*Small Scale Mapping**

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THE title of this paper is, of course, far too universal; one could write a book of several hundred pages on this subject. This paper will include only a few words about the methods and related equipment now available to the photogrammetrist who is busy with small-scale mapping projects. When discussing maps at small scales I think of maps of from 1:25,000 down to 1:100,000 and eliminating the even smaller scales, since so-called cartographic maps on those small scales are usually compiled by reduction from larger scales and with appropriate generalization of the contents. In other words, the author has in mind for this paper maps which can be plotted directly from aerial photographs by one of the many photogrammetric methods.

The basic material for a photogrammetrically produced map is the photographic negative. This must contain not only all the topographic detail to be represented on the map, but also it must allow an easy and accurate identification of these objects. Hence, the first important question when a photogrammetric mapping project is studied is: What must be the photograph-scale? The immediate answer is: As small as possible for economical reasons.

The first somewhat complete paper on this matter was published by Dr. Otto v. Gruber in 1937. He established a mathematical relationship between map-scale and the corresponding photograph-scale. He chose the general form of the formula:

$$S_p = C_1 \cdot S_m^{C_2}$$

and found from experiences in different countries the values C_1 to about 120 and $C_2 = 0.5$. Therefrom,

$$S_p = 120\sqrt{S_m}$$

The same problem was thoroughly investigated in 1954 by Dr. Viktor Heissler from the Institute of Technology in Hannover. Heissler found the above formula fully justified, but changed the factor from 120 to about 200, this increase being made possible by the improvements in the manufacture of camera lenses and of the photographic emulsions. Today, it is possible to go still further and to adopt the formula

$$S_p = 250\sqrt{S_m}$$

However, the planner should not adhere strictly to the factor 250, but use a higher or lower value according to the purpose of the map, the requirements in accuracy, the quality of the camera, the base-height ratio, etc. It is obvious that the quality of the lens is of primary importance; the best lens is the most economic one.

The photograph-scale fixes the flight altitude for a given focal-length of the camera. Applying the above mentioned formula for a 50,000 map, for instance, would require a photo-scale of 1:56,000 thereby resulting in a flight altitude of 28,000 ft. with a 6-inch camera. This is rather high and needs special aircraft and auxiliary equipment for the air crew. A shorter focal-length is therefore desirable, maintaining, of course, the same standard picture size.

Here is where the super-wide-angle camera has its place. A focal-length of 3½ inch, for instance, as it is the case with the WILD Super-Aviogon lens, designed by Dr. L. Bertele, reduces the flight altitudes by 42% in comparison to a 6-inch lens, bringing it down to 16,000 ft. for the mentioned example.

The reduction of the necessary flight altitudes is then of great advantage when the maximum altitude is limited by a low ceiling of the aircraft, or by a low formation of clouds; the photographs will then be close to the right scale. In some cases the size of the publication sheets of the map determines the area to be covered by one stereoscopic model. Here again the flight altitude is fixed by the required photograph scale and the focal-length of the camera.

In the future the practical photogrammetrist will certainly prefer the camera with the widest angular field provided the image definition is equal to that of customary cameras. It may be of interest to know that the resolv-

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ing power of the WILD Super-Aviogon is better than that of the 90° Aviogon with the exception perhaps of the extreme corners. The figures in lines-per-millimetre are as follows:

Angle:	0	5	10	15	20	25	30	35	40	45	50	55	60°
Lines/mm.:	81	80	76	71	60	48	43	38	39	45	31	13	11

This represents the average between radial and tangential lines measured on five different Super-Aviogon lenses.

In this connection mention also might be made of the law of distribution of light from the center to the corner of a photographic image. The correct formula is

$$E = x \cdot Q \cdot \cos^4 \phi$$

The light intensity is primarily proportional to the fourth power of cos ϕ. But it is also proportional to the size of the entrance pupil.

From four images of the entrance pupil of Super-Aviogon, at 0, 50, 100 and 130 mm. one obtains the following figures:

Distance from center:	0	50	100	130 mm.
Area of entrance pupil:	1	1.15	1.58	1.86
Angle of incidence ϕ:	0	29.3	48.4	55.6°
cos ϕ:	1	0.872	0.664	0.565
$\cos^4 \phi$:	1	0.578	0.194	0.102
$E = area \cdot \cos^4 \phi$:	1	0.665	0.307	0.190

From these last figures, using the expressions

$$E = \cos^x \phi \quad \text{and} \quad x = \frac{\log E}{\log \cos \phi}$$

it follows:

log E:	0	−0.176	−0.514	−0.723
log cos ϕ:	0	−0.060	−0.177	−0.247
x:		2.93	2.90	2.92

$$E = x \cdot \cos^{2.9} \phi$$

Coming now to discussions of photogrammetric map compilation from small-scale superwide-angle photographs, two sets of instruments are actually in use or will be available in the course of this year. According to the required map-scale and map-accuracy one or the other equipment will be preferred. The following procedure and equipment are considered as normal for small publication scales.

The first step is a reduction of the original negatives to half of their size, i.e., to $4\frac{1}{2}$ by $4\frac{1}{2}$ inches. This can be done in any universal diapositive printer.

Simultaneously with the reduction, the negative should be corrected for lens distortion and, if so desired, also for earth curvature and refraction. Distortion compensation is no problem and well known. For the treatment of earth curvature and atmospheric refraction the following formula is applied:

$$\Delta x = \frac{1 - k}{2r} \cdot \frac{H}{f^2} \cdot x^3$$

It is

k = refraction factor = 0.13
r = earth radius
H = flight altitude
f = focal length of camera
x = distance from the principal point.

$$\Delta x = 2.08 \cdot 10^{-8} \cdot \frac{H}{f^2} \cdot x^3 \text{ millimetres,}$$

if H is taken in feet and f and x in millimetres.

The image displacement Δx, of course, varies with the flight altitude, and different correction plates have to be used for different altitudes. If a displacement of ±10 microns is tolerated, a separate correction plate must be used for every 5,000 ft. variation in flight altitude.

Obviously, the reduction printer must be of such a quality that there is no loss of image definition through the process of projection. Since for small-scale mapping there are never sufficient ground-control points available for levelling and scaling of the models, the method of aerial triangulation or control extension is applied. For this purpose the transfer points from model-to-model should be selected and marked on the diapositives in advance.

On photographs with little detail, such as of desert areas, or with uniform coverage of the ground as with virgin forest, transfer points are not easily recognized and described. In fact, with a point transfer device there is no description or sketch of the location necessary, and with the points pricked on the diapositive no identification error is possible. A point transferring device is also of great advantage for the transfer of pass-points from one strip to the neighboring parallel strip. The third step is the aerial triangulation proper. So far there is only one instrument on the market which takes super-wide-angle photographs, the WILD A9 Autograph. It uses the half-size diapositives. This instrument is already in use in the U. S. and details have been published.

Once the triangulation had been completed, people used to rack their brains to find the most suitable and most reliable method of compensating the errors found at the various control points within the strip, and even more so, if there was a combination of several parallel strips to be tied together. The graphical method used by the Army Map Service and the U. S. Coast & Geodetic Survey is a very efficient method for single strips; For a block of strips the best answer is probably given by Dr. H. G. Jerie's Analogue Computer, designed in collaboration with the ITC in Delft, a combined numerical and mechanical method. It guarantees a uniform distribution of the residual errors and requires a minimum number of control points.

Compensation being completed and the final coordinates of the transfer points established, the plotting of the detail can be done on the A9 Autograph or preferably and more economically with the WILD B9 Aviograph. This B9 is a new desk model plotting instrument which can be defined as something between a second order and a third order instrument.

It uses mechanical means to give a rigorous solution to the geometric problem involved. One-half size diapositives are used in the A-9 but it is less universal and limited in its range. It is built for a constant focal length of ± 3 mm. and the range of the vertical motion is such that the ratio between the original negative scale and plotting scale can be varied between 0.9 and 1.6. This is in line with the previously mentioned law:

$$S_p = 250\sqrt{S_m}.$$

When the previously mentioned law is observed, the B-9 Aviograph allows the direct plotting of scales from 1:25,000 to 1:75,000. The diapositives are viewed under a magnification of 7.5.

If for some reason a higher accuracy is required, a second procedure is preferred entailing the use of quite a different set of instruments. Here, original size diapositives are used.

Preparation of these diapositives on the Point Transfer Device will be the same.

But the pictures will then be placed into a Precision Stereocomparator, where the image coordinates and x and y-parallaxes are measured.

An electronic computer does the aerial triangulation in no time, together with the compensation of the entire job and supplies any other data which may be of interest.

For detail plotting yet another new small plotter, the WILD B8 Aviograph has been designed. It takes the original 9×9 inch diapositives which can be plotted with the use of a pair of correction plates for radial distortions. The range of enlargement from photograph scale to model scale extends from 1.7 to 2.3. times. Besides, there will be a sort of a pantograph for further enlargement or reduction from model scale to compilation scale. The magnification of the observation system is 6 times, and, quite obviously, we shall maintain the well approved method of perpendicular observation of the photographs. The B8 cannot only be used for super-wide-angle photographs, but also accommodates ordinary wide-angle pictures with 6-inch focal length. This fact and the presence of a pantograph make the B8 a more versatile instrument than the B9 Aviograph. Both the B8 and the B9 will be exhibited at the coming International Photogrammetric Congress in London.

For smaller areas where there is sufficient ground control available, no aerial triangulation will be necessary and the entire work can be done on the B8 Aviograph, as, for instance, for preliminary technical plans for civil engineering projects. There is no lack of occasions where super-wide-angle photography is of great advantage.