value of the height-error to be equal to 75 mm, which may be neglected for all practical purposes. It should be borne in mind that the proper compensating effect of the absolute orientation would give even smaller values than those found here.

2.3.1 The x-parallax caused by dl and dt (of the left projector) are according to (3) a sum of the two components. Their influence on the height-measurement is not identical.

Supposing the dimensions of the model to be $b=d=100$ mm. in the negative plane, we obtain the following relations:

$$0 \leq \frac{x}{s} \leq 0.6$$
$$0.8 \leq \frac{g-y}{s} \leq 1.0 \tag{24}$$

From (24) we deduce that the coefficient $(g-y)/s$ is nearly constant over the whole of the negative. Therefore the transversal inclination dt has practically no influence on the heights. It only affects the reference plane of the heights, a fact which is of no importance.

The coefficient of the second component is variable. Analysing the deformation of the model we need to deal with the influence of dl only.

The variation of the x-parallaxes due to dl is best represented graphically. Computing the x-parallaxes at points distributed over the whole of the model, and drawing lines of equal-x-parallaxes results in the Figure 2 which shows how lack of adjustment affects the heights in the model.

From Figure 2 it is clear that the reference plane of the height-measurement is deformed. Instead of a plane a curved surface occurs, which is, however, largely compensated by the absolute orientation.

The height difference is given as a function of small changes in elements of absolute orientation in the coordinate-system of the A7 by the familiar formula:

$$\Delta h = \Delta h_0 - X\,\Delta\Phi + Y\Delta\Omega \tag{25}$$

For the absolute orientation it is recommended to use five points, four at the corners and one in the centre of the model. Introducing the coordinates of these points in equation (25) observation equations are formed, their free elements being found from the diagram by multiplying the x-parallaxes by $f/b=5/3$, in accordance with the relation between small x-parallaxes and height-differences in a wide-angle model. (The heights are given in the model scale.)

From the observation equations a system of normal equations is derived, which upon solution give:

$$\Delta h_0 = 0.2dl$$
$$\Delta\phi = -\frac{0.8}{b}\,dl \tag{26}$$
$$\Delta\Omega = \frac{0.2}{d}\,dl$$

The residual height-error at any point in the model after the compensation by the elements of the absolute orientation found above will be:

$$\Delta h_r = \Delta h_i + \left(0.2dl - \frac{0.8}{b} dlX_i + \frac{0.2}{d} dlY_i\right) \tag{27}$$

The elements 0.2 dl and $(0.2_i dy/d)dl$ are small, so the residual height-error is written as follows:

$$\Delta h_r = \Delta h_i - \frac{0.8X_i}{b} dl \tag{28}$$

From (28) and the diagram we deduce that the residual height-error at an arbitrarily chosen point in the model due to lack of adjustment is very small and may be neglected for all purposes.

Summary

In the first section of this paper a simple method to test the Autograph A7 for the presence of width and inclination errors dt and dl is indicated. Linear equations are deduced from which the instrumental errors may be found, and the results of the measurements carried out in the instrument corrected if necessary.

The second section deals with the influence of the inclination errors on a single model.

The inclinations effect the relative orientation, mainly the elements $\Delta\kappa'$. This results in a swing of the restituted model in its plane (and leads to a second degree curvature of a strip, when carrying out an aero-triangulation in the instrument).

The deduced relation between the instrumental errors and the standard errors in the elements of the relative orientation provides a criterion for the adjustment.

The inclinations have practically no influence on the height measurements in the model.

This paper does not discuss the influence of the inclination errors on the X, Y coordinates in the model. This is easily derived according to the customary procedure. The relation between errors in the elements of orientation and heights in the model and the x, y, coordinates are known and it is a simple matter to derive the influence on the planimetry from the contents of this paper.

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