*Analytical Aerotriangulation Applied to Photogrammetric Mapping for Highway Design**

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ABSTRACT: *Analytical aerotriangulation may soon eliminate much of the ground control now considered necessary for aerial photography. Analytical aerotriangulation by cantilever extension and by bridging was applied to a short strip of five actual aerial photographs. A comparison of the computed position of points with the actual values provides a measure of the reliability of the methods employed.*

The results indicate that the analytical cantilever extension method could be used to establish control for mapping used in highway design where a low order of accuracy is sufficient. In its present form, the application of analytical bridging to photogrammetric mapping is not feasible. The accuracy which was obtained by analytical bridging indicates that further research in this field is justified.

INTRODUCTION

IN RECENT years various solutions to problems in analytical aerotriangulation have been developed. In this investigation analytical methods were employed to extend control and compute the position of points in a short strip of actual aerial photography. The information obtained from these tests provided sufficient data to determine the accuracy of the methods and the feasibility of applying analytical techniques to photogrammetric highway mapping.

Maps and plans produced photogrammetrically, for highway design, must meet specific standards of accuracy. The U. S. Bureau of Public Roads has issued recommendations for the standards of accuracy for all types of highway design projects.[1] To satisfy the requirements of this Bureau, an adequate number of surveyed ground-control points is needed in each stereoscopic model. Four vertical and two horizontal ground-control points are generally considered desirable. Consequently, extensive ground-control networks are necessary for compilation. Even when modern electronic surveying instruments are employed, the cost of establishing the ground-control is from 30% to 50% of the entire cost of the map.

The amount of ground-control which might be eliminated is subject to debate. Some form of horizontal and vertical-control will eventually be required for the final stake-out of the highway. To satisfy this requirement, a primary traverse is usually run throughout the length of the strip under study. Naturally, an effort is made to place these traverse points where judgment indicates they might be of further use with respect to the eventual highway stake-out. However, excessive concentration in this direction can lead to much waste, since the final highway location may be far removed from these points. Therefore, the principal objective in establishing the primary-control traverse should be to obtain an accurate speedy closure throughout the length of the strip, placing the points where they will be useful in stereocompilation. By using modern surveying instruments, a primary traverse may be established quickly and accurately with the traverse points often from 1 to 5 miles apart. This leaves large gaps which contain no ground control points for controlling photo models. Supplementary surveys must be run to establish addition control.

One technique which is employed to reduce control is aerotriangulation. This by cantilever extension or bridging may be accom-

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plished either by instrumental or analytical methods. For instrumental aerotriangulation, first-order plotting equipment is used to provide the best results.

Recently, analytical techniques for establishing control, which are theoretically correct, have been developed, both for the cantilever extension and bridging. One of the advantages of analytical aerotriangulation is that corrections for lens distortion, film shrinkage, and camera calibration may be applied to the observed values before computation. Subsequently, a least squares adjustment of a multiplicity of values may be made to obtain the best results.

From July 1955 through October 1958, a series of reports were issued at Cornell University in which a method for the special case of cantilever extension and a solution to the general problem in aerotriangulation were developed. These reports were written under the direction of Professor Arthur J. McNair in the Department of Civil Engineering at Cornell University.[2,3,4,5,6,7]

The cantilever method has been tested with actual photography. The solution to the general aerotriangulation problem, known as bridging, had not been tested with actual data at the time of this study. In this paper the Cornell methods were employed to calculate the position of control points for a highway location. The computed positions were then compared with the actual locations of the points as determined by ground surveys. This comparison: (1) provided a measure of the accuracy of each method when applied to actual photography; and (2) allowed a determination of the feasibility of applying these methods to photogrammetric mapping for highway design.

Data Acquisition and Preparation

Photography from various locations was considered for test purposes. A strip of photographs taken in western Pennsylvania for a highway location was ultimately selected. The procedure for preparation and collection of data was divided into the following phases:

A. Study of available photography to select a test section.
B. Determination of the photographic coordinates.
C. Preparation of the ground-control data for computation.
D. Preparation of the estimates.

With the above steps completed, the computation of control extension could proceed.

A. Study of Available Photography to Select a Test Section

The photography from which the test section was chosen consists of 41 photographs used to compile the plans for the design of a section of Interstate Route 34 north of Pittsburgh, Pa. The map was compiled instrumentally by the Aerial Map Service Corporation of Pittsburgh, and was used by Richardson-Gordon & Associates of Pittsburgh, the firm engaged to design the highway. This part of Interstate 34 extends from downtown Pittsburgh to the Perry Interchange of the Pennsylvania Turnpike, and covers a distance of approximately eighteen miles. The photography was taken from an elevation of about 6,000 feet above sea level on May 23, 1958. The camera used was a Fairchild Type K-17 Camera with a Bausch and Lomb Metrogon lens having a calibrated focal length of 153.59 mm.

Horizontal control was established after the photography was taken and consisted of two stages: (1) a primary traverse established through the entire length of the proposed route; and (2) supplemental surveys run to locate picture control points. All horizontal points were located by closed traverse. Coordinates of these horizontal ground-control points were available in Pennsylvania State Plane Coordinates. Vertical-control was established by differential leveling.

The entire strip was studied and a five photograph strip was selected. Emphasis was placed on a fairly dense distribution of ground-control in photographs having images easily identified. This section lies about one mile south of the Perry Interchange of the Pennsylvania Turnpike. Figures 1, 2, and 3 show the location of the proposed route, the layout of the primary traverse and the test photography. The test strip and distribution of horizontal and vertical control points are shown in Figure 3.

B. Determination of the Photographic Coordinates

Measurement of the photographic coordinates was performed on a Mann Monocular Comparator. This instrument was made available through the courtesy of Professor Arthur H. Faulds of the Civil Engineering Department, Syracuse University, Syracuse, New York.

Glass plate diapositives must be used when making measurements on the Mann Comparator. For this experiment, the same Kelsh plates used for the map compilation were available.

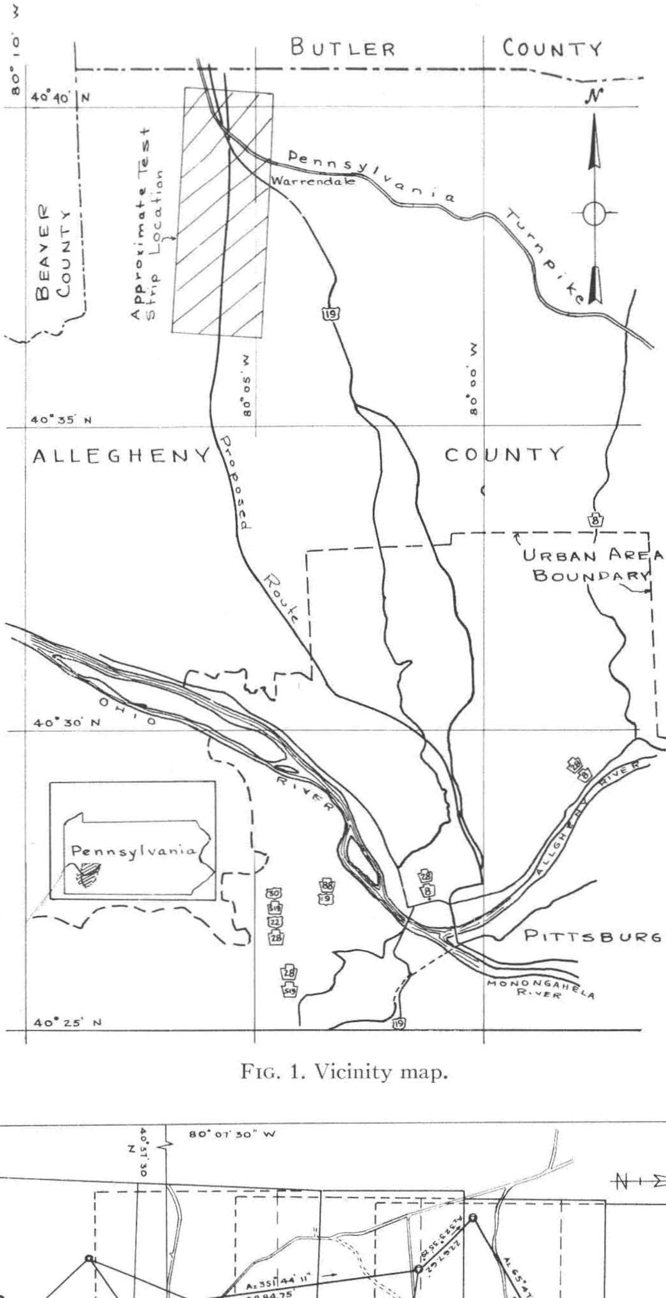

FIG. 1. Vicinity map.

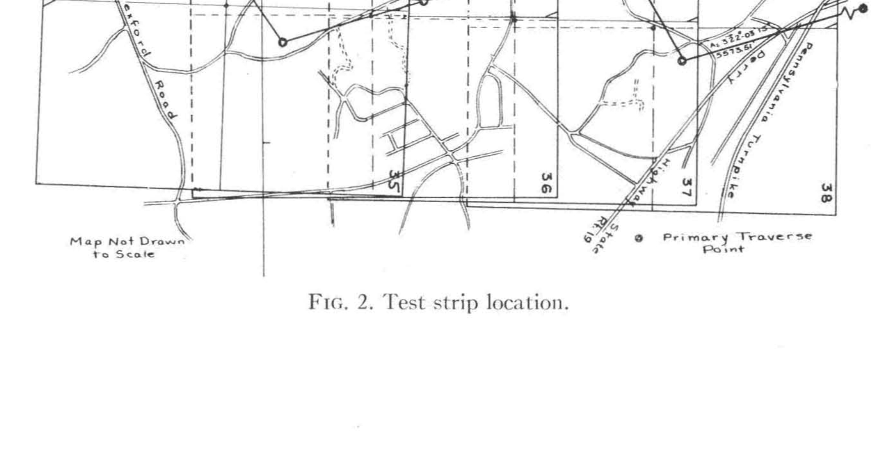

FIG. 2. Test strip location.

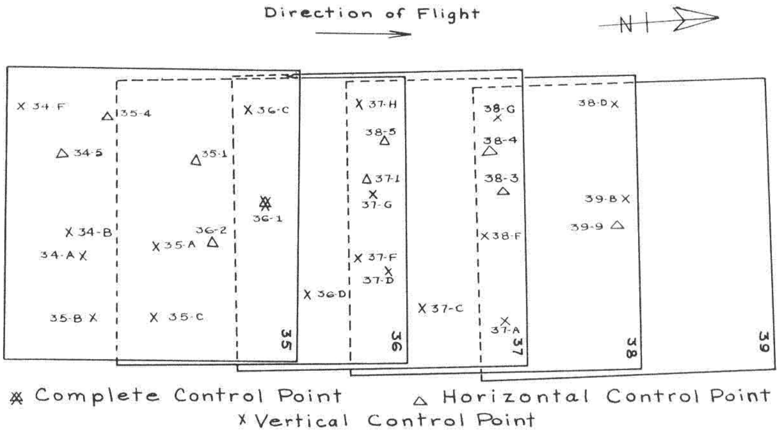

FIG. 3. Test strip and distribution of horizontal and vertical control points.

A total of 73 measurements were made to image points on five photographs containing 29 ground-control points. Forty clock hours were required to complete these measurements.

Since the Mann Comparator is a monocular instrument, the chief source of errors lay in the identification of the same point appearing in the overlap of three photographs. Another source of difficulty was in being positive that the point being observed actually was the object located in the field. A summary of the errors believed present in the photographic coordinates is as follows:

Instrumental Error	±0.001 mm.
Identification of a point on a single photograph	±0.004 mm.
Identification of the correct point on a single photo	±0.050 mm.

The Standard Error $=\sqrt{(0.001)^2+(0.004)^2+(0.050)^2}$

$=0.050$ mm.

Maximum Error $=0.150$ mm.

These estimates were made assuming a truly vertical photograph. The station position was interpolated using the photograph in conjunction with the position of ground-control points which could be identified on a quadrangle map. A detailed procedure for arriving at the estimates is given in the Cornell Final Report.[8]

The values of measured photographic-coordinates, the ground-control data and estimates of exposure station position were then fed to the electronic computer. A detailed description of the input format and operating procedure for the cantilever program is contained in the Cornell Report on Cantilever Extension for Convergent Photography.[9] A similar description for the bridge program is found in the Cornell Final Report.[10]

C. PREPARATION OF GROUND CONTROL DATA FOR COMPUTATION

Vertical ground-control points were available in feet above sea level and the elevations were used directly. The horizontal points were given in Pennsylvania State Plane Coordinates. For reasons best known only to the electronic computer, these values had to be converted to latitude and longitude and from that to geocentric coordinates.

D. PREPARATION OF ESTIMATES

The computing system solves a series of simultaneous linear equations, employing the Newton method of approximation. Therefore it was necessary to make a first estimate of exposure station position and orientation.

COMPUTATIONS AND PRESENTATION OF RESULTS

Computations were arranged to test the performance of the Cornell bridging method when used with actual data, and to determine the accuracy which could be obtained using both the cantilever and the bridging solutions. This program was divided into two major phases:

A. An analytical extension of control by the cantilever method to check the accuracy of the photographic measure-

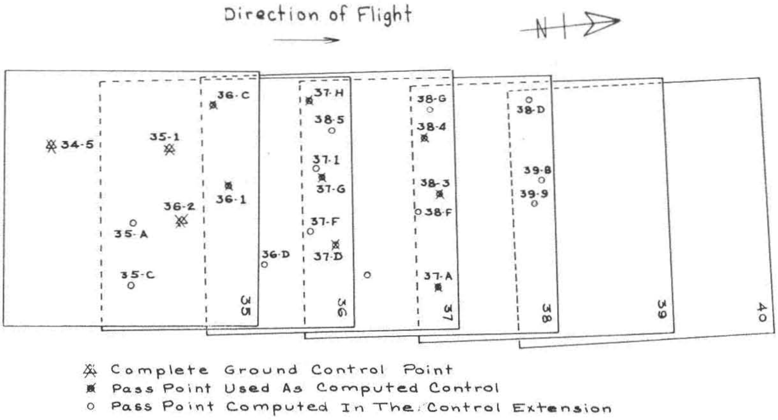

FIG. 4. Distribution of control points.

ments and ground control data. This control extension was also used to determine the feasibility of employing the Cornell cantilever technique for highway mapping.

B. Tests with two and three photograph bridges, including the computation of ground points to check the accuracy of the bridging method when used with actual data.

All computations were carried out at the Cornell Computing Center on the IBM 650 electronic drum calculator. Program decks were available for both the cantilever and bridge solutions and a supplementary program was written for the computation of additional ground points. Calculations for the cantilever extension of control were carried out first.

A. EXTENSION OF CONTROL BY THE CANTILEVER METHOD

Control was extended through a four-model strip using the analytical cantilever method. The distribution of control in the photographs is illustrated in Figure 4. When using the cantilever method it is necessary to have a minimum of three complete ground control points in the initial model. These points are indicated in Figure 4. The horizontal and vertical positions of the balance of the points were then computed. The maximum errors, occurring in the third and fourth models, for the unadjusted cantilever extension were 27.00 feet in elevation and 16.46 feet in horizontal position.

When an instrumental cantilever extension is attempted, an empirical or mathematical adjustment is usually made to reduce the errors at the end of the strip. The spline adjustment is one of the more common techniques employed. Consequently, the spline method was used to adjust this analytical extension. The results from the adjusted strip are compared with the unadjusted values in Table 1. The maximum errors in the adjusted extension were 9.00 feet in elevation and 2.39 feet in horizontal position.

As a by-product of the analytical method the tilt and swing of each photograph is determined. This enables precise evaluation of the quality of the photography. The photography under study proved to be of high quality (Table 2).

B. CONTROL EXTENSION BY BRIDGING

Several control extensions using various combinations of ground control were computed (Figures 5 and 6). This furnished a comparison of the strength of the extension provided by different geometric layouts of initial control.

Test Case IIa provided the best values. The maximum vertical displacement was 10.23 feet with a maximum horizontal error of 1.05 feet. No further adjustment was necessary since the bridging method incorporates a least squares adjustment in the simultaneous solution of equations to establish control. The errors which occurred in Test Case IIa are tabulated in Table 1 and are compared with the results from both the unadjusted and adjusted cantilever extensions. The probable error of vertical displacement for each was respectively ± 3.83, ± 7.76 and ± 2.97 feet. Although slightly less accurate, the results from the bridge solution required no

TABLE 1

COMPARISON OF RESULTS FROM CANTILEVER AND BRIDGE CONTROL EXTENSIONS IN A SINGLE MODEL

Point	*Adjusted Cantilever Extension*			*Unadjusted Cantilever Extension*			*Bridge*		
	h	Δ*y*	Δ*x*	*h*	Δ*y*	Δ*x*	*h*	Δ*y*	Δ*x*
36-C	0.00			− 2.20			− 0.05		
36-D	−4.58			− 5.42			+ 7.05		
37-D	+2.32			+ 5.32			− 1.69		
37-F	−2.50			− 8.50			+ 0.02*		
37-G	−0.40			+ 8.30			− 0.73		
37-1	−0.33	+0.94	+2.12	+ 6.83	− 8.96	−1.68	+ 0.97	+0.55*	−0.39*
38-5	−6.20	−1.00	+0.77	+ 5.39	−11.30	−3.23	− 7.16	−1.05	−0.22
37-H	−6.07			+ 7.05			0.00*		
36-1	−9.00	+2.39	−1.72	+12.36	− 3.11	−3.34	+10.23	+0.94*	−0.72*
36-A							+ 3.90*		
Probable Error	±2.97			± 7.76			± 3.83		

* Point used to compute bridge and not included in analysis.

additional adjustment as was the case with the cantilever extension.

COST OF ANALYTICAL AEROTRIANGULATION

The feasibility of applying analytical aerotriangulation to practical problems will eventually be decided on the basis of the expense involved. Regardless of the degree of accuracy attained, analytical methods will not be adopted if establishing control by conventional survey methods is less expensive. A comparison of the cost involved in each method was made.

Since the cantilever method of analytical aerotriangulation has been tested with many strips and is suitable for production runs, the data given are a reasonable estimate of what might be expected for initial control extensions. Continued use of this method would result in more efficient operation and lead to a further reduction in the cost.

The bridge program, as coded for electronic computers in 1958, requires manual intervention and accurate estimates of exposure station position which are sometimes difficult to obtain. Since this program was not in final form for production runs cost figures were not estimated.

A. COST OF CONTROL ESTABLISHED BY SURVEY METHODS

On the basis of data available for establishing control through the entire 12 mile strip of photography, it is estimated that five days would be required to provide control for the four photo-test portion by conventional survey methods. The 1958 rate for a four-man party in the Pittsburgh area is $150.00 per

TABLE 2

TILT AND HEADING FROM CANTILEVER DATA

Photo	*x Tilt*	*y Tilt*	*Heading (H)*
35	−0°–34′–16″	−0°–25′–26″	0°–54′–46″
36	−0°–03′–58″	−1°–36′–20″	0°–25′–45″
37	−1°–37′–29″	−0°–27′–05″	1°–21′–04″
38	−0°–43′–43″	−1°–22′–17″	0°–59′–03″
Estimates	0°–00′–00″	0°–00′–00″	3°–34′–00

Heading—Clockwise direction from north of the $+x$ photographic axis.

x Tilt—The vertical angle between the y-photographic axis and a horizontal plane. A plus x-tilt is that due to the left wing of the aircraft being lowered.

y Tilt—The vertical angle between the x-photographic axis and a horizontal plane. A plus y-tilt is that which is due to the nose of the aircraft being lowered.

day including overhead and profit. In addition to the field work, office computation was also necessary. The total cost is listed below:

Field party for five days at $150.00 per day	$750.00
Office computation to determine coordinates of points in a four-photograph strip	100.00
Total cost	$850.00

B. COST OF ANALYTICAL CANTILEVER EXTENSION

A breakdown of costs of analytical cantilever extension is as follows:

Field work to establish complete control points in initial model	$140.00
Measurements of photo coordinates. Two technicians 12 hr. at $2.50/hr.	60.00
Assemble the data—16 hr. at $4.00/hr.	64.00
Punch cards as input to the IBM 650 4 hr. at $2.50/hr.	10.00
Computing on the IBM 650—1.5 hr. at $75.00/hr.	112.50
Analyze the results—8 hr. at $4.00/hr.	32.00
Total actual cost	418.50
Plus 100% for overhead and plus 10% for profit	460.35
Total cost	$878.35

For the job considered in this study, extension of control analytically costs 3.2% more than establishing control by the conventional field methods. In remote inaccessible areas, an adjusted cantilever extension has further advantages over ground survey techniques. If the bridge method can be perfected, it contains advantages over the cantilever method, either with or without adjustment, and should be in an even better competitive position with conventional methods.

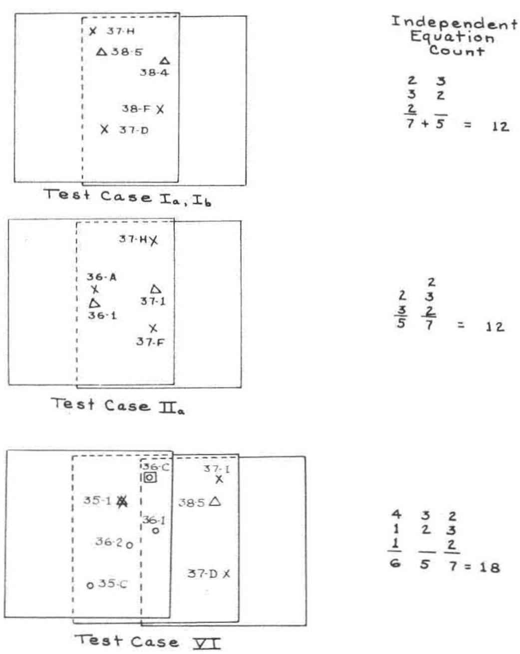

FIG. 5. Control extensions using various combinations of ground control.

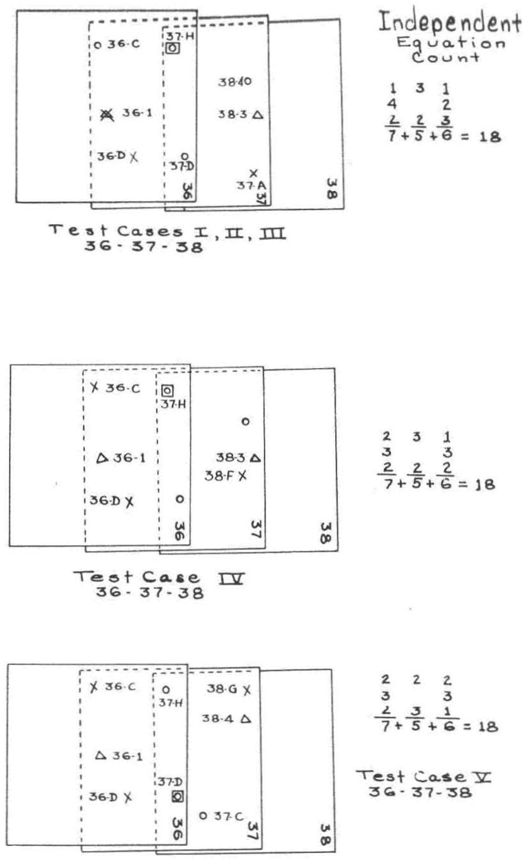

FIG. 6. Control extensions using various combinations of ground control.

CONCLUSIONS

As originally stated, the purpose of this study was to determine: (1) the accuracy of analytical methods when used with actual photography and ; (2) the feasibility of applying these methods to photogrammetric mapping for highway design. Sufficient results were obtained to draw conclusions with regard to both the cantilever and bridging methods of analytical aerotriangulation.

The cantilever method of extension was employed to calculate positions of points. The cantilever after adjustment had a probable error in elevation of ±2.97 feet and a maximum error in horizontal position of 2.39 feet. Although this accuracy does not satisfy the Bureau of Public Roads recommendations for the Interstate System, it is adequate for highway locations where a high-order of accuracy is not required. For example, this technique is presently applicable to projects in remote inaccessible areas where the cost of establishing control by conventional survey methods is high.

The bridge program contains advantages over the cantilever method in that any type of ground-control point wherever it appears in the strip may be used for the control extension. However, results from the test cases in this study indicate the need for very accurate estimates of station position and orientation in order to converge on a solution. Certain test cases diverged although close estimates were employed. In spite of this, the quality of the results show that the method has great potential. In order to improve the bridge program the following steps should be taken: (1) the solution should be examined to discover any weakness which might cause divergence with certain combinations of ground-control points; (2) the effects of systematic and accidental errors on the computing program should be thoroughly investigated; and (3) the program should be re-written to allow for more efficient operation and the use of redundant control data.

The effects of errors in the photo coordinate measurements cannot be overlooked. It is felt that accumulations of these errors were responsible for a major portion of the resulting displacements in the positions of calculated control points. Measurements should be made by experienced personnel preferably with a stereo comparator. Better comparators have now become available for performing this measuring. Incorporation of redundant data in the least squares solution of the bridge program will further improve the accuracy.

This first study applying recently developed methods of analytical aerotriangulation to actual photography for highway mapping proved very encouraging. Already analytical techniques appear to be competitive with ground-survey methods. Further development and applications of analytical methods to photogrammetric highway mapping are certainly warranted.

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