3. Pennington, John T., "Photogrammetric Plotting Instruments," MANUAL OF PHOTOGRAMMETRY, Am. Soc. of Photogrammetry, pp. 633 and 636, 1952.
4. ——— "Photogrammetric Plotting Instruments, MANUAL OF PHOTOGRAMMETRY, Am. Soc. of Photogrammetry, p. 639, 1952.
5. Harris, William D., "Aerotriangulation Adjustment of Instrument Data by Computational Methods, *Technical Bulletin No. 1, Coast and Geodetic Survey*, Washington 25, D. C., 1958.
6. Shu, Dr. C. S., and McNair, A. J., "Determination of Geographic Coordinates, Flight Heights, and True Orientations for Extensions of Strips of Aerial Photographs," PHOTOGRAMMETRIC ENGINEERING, XXII, no. 4, p. 637, 1956.
7. Schmid, Dr. Hellmut, "An Analytical Treatment of the Orientation of a Photogrammetric Camera," PHOTOGRAMMETRIC ENGINEERING, XX, no. 5, p. 765, 1954.
8. Hobrough, Gilbert Louis, "Automatic Stereo Plotting," PHOTOGRAMMETRIC ENGINEERING, XXV, No. 5, p. 763, 1959.
9. Cude, William C., "Automatic and Semiautomatic Mapping," PHOTOGRAMMETRIC ENGINEERING, XXVI, no. 2, p. 303, 1960.
10. Southard, Rupert B., Jr., "Orthophotography —Its Techniques and Applications," PHOTOGRAMMETRIC ENGINEERING, XXIV, no. 3, p. 443, 1958.
11. Vasilevskis, S., "Large-field Photographic Astrometry," PHOTOGRAMMETRIC ENGINEERING, XXV, no. 3, p. 463, 1959.
12. Advertisement, Nistri, PHOTOGRAMMETRIC ENGINEERING, XXIV, no. 3, p. 328, 1958.
13. Advertisement, Wild, PHOTOGRAMMETRIC ENGINEERING, XXVII, no. 2, p. 212, 1961.
14. Lorenz, Gilbert, "Automatic Mapping Developments, Geodesy in Space Age," symposium, Ohio State University, February, 1961, (publication forthcoming).
15. Gilleland Instrument Company, "Projector Plots Path of Moving Objects," *Product Engineering*, May 30, 1960.

Maximum Bridging Distance in Spatial Aerotriangulation

H. M. KARARA,
Asst. Prof. of Civil Engineering,
Univ. of Illinois, Urbana, Ill.

ABSTRACT: *To ensure a specified accuracy in aerotriangulated control (photogrammetric-control) on which the stereoplotting of individual stereomodels is based, the bridged distances have to be confined within certain limits. This paper presents a universal chart for the determination of the maximum allowable bridging distance for wide ranges in the parameters affecting the accuracy of spatial aerotriangulations. The chart also indicates the expected accuracy in aerotriangulated elevations. The maximum bridging distance and the expected accuracy in elevations are also given by two practical formulae presented in this paper. The planning and design of flight charts for small-scale photogrammetric mapping of extensive areas could be made considerably easier and safer with the help of the presented* mbd *chart or formulae.*

THE use of aerotriangulation in connection with small-scale mapping of extensive areas has already been established and well-proven in practice. There are different ways and methods for the execution and adjustment of aerotriangulation. In most cases, the end product of such spatial aerotriangulations is what is generally termed "photogrammetric-control" and used to control the stereoplotting of individual models. As one can expect, the accuracy of such control effected through aerotriangulation is somewhat less than that of terrestrially determined ground-control. To ensure a specified accuracy in photogrammetric-control, the bridged distance has to be confined within certain limits. The maximum bridging distance (*mbd*) for a specific project (see Figure 1 for a long strip and Figure 2 for a block of long strips) depends, among other things, on the following parameters: standards of accuracy required, photo-scale, forward overlap, quality of the pictures, reliability of the photogrammetric measurements (a function of the stereoscopic

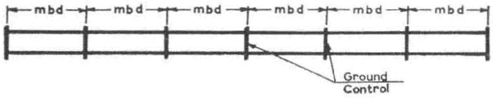

FIG. 1. Maximum bridging distance for a specific project for a long strip.

machine and operator), the method of relative orientation, and the methods used in executing and adjusting aerotriangulations.

The idea of designing a chart that would help in determining the maximum distance allowable for bridging was brought up by the author in 1956 (Karara, 1956) and has proven to be of much help in planning small-scale mapping for extensive areas. The basic idea lies in studying the propagation of the errors (in planimetry and elevation) remaining after the adjustment of aerotriangulations and confining these errors within the allowable tolerances through restricting the bridged distances to certain limits.

The *mdb* chart published in 1956 (see Karara, 1956) was restricted to one focal-length. In this paper, a more universal *mbd* chart is presented. In addition to this chart, universal equations are also given for the determination of the maximum allowable bridging distance, as well as the expected accuracy of the deduced elevations.

In general, aerotriangulation is applied only in case of small-scale mapping. For this reason, the 1956 chart as well as the chart given in Figure 3 of this paper deal only with map-scales ranging from 1:50,000 to 1,000,000. In case of such small scales, the accuracy in planimetry is generally the decisive factor in determining the maximum distance to be bridged. In odd cases, however,

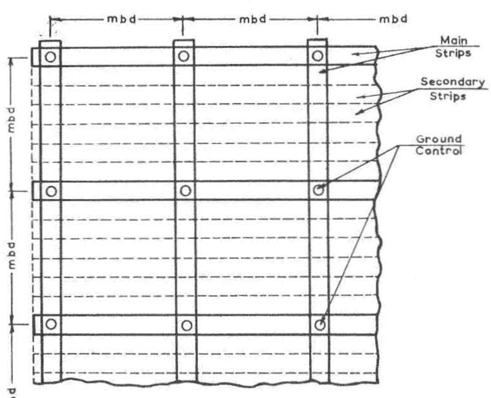

FIG. 2. Maximum bridging distance for a block of long strips.

where the accuracy of aerotriangulated elevations is, for some special reason, the critical value, the *mbd* chart and equations have to be used in a slightly different way. Case 1 of the two examples given below gives details on that point.

Figure 3 shows a universal chart for *mbd*, completed recently at the University of Illinois, for the case of 60% longitudinal overlap. This chart is worked out for the following restrictions in the different parameters:

1. Accuracy requirement in planimetry: 0.2″ (or 0.5 mm.) measured on the publication scale of the map.
2. Accuracy of measurement of parallax with stereoscopic machine = ±0.01 mm. (in image-plane).
3. Relative Orientation: According to Bachmann[1] (see Bachmann, 1946, pp. 56–57).
4. Aerotriangulation: Instrumental conjunction of successive photographs,[2] generally termed "Aeropolygon."[1]
5. Control and Adjustment: Cross Bases Method[1] (see Karara, 1956 or 1957).
6. The chart could be used for aerial cameras with principal-distance between 3″ and 12″.
7. Flight altitudes ranging between 3,000′ and 30,000′.
8. Map scales between 1:50,000 and 1:1,000,000 are considered in the chart.

It should be emphasized that the map scale referred to herein is not necessarily that of the direct stereo-compilation; it could be reduced (photographically or otherwise) from the most favorable mapping scale that best suits the photo-scale and the stereoplotter.

Figure 3 not only gives *mbd*, but it also determines the expected accuracy in elevations. Instructions for the use of the chart are given in the figure.

Following the same basic ideas, used to develop the chart, the following formulae for *mbd* have been derived.

$$mbd \text{ (in feet)} = 0.43B\sqrt{\frac{\mu f S}{\mu_0 Z}}$$

$$\text{or} \quad mbd \text{ (in meters)} = 0.047B\sqrt{\frac{\mu f S}{\mu_0 Z}}$$

[1] For all practical purposes, however, any other method could be used. Experience has shown that there is no substantial difference in accuracy between the different methods.

[2] Better known by its German name "Folgebildanschluss." For details see Brandenberger, 1956, Chapter II.

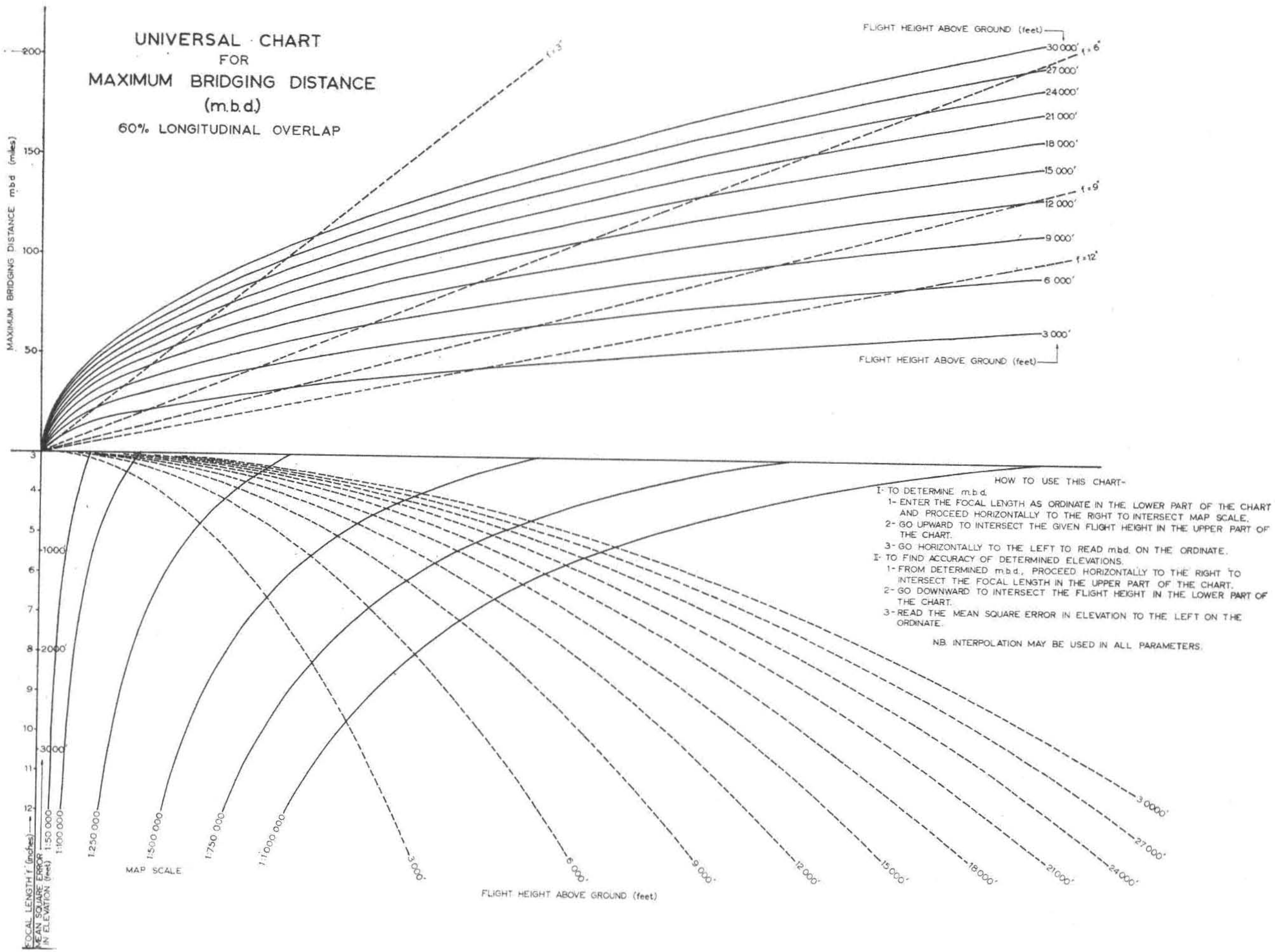

FIG. 3. Universal chart for maximum bridging distance. 60% longitudinal overlap.

where

S is the reciprocal of the map-scale (map scale number)
f is the focal-length of the lens in inches or millimeters
B is the average length of aerial base in feet or meters
Z is the average flight height above ground in feet or meters
μ is the tolerance in planimetry (in inches or millimeters) measured on the publication scale of the map
μ_0 is the accuracy of measurement of parallax in the stereo-machine (in the image plane) in inches or millimeters

The expected accuracy (mean square error) in the derived elevations of aerotriangulated points is given by the following formula, derived following the same basic ideas as the chart in Figure 3.

$$\mu_H \simeq 2\mu_0 \frac{Z^2}{Bf}(4.35 - 1.25N + 0.375N^2 - 0.0625N^3 + 0.015625N^4)^{1/2}$$

where:

N is the number of stereoscopic models in the bridged distance.
(other symbols as given above). In this equation μ_0 and f should be in inches or millimeters, while Z and B should be in feet or meters. In such a case μ_H will be in feet or meters.

It must be underlined that while the chart in Figure 3 has some limitations in the ranges of the parameters, the three formulae shown above are free from such restrictions. As an example; if the longitudinal overlap is 70% instead of 60%, another chart has to be compiled or deduced. The formulae, however, can take care of that since the aerial base B appears as such in the equations.

Strictly speaking, the above described chart and formulae are valid only for the case where the Bachmann's method of relative orientation, the aeropolygon method of triangulation and the Cross Bases Method are used. For all practical purposes, however, the chart and the formulae given above could be used for other methods of relative orientation, triangulation, control, and adjustment. This generalization is based on the fact that experience has shown that there is no significant difference in the accuracy of photogrammetric work executed according to the different methods known today. Should a drastically new method of relative orientation, of triangulation, of control, or of adjustment be devised, it then would be wise to examine the propagation of the errors remaining after the adjustment before adopting the *mbd* chart or the formulae.

No attempt is made in this paper to show the mathematical manipulations used in deriving the chart or the formulae, since such a topic is of limited interest to most readers. Full account of the derivations of the chart and the formulae will be included in a research report to be published in the near future by the University of Illinois. The basic ideas and the fundamental operations have been published by the author in 1956 (Karara, 1956).

EXAMPLES:

The following two examples demonstrate how the *mbd* chart and formulae are used:

Case 1:

Given: 9″×9″ pictures, focal-length 8.25″, 60% longitudinal-overlap, flight-height above ground = 27,000′, required map-scale 1:100,000 (not necessarily through direct compilation), required accuracy in planimetry 1/50″, accuracy of stereoscopic measurement of parallaxes in the image-plane = ±0.01 mm.

Wanted: maximum safe bridging distance (*mbd*) and estimated accuracy of deduced elevations (μ_H).

Solution:

$$mbd \text{ (in feet)} = 0.43B\sqrt{\frac{\mu f S}{\mu_0 Z}}$$

$$B = 40\% \text{ of } \frac{9''}{8.25''} \times 27{,}000 = 11{,}782 \text{ ft.}$$

$$\mu = 0.02''$$

$$\mu_0 = \pm 0.01 \text{ mm.} = 0.00039''$$

$$Z = 27{,}000'$$

$$f = 8.25''$$

$$S = 100.000$$

$$mbd = 0.43 \times 11{,}782\sqrt{\frac{0.02 \times 8.25 \times 100.000}{0.00039 \times 27{,}000}}$$

$$= 199{,}600 \text{ ft.}$$

$$mbd = 37.8 \text{ miles}$$

$$\mu_H \simeq 2\mu_0 \frac{Z^2}{Bf}(4.35 - 1.25N + 0.375N^2 - 0.0625N^3 + 0.015625N^4)^{1/2}$$

$$N = \frac{(mbd)}{B} = \simeq \frac{199{,}600}{11{,}782} = 16.94 \simeq 17 \text{ stereomodels}$$

$$\mu_H \pm 193 \text{ feet}$$

The values for *mbd* and μ_H derived from Figure 3 agree with the above computed values.

N.B. If the value obtained for μ_H is, for some reason, considered too high for the purpose of the map, then another value for *mbd* could be determined backwards from a specified μ_H. As an example, if μ_H in this case should not exceed 150 ft., chart 3 gives the *mbd* to be 35 miles.[3] Note that the μ_H equation can also give the number of models (N) to satisfy the condition that μ_H is not to exceed ± 150 ft. In this case $N=16$ stereomodels. This circumstance, however happens only in seldom cases, since in small-scale mapping, the accuracy in planimetry is generally the decisive factor as mentioned before.

Case 2:

Given: 23×23 cms. pictures, $f=88.5$ mm., $h=10{,}000$ meters above ground, 60% longitudinal overlap, required map-scale 1:1,000,000 (reduced from the most favorable plotting-scale), required accuracy in planimetry = 0.1 mm., accuracy of stereoscopic measurement of parallaxes in image-plane $= \pm 0.01$ mm.

Solution:

$$mbd \text{ (in meters)} = 0.047B\sqrt{\frac{\mu f S}{\mu_0 Z}}$$

$$B = 40\% \text{ of } \frac{230}{88.5} \times 10{,}000 = 10{,}395 \text{ meters}$$

$$\mu = 0.1 \text{ mm.}$$

$$\mu_0 = 0.01 \text{ mm.}$$

$$f = 88.5 \text{ mm.}$$

$$Z = 10{,}000 \text{ meters}$$

$$S = 1{,}000{,}000$$

mbd (in meters)

$$= 0.047 \times 10{,}395\sqrt{\frac{0.1 \times 88.5 \times 1{,}000{,}000}{0.01 \times 10{,}000}}$$

$$= 145348\text{m say } 145 \text{ km}$$

$$\mu_H \simeq 2\mu_0 \frac{Z^2}{Bf}(4.35 - 1.25N + 0.375N^2 - 0.062N^3 + 0.015625N^4)^{1/2}$$

$$N \simeq \frac{(mbd)}{B} = \frac{145348}{10395} = 13.98 \simeq 14 \text{ stereomodels}$$

$$\mu_H \simeq \pm 2 \times 0.01 \frac{(10{,}000)^2}{88.5(10395)}[4.35 - 1.25 \times 14 + 0.375(14)^2 - 0.0625(14)^3 + 0.015625(14)^4]^{1/2}$$

$$\mu_H \simeq \pm 48 \text{ meters}$$

[3] In such a case, proceed as follows: 1) enter μ_H as ordinate in the lower part of the Chart and proceed horizontally to the right to intersect the flight height. 2) go upward to intersect the focal-length f. 3) read *mbd* on the ordinate.

The values for *mbd* and μ_H derived from chart 3 agree with the above deduced values.

In this example, some type of ground-control has to be established every 145 kilometers to ensure precise photogrammetric control satisfying the above mentioned specifications.

Thorough planning for aerotriangulation is absolutely essential if the photogrammetric control has to satisfy certain accuracy requirements. The author is inclined to believe that one of the most important reasons behind the unsatisfactory results that are sometimes obtained from some of the spatial aerotriangulation projects lies in the fact that the bridged distances in such cases exceed by far the maximum allowable bridging distance.

Practitioners dealing with small-scale mapping of extensive areas are invited to use the chart shown on Figure 3 as well as the *mbd* and μ_H equations. Constructive comments or suggestions will be appreciated by the author.

ACKNOWLEDGEMENT

The author thanks Mr. A. A. Elassal, Graduate Student at the University of Illinois for his assistance in preparing this paper. Mr. Elassal played a major role in designing the chart shown in Figure 3.

REFERENCES

1. Bachmann, W. K., "Etudes sur la Photogrammetrie Aérienne." Copyright 1946 by S. A. de Vente des Instruments de Géodésie H. Wild, Herrbrugg (SG), Switzerland.
2. Brandenberger, A. J., "The Practice of Spatial Aerial Triangulation," 1956, Photogrammetric Institute of the Swiss Federal Institute of Technology, Zurich, Switzerland.
3. Karara, H. M., "Fehlerfortpflanzung und Ausgleichung von Aerotriangulationsstreifen mit gemessenen Querstrecken." Diss. E.T.H. Zurich, Switzerland, 1956.
4. Karara, H. M., "Cross-Bases Method in Aerial Triangulation," PHOTOGRAMMETRIC ENGINEERING, Vol. XXIII, No. 5, December 1957.
5. Karara, H. M., "About the Character of Errors in Spatial Aerotriangulation," PHOTOGRAMMETRIC ENGINEERING, Vol. XXV, No. 3, June 1959, and Vol. XXVI, No. 2, April 1960.
6. Karara, H. M., "Reduction of the Effect of the Deflection of the Vertical on Photogrammetric Work in Geodetically Unexplored Regions," "(The Cross-Bases Method)," PHOTOGRAMMETRIC ENGINEERING, Vol. XXVI, No. 1, March 1960.
7. Schermerhorn, W., "Results of a Pre-War Aerial Triangulation of the Large Area, Photographed for Geological Purposes." *Anales de la IV Reunion Panamericans de Conulta Sobre Cartografia*, Buenos Aires, 1948.