Aerial Triangulation Strip Adjustment with Independent Geodetic Control

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ABSTRACT: *This paper discusses the adjustment of strip triangulation performed according to the aerial polygon method on the Wild A-7, using two independent base lines and azimuths for horizontal control. Relative vertical control for the strip consists of four elevations in each of the first and last models. The resulting errors found in this test strip are shown, and the limitations of the method for obtaining control for small scale maps are discussed.*

PHOTOGRAPHIC DATA

THE area of study, in Laufen-Bauma, Switzerland, is from photography flown for the OEEPE/Committee A, Schweizer Block in 1956. The photography was made with a Wild RC7a camera of focal length $f=100.26$ mm. at an elevation of about 6,000 m. with a photo scale of about 1:60,000.

AERIAL TRIANGULATION

Utilizing the photographic data and the furnished diapositives, the first model was oriented in the A-7 using a model scale $M_M=1:15{,}000$. With the data given, this results in an instrument $h^*=400$ mm. and, assuming 60% overlap, the approximate $b^*=220$ mm. as calculated from the equation:

$$b^* = \frac{h^*(1-u)a}{f},$$

where u is the overlap and a is the diapositive format size (140 mm. in this case). For relative orientation, the over-correction factor used in ω is calculated from: $n=(2f/a-20 \text{ mm.})^2 \cong 2.8$. The relative and absolute orientation of the first model are carried out in the usual manner, using the distance of the first base D_A to scale the model, and the four given elevations for levelling. In some instances it is advisable to orient the x-axis parallel to the strip axis thus avoiding excessive "by" motion. If this is desired, a simple determination of x_1 can easily be carried out (1). It is also possible to orient the machine axes to the astronomic azimuth of the first base with similar procedure. Normally the scaling is checked until the residual scale error is $\leq 0.02\%$ in the first model in normal practice; however, in this strip no effort was made to reach that desirable value, as compensations can theoretically be made by corrections. The leveling accuracy should reach

$$m_H = \pm \frac{h}{10{,}000}$$

or in the case of this work ≤ 0.6 meter. In model connection, the technique of selection of three transfer points was used and the connection was carried out in accordance with the methods described by Brandenberger (1). The resulting tables I to IV are given in the Appendix of this paper and form the basic data on which the adjustment and error analysis is made. The work of aerial triangulation was performed at the Ohio State University by Nöel Poulin as a portion of his work in the Division of

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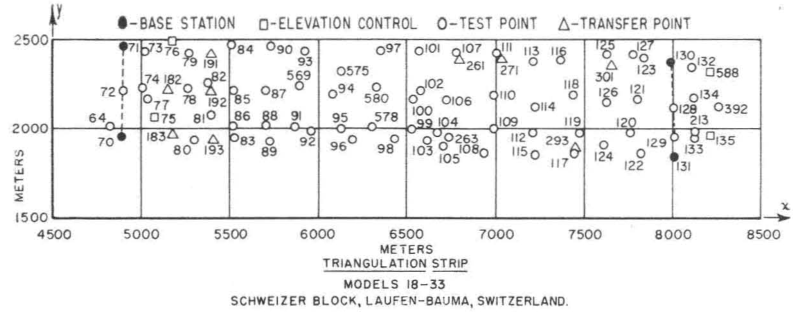

FIG. 1. Triangulation strip.

Geodetic Sciences. Figure 1 shows the triangulation strip with the control and intermediate check points.

Strip Adjustment

The strip is adjusted using as the only control the base and azimuth in the first model and the base and azimuth in the last model for horizontal positions. In addition the relative elevations in the first and last models will be used to analyze the vertical deformations developed in the aerial polygon method and the effects of earth curvature. The method is briefly described by Brandenberger in (2), for a specific case, but the equations are not complete for use in general terms.

Derivation

Let us assume that we have the following given data as shown in Figure 2:

1. Two distances D_A and D_E at both ends of the strip, and approximately $\perp$ to the strip.

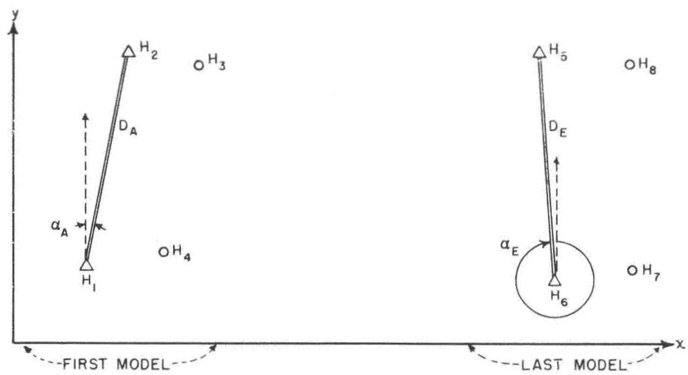

FIG. 2. Geodetic control elements.

2. Two azimuths α_A and α_E of the bases in a plane grid system.
3. Four relative elevations in the first and last models denoted by $H_1, H_2, \cdots H_8$.

Note that there need be no geodetic connection between the horizontal position data, but that the levels must be on the same datum. It is not essential that the elevation points be coincident with the horizontal control, but if the bases are measured by Tellurometer or most other geodetic means, it will be necessary to reduce them to a reference surface which will require the relative elevation at these points. It is also possible to utilize only three non-colinear elevations in each of the first and last models; however, this work was carried out with four.

If one considers a longitudinal section at y equal to a constant, a falsified profile will be obtained caused by errors of the triangulation in the autograph. The corrections following the laws of error propagation are assumed as:

$$\Delta x = a_0 + a_1 x + a_2 x^2 \cdots \tag{1a}$$

$$\Delta y = c_0 + c_1 x + c_2 x^2 \cdots \tag{1b}$$

$$\Delta H = d_0 + d_1 X + d_2 X^2 \cdots \tag{1c}$$

These laws apply for shorter strips usually up to 20 models, or about 50 km. in length; for longer strips higher order terms should be included. In order to determine the constants in the correction equations we must utilize the given data. In formulas 1a and 1b the constant coefficients a_0 and c_0 can be made zero if the origin is assumed close to D_A, as the machine system will later be transformed to a ground system N and E. Thus we can write:

$$\Delta x = a_1 x + a_2 x^2 \tag{2a}$$

$$\Delta y = c_1 x + c_2 x^2 \tag{2b}$$

If these equations (1) or (2) are differentiated with respect to x we have:

$$\frac{d\Delta x}{dx} = a_1 + 2a_2 x = \delta_s \tag{3a}$$

$$\frac{d\Delta y}{dx} = c_1 + 2c_2 x = \delta_\alpha \tag{3b}$$

Note that δ_s is the scale correction and δ_α is the azimuth correction, and that a_1 and c_1 are the scale and azimuth corrections in the first model as can easily be shown by putting x equal to zero for the first model. The azimuth correction in the first model, c_1, can however be taken into account in the transformation equations converting the final machine x and y coordinates into N and E ground coordinates. If the first model were accurately scaled, a_1 would be close to zero and could be neglected; however, in this work this was not the case so this term must be included. In practice, a_2 can be solved for using the scale correction on the end base D_E (i.e., solve for the constant using the x values of the mean base position), and the constant c_2 by using the relative change in azimuth as determined from machine coordinates and astronomic observation corrected to grid azimuths.

Due to the scale error, the y coordinates must also be corrected according to the general formula

$$\Delta y_s = g_0 y + g_1 xy \tag{4}$$

Differentiation of equation 4 with respect to y leads to

$$\frac{d\Delta y}{dy} = g_0 + g_1 x = \delta_s \tag{5}$$

where again g_0 is the scale correction in the first model and thus

$$g_0 = a_1 \tag{6}$$

and since

$$g_1 x = \delta_s - a_1, \qquad \text{and from equation 3a} \quad 2a_2 x = \delta_s - a_1;$$

$$g_1 = 2a_2 \tag{7}$$

Due to the azimuth error, the x coordinate must also be corrected by the general equation

$$\Delta x = e_0 y + e_1 xy \tag{8}$$

Again differentiation yields

$$\frac{d\Delta x}{dy} = e_0 + e_1 x = \delta_\alpha \tag{9}$$

where we can also show that

$$e_0 = c_1 \tag{10}$$

and

$$e_1 = 2c_2 \tag{11}$$

Combining these elements and using x_1 as the initial coordinate on the first base D_A at the crossing of the base and the line y equals a constant $= y_0$, based on the sketch. Generally this passes close to the mean position on the bases; thus we get, using all terms:

$$\Delta x = a_0 + a_1(x - x_1) + a_2(x - x_1)^2 + c_1(y - y_1) + 2c_2(x - x_1)(y - y_1) \tag{12a}$$

$$\Delta y = c_0 - c_1(x - x_1) - c_2(x - x_1)^2 + a_1(y - y_1) + 2a_2(x - x_1)(y - y_1) \tag{12b}$$

Note particularly the negative signs used in c_1 and c_2; in the equation for Δy this is caused by a plus azimuth correction resulting in a minus y correction. Since we have said that for our case we shall let a_0, c_0, and c_1 be $=0$ as they will be handled by transformation equations, our working equations become

$$\Delta x = a_1(x - x_1) + a_2(x - x_1)^2 + 2c_2(x - x_1)(y - y_1) \tag{13a}$$

$$\Delta y = - c_2(x - x_1)^2 + a_1(y - y_1) + 2a_2(x - x_1)(y - y_1) \tag{13b}$$

In deriving the corrections to the elevations, we go back to equation 1c and proceed to determine the constants involved by analysis of the longitudinal tilts (ϕ) of the first and last models through comparison with the given and autograph elevations. If we look at Figure 3, we see that correction graphs of the ΔH errors have been prepared based on the available elevation control points. By interpolation, the ϕ and ω errors at the lines y_0 and X_1 and X_E can be determined. Note that X_1 and X_E must be in meters on ground scale as ΔH is recorded in meters by the A-7. The values of these tilts are expressed in radians and called $\delta\phi_1$, and $\delta\omega_1$, in the first model and $\delta\phi_E$ and $\delta\omega_E$ in the end model. If equation 1c is differentiated with respect to x, we obtain

$$\frac{d\Delta H}{dx} = d_1 + 2d_2 x = \delta_\phi \tag{14}$$

where we have for x equals zero,

$$d_1 = \delta\phi_1 \tag{15}$$

and for x equals x_E, we find

$$d_2 = \frac{\delta\phi_E - d_1}{2X_E} \tag{16}$$

In addition to the effect of longitudinal tilt, we have the torsion correction which is a function of the lateral (ω) tilt and may be expressed by the equation:

$$\Delta H_t = i_0 Y + i_1 XY \tag{17}$$

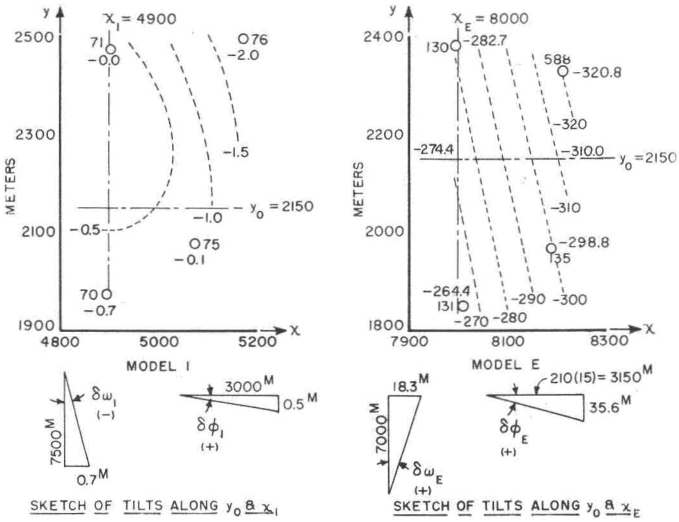

FIG. 3. ΔH error graphs.

When differentiated with respect to Y, this equation yields:

$$\frac{d\Delta H_t}{dy} = i_0 + i_1 X = \delta_\omega$$

Substitution of $X=0$ gives

$$i_0 = \delta\omega_1 \tag{18}$$

and when X equals X_E:

$$i_1 = \frac{\delta\omega_E - i_0}{X_E} \tag{19}$$

Thus the whole correction equation for H becomes, when combined in simplified form:

$$\Delta H = d_0 + i_0(Y - Y_1) + (X - X_1)[d_2(X - X_1) + i_1(Y - Y_1) + d_1]$$

where in this instance d_0 is the mean correction to the four points in the first model that is necessary if the first model is not leveled to the standard tolerance. Note again that since H is in meters from the autograph, the X and Y terms must as well be in meters. It is probably easiest to accomplish this by multiplication of the x and y terms previously determined by the scale number/1,000. In the case of our problem this will mean multiplication by 15.

After the adjustment of the strip according to these derived correction equations, the comparison with the computed X, Y and H coordinates with the factual test points can be made, and the errors in the method and assumptions found.

COMPUTATIONS FOR GIVEN DATA

BASE DISTANCES AND δ_S

Given distance △70–△71 $D_A = 7{,}551.26$ m.→503.42 mm. in the model
Given distance △130–△131 $D_E = 8{,}001.40$ m.→533.43 mm. in the model
Based on Table IV

$$d_A = 10.45^2 + 502.77^2 = 502.88 \text{ mm.}$$

$$d_E = 16.21^2 + 533.57^2 = 533.82 \text{ mm.}$$

Thus the scale factors are:

$$SF_A = \frac{503.42}{502.88} = 1.001073814$$

$$SF_E = \frac{533.43}{533.82} = 0.999269416$$

Since the corrections equal the scale factor minus one:

$$\delta S_A = 0.001073814$$

$$\delta S_E = 0.000730584$$

AZIMUTHS AND δ_α

This correction is best computed with a sketch; however, an equation can be developed (3) from the geometry.

Given azimuth of D_A 359°05′05″

Given azimuth of D_E 356°02′26″

From the machine coordinates of Table IV we find

$$\text{azimuth of } d_A = 1°11'39''$$

$$\text{azimuth of } d_E = -\ 1°44'25''$$

Thus the angle between the ground and autograph azimuths are

$$\theta_A = 2°06'34''$$

$$\theta_E = 2°13'09''$$

Since these should be the same if there were no relative rotation, there is relative rotation that must be corrected by rotating the model through $-06'35'' = \delta\alpha_E$ in angle units or by conversion to radians:

$$\delta\alpha_E = -\frac{6.5833}{3{,}437.8} = 0.00191497$$

LONGITUDINAL AND LATERAL TILTS

Based on Figure 3 the values obtained are:

From model 1.

$$\delta\phi_1 = \frac{+.5}{3{,}000} = +\ 0.000167 \text{ radian}$$

$$\delta\omega_1 = \frac{-.7}{7{,}500} = -\ 0.000093 \text{ radian}$$

From model E

$$\delta\phi_E = \frac{+35.6}{21(15{,}000)} = +\ 0.0113 \text{ radian}$$

$$\delta\omega_E = \frac{18.3}{8{,}000} = +\ 0.0023 \text{ radian}$$

COMPUTATION OF CORRECTION CONSTANTS

From the above calculations, the constants for the correction equations can be computed using the previously derived equations. Based on Figure 3, we will use

$$x_0 = 4{,}900, \qquad y_0 = 2{,}150$$
$$x_E = 8{,}000, \quad \text{and thus}$$
$$(x_E - x_0) = 3{,}100$$

From Eq. 3a:

$$\underline{a_1 = \delta S_A = +\,0.001073814}$$

$$a_2 = \frac{\delta S_A - a_1}{2(x_E - x_0)} = \frac{-0.001804398}{2(3{,}100)} = \underline{-2.91032 \times 10^{-7}}$$

From Eq. 3b: Assume $c_1 = 0$
Then

$$c_2 = \frac{\delta \alpha_E}{2(x_E - x_0)} = \frac{-0.00191497}{2(3{,}100)}$$
$$\underline{c_2 = -\,3.08867 \times 10^{-7}}$$

From Eq. 6:

$$g_0 = a_1 = \underline{0.001073814}$$

From Eq. 7:

$$g_1 = 2a_2 = \underline{-0.000000582064}$$

From Eq. 11:

$$e_1 = 2c_2 = \underline{-0.000000617734}$$

Note that a_0, c_0, c_1, e_0 equal 0 for this case.
From Eq. 15:

$$d_1 = \delta\phi_1 = +\,0.000167$$

From Eq. 16:

$$d_2 = \frac{\delta\phi_E - d_1}{2(X_E - X)} = \frac{0.01113}{2(3{,}100)15} = \underline{0.0000001197}$$

In addition, from model 1,

$$d_0 = \frac{-\,.7 - .7 - 2.0 - 0}{4} = \underline{-.8\,\text{m.}}$$

From Eq. 18:

$$i_0 = \delta\omega_1 = \underline{-\,0.000093}$$

From Eq. 19:

$$i_1 = \frac{0.002393}{3{,}100(15)} = \underline{5.146 \times 10^{-8}}$$

This results in the following table of correction equation constants:

$$a_1 = 0.001073814$$
$$a_2 = -\ 0.000000291032$$
$$2a_2 = -\ 0.000000582064$$
$$c_2 = -\ 0.000000308867$$
$$2c_2 = -\ 0.000000617734$$
$$d_0 = -\ 0.8 \text{ m.}$$
$$d_1 = 0.000167$$
$$d_2 = 0.0000001197$$
$$i_0 = -\ 0.000093$$
$$i_1 = 0.0000005146$$

In these constants no attempt has been made to analyze the best number of significant figures as all work is done on a calculator and rounding will be left for the final results.

SAMPLE TABULATION

Original data from Table IV of the Appendix:

Pt.	70	71	130	131
x	4,890.66	4,901.14	7,992.34	8,008.55
y	1,971.82	2,474.59	2,383.76	1,850.19
$x-x_1$	−9.34	1.14	3,092.34	3,108.55
$y-y_1$	−178.18	324.59	233.76	−299.81
$(x-x_1)^2$	87.2356	1.2996	9,562,566.6756	9,663,083.1025
$(x-x_1)(y-y_1)$	1,664.2012	370.0326	722,865.3984	−931,974.3755
$a_1(x-x_1)$	−0.010	0.001	3.321	3.338
$a_2(x-x_1)^2$	−0.000	−0.000	−2.783	−2.812
$2c_2(x-x_1)(y-y_1)$	−0.001	0.000	−0.447	+0.576
Δx	−0.011	0.001	0.091	1.102
$-c_2(x-x_1)^2$	0.000	0.000	2.954	2.985
$a_1(y-y_1)$	−0.191	0.349	0.251	−0.322
$2a_2(x-x_1)(y-y_1)$	−0.001	−0.000	−0.421	0.542
Δy	−0.192	0.349	2.784	3.205
$X-X_1$	−140.10	17.10	46,385.10	46,628.25
$Y-Y_1$	−2,672.70	4,868.85	3,506.40	−4,497.15
$[C]$	0.000016	0.000423	0.005903	0.005520
$(X-X_1)[C]$	−0.002	0.006	273.811	257.388
$i_0(Y-Y_1)$	0.249	−0.453	−0.326	0.418
ΔH	1.0	0.4	274.3	258.6
		Computed autograph values		
x	4,890.65	4,901.14	7,992.43	8,009.65
y	1,971.63	2,474.94	2,386.54	1,853.40
H	419.7	560.9	521.7	465.5
true H	419.4	560.5	530.1	471.3
Error e_H (Meters)	+0.3	+0.4	−8.4	−5.8

TRANSFORMATION EQUATIONS

The transformation equations used are called the Two-point Coordinate Transformation Equations which assume orthogonal coordinates (4). The general form is:

$$N = Ax + By + C'$$
$$E = Ay - Bx + C''$$

where

$$Y' = Y_1 - Y_2, \qquad X' = X_1 - X_2, \qquad x' = x_1 - x_2, \qquad y' = y_1 - y_2$$

$$m = x'^2 + y'^2$$

$$n = x'X' + y'Y'$$

$$p = y'X' - x'Y'$$

$$A = n/m, \qquad B = p/m$$

$$C' = X_1 - Ax_1 - By_1 = X_2 - Ax_2 - By_2$$

$$C'' = Y_1 + Bx_1 - Ay_1 = Y_2 + Bx_2 - Ay_2$$

In the case of this problem using the coordinates as given in the Schweizer Block data:

Pt.	X	Y
70	244,905.40	650,506.80
71	252,455.70	650,386.20

We can compute:

$$Y' = -7{,}550.30, \qquad X' = 120.60$$

$$y' = -503.31, \qquad x' = -10.49$$

$$m = 253{,}430.9962, \qquad n = 3{,}798{,}876.399, \qquad p = -139{,}901.833$$

$$A = 14.989786, \qquad B = -0.552031263, \qquad C' = 578{,}285.40$$

$$C'' = 212{,}651.30$$

Using these transformation constants, we obtain the following values and checks on the given data:

Pt	Calculated		Given Coords.		Errors (meters)	
	E	N	E	N	e_x	e_y
70	650,506.80	244,905.40	650,506.8	244,905.4	0.0	0.0
130	696,772.77	252,837.10	696,792.0	252,821.0	+19.27	−16.1
131	697,325.20	244,854.90	697,344.5	244,838.5	+19.3	−16.4

The complete tabulation of data and computations, are given in the Appendix, Tables I–X.

Conclusions

In utilizing work of this nature, it is of utmost importance to have an idea of the errors to be expected so that the work system can be properly designed. According to Brandenberger (2) the standard closing error for this type of strip with similar control and length is $m_x = m_y = \pm 10.3$ m. and the error in elevation is $m_H = \pm 12.0$ meters. If the closing error is defined as the error at the end of the strip, we have from our case as shown in Figure 4, the following closing errors:

$$e_{x_E} = -16 \text{ m.}$$

$$e_{y_E} = +14 \text{ m.}$$

$$e_{H_E} = -12 \text{ m.}$$

It should be noted, however, that the error, as shown in Figure 4, is not linear in x for all cases. The curve shows that the expected error in this case does not greatly exceed 20 meters and has an average value close to those of the standard errors shown by Brandenberger. Since this work was done without the normal standard care in the orientation of the first model, and as it was also done for experience and training

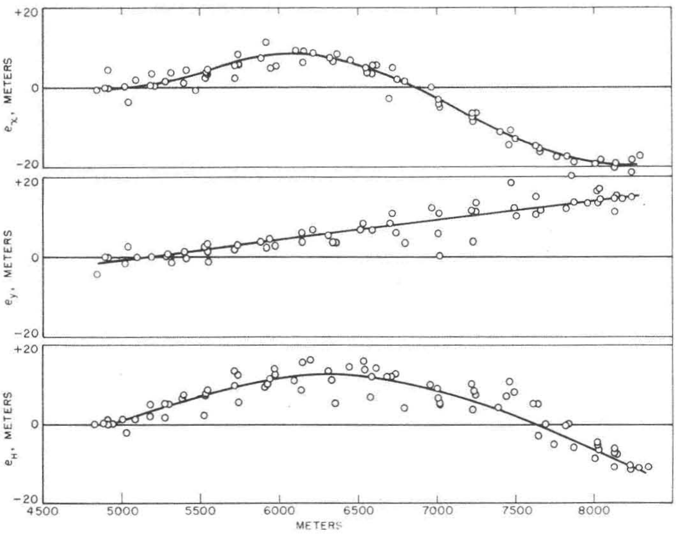

FIG. 4. Error distribution curves.

on the A-7, it logically follows that one would expect the errors to be higher than those found by Brandenberger. Even in this case, utilizing the minimum of control, the errors are acceptable [5] for control of small-scale mapping in the order of 1:50,000 or 1:100,000 for position, and for plotting of approximately 50 meter contours. The advantage of the method lies in the minimum geodetic control required and thus the saving in cost, particularly in adaption for mapping in undeveloped areas of the world, or in preparation of maps of areas in which geodetic control is unavailable or physically unattainable. The extension of this technique to longer strips in areas where aerial inspections were allowed would give good mapping of countries now lying behind closed borders, utilizing only geodetic control in free world areas.

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APPENDIX*

* Only Tables IV and X of the original computations are included.

(*See next page for tables*)

Appendix Table IV

Point	*x*	*y*	*z*	*Remarks*	*Point*	*x*	*y*	*z*	*Remarks*
71	4,901.14	2,474.59	560.5	1 model only	261	6,779.22	2,405.47	432.0	Control
72	4,904.77	2,227.43	406.3	1 model only	107	6,776.20	2,440.60	403.6	1 model only
69	4,836.36	2,028.61	408.9	1 model only	105	6,701.40	1,912.52	424.3	
70	4,890.66	1,971.82	418.7	1 model only	236	6,737.64	1,964.55	591.1	Control
183	5,176.05	1,982.10	447.5	Control	104	6,684.14	1,987.18	532.1	
75	5,075.55	2,076.89	424.5		106	6,719.39	2,170.85	363.4	
77	5,178.95	2,042.62	448.3		111	7,007.34	2,426.29	331.5	1 model only
74	5,008.35	2,245.69	397.7		271	7,017.79	2,402.13	320.3	Control
73	5,027.59	2,440.76	393.9		108	6,946.61	1,888.54	323.5	
76	5,171.78	2,496.80	346.9		109	6,996.64	2,012.38	306.6	
182	5,157.17	2,228.73	407.3	Control	110	6,989.99	2,200.95	288.2	
78	5,267.81	2,239.63	412.3		113	7,213.57	2,380.56	272.5	
82	5,387.73	2,278.11	417.2		115	7,227.15	1,868.56	325.0	
191	5,394.03	2,429.34	417.3	Control	112	7,205.32	1,985.51	326.5	
79	5,270.76	2,443.08	387.5		114	7,221.49	2,128.04	476.9	
80	5,302.90	1,945.92	484.2		116	7,371.42	2,391.87	285.7	
193	5,395.11	1,949.37	563.2	Control	117	7,445.91	1,873.26	468.8	1 model only
81	5,394.27	2,097.02	455.4		293	7,449.66	1,893.52	471.6	Control
192	5,394.27	2,232.19	417.9	Control	119	7,472.29	1,987.50	441.0	
84	5,517.31	2,475.81	400.2		118	7,439.15	2,204.70	262.0	
83	5,532.68	1,962.46	441.1		125	7,631.18	2,430.15	250.1	
86	5,532.16	2,031.86	425.3		301	7,658.07	2,361.54	279.2	Control
85	5,529.06	2,215.20	414.9		124	7,602.76	1,914.29	379.0	
90	5,736.18	2,472.72	342.0		126	7,630.62	2,153.94	240.9	
89	5,725.85	1,944.10	407.4		123	7,843.70	2,400.43	261.6	
88	5,707.45	2,032.01	445.7		127	7,719.05	2,410.46	254.5	1 model only
87	5,702.88	2,224.53	527.4		122	7,826.62	1,881.44	207.3	
91	5,868.55	2,028.89	400.1		120	7,764.38	1,989.21	216.1	
92	5,962.31	1,994.31	371.2		121	7,801.37	2,167.16	226.4	

Appendix Table X

Point	*Errors*			*Point*	*Errors*		
	e_x	e_y	e_H		e_x	e_y	e_H
71	0	0	+ .4	72	+ 4.65	x	+ 1.3
69	− .33	− 4.24	+ .4	70	0	0	+ .3
75	+ 2.08	− .1	+ 1.3	77	+ 3.80	− .2	+ 5.0
74	+ .20	− 1.76	+ 1.5	73	− 3.89	+ 2.61	− 2.1
76	+ .42	− .01	+ 2.1	78	+ 1.73	+ .06	+ 5.3
82	+ 1.33	+ 1.51	+ 6.3	79	+ .59	+ .78	+ 1.9
80	+ 3.99	− 1.46	+ 5.6	81	+ 4.31	− .9	+ 7.0
84	+ 2.52	+ 2.27	+ 2.4	83	+ 3.25	− 1.23	+ 8.4
86	+ 4.87	+ 1.21	+ 7.8	85	+ 3.97	+ 3.23	+ 7.0
90	+ 5.91	+ 2.63	+ 5.6	89	+ 8.45	+ 2.48	+12.3
88	+ 5.74	+ 1.94	+13.6	87	+ 2.2	+ 1.85	+ 9.5
91	+ 7.76	+ 3.66	+ 9.9	92	+ 5.67	+ 2.27	+12.4
93	+ 4.66	+ 4.15	+11.5	567	+11.54	+ 2.24	+ 9.4
575	+ 8.89	+ 3.76	+ 8.4	96	+ 8.64	+ 6.34	+16.0
95	+ 6.1	+ 5.49	+15.7	94	+ 9.04	+ 4.48	+10.9
97	+ 8.26	+ 3.89	+ 5.3	578	+ 7.7	+ 4.97	+13.3
580	+ 6.63	+ 3.49	+11.2	101	+ 3.6	− 4.75	+ 6.9
98	+ 6.6	+ 7.12	+14.2	103	+ 5.69	+ 6.15	+14.3
99	+ 4.80	+ 7.9	+15.9	102	+ 5.61	+ 6.35	+12.1
100	+ 3.58	+ 6.48	+13.5	107	+ 1.55	+ 3.19	+ 4.0
105	+ 4.97	+10.83	+11.9	104	− 3.06	+ 7.89	+11.9
106	+ 1.92	+ 5.82	+12.6	111	− 4.85	+ .31	+ 5.3
108	− .43	+12.17	+ 9.9	109	− 3.08	+10.43	+ 9.1
110	− 4.44	+ 5.55	+ 6.1	113	− 8.56	+ 3.78	+ 3.8
115	− 7.4	+15.59	+ 8.2	112	− 6.7	+11.60	+10.0
114	− 7.39	+10.92	+ 7.8	116	−11.23	x	+ 4.1
117	−10.98	+18.13	+10.7	119	−12.96	+11.97	+ 8.1
118	−14.64	+ 9.88	+ 7.3	125	−16.01	+10.12	− 3.1
124	−14.77	+14.93	+ 5.0	126	−15.86	+11.74	+ 4.9
123	−18.92	+13.77	− 5.4	127	−17.82	x	− 5.0
122	−22.59	x	+ 0.3	120	− 4.12	x	+ 0.6
121	−17.91	+11.94	− 0.3	130	−19.23	+16.1	− 8.4
131	−19.3	+16.25	− 5.8	129	−18.39	+14.1	− 4.9
128	−17.25	+13.28	− 4.5	133	−20.63	+11.17	− 5.9
213	−16.50	+15.17	− 7.2	135	−18.60	+14.72	−11.1
592	−17.49	x	−10.8	134	−19.35	+14.25	− 6.9
588	−17.13	+14.89	−10.1	132	−17.97	+14.47	−10.4