Investigations of the Weights of Image Coordinates in Aerial Photographs

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ABSTRACT: *Up to now coordinates and coordinate differences (parallaxes), measured in photographic images are generally assumed to have equal weights. It is, however, quite obvious that this assumption is doubtful, at least if the image coordinates are measured in instruments with orthogonal observation of the images (stereocomparators, instruments with mechanical projection). It is not possible to make the image surface exactly flat, nor is it possible to determine the actual undulations exactly because of the inevitable measuring errors. Furthermore, the real positions of the image details (contrasts) within the emulsion are not exactly known. Therefore, due to the nature of central projection of the photographic imaging procedure the image coordinates, measured orthogonally to the image surface, must become influenced. The errors of the image coordinates must grow with the radial distance from the center point. In other words, the weights of the image coordinates must decrease with the radial distance of the points from the center of the image. For a determination of the weight distribution it is suitable to determine the standard errors of the image coordinates preferably from an adjustment according to the method of least squares. Weights are then defined as inverse proportional to the squares of the standard errors (variances).*

Some results of performed experiments are demonstrated below.

STANDARD ERRORS OF UNIT WEIGHT OF IMAGE COORDINATES

INVESTIGATIONS into the basic accuracy of image coordinates of aerial and terrestrial photographs have been performed at the Division of Photogrammetry of the R. Institute of Technology in Stockholm. Some of the results have been reported to the London Congress 1960 (Hallert, 1960).

The basic accuracy of photographs from aerial cameras has been tested over test fields following vertical photography from high towers and from the air (flying altitude about 5,000 meters above the ground).

From the tests the most important systematic errors of the photographs (the radial distortion and the affine deformations) have been determined, and further the residual errors have been estimated as standard errors of unit weight of the image coordinate measurements, according to the method of least squares. The adjustment was performed for point combinations in circles with different radii around the principal point of the photographs. Consequently the variation of the standard errors of unit weight with varying radii can be used as an indication of the possible weight variation of the image coordinates.

The result of the performed investigations will briefly be demonstrated herein concerning wide-angle cameras (Aviogon c = 152 mm.) and for film.

WEIGHT TESTS FROM A HIGH TOWER

The author has previously demonstrated the averages of the standard errors of unit weight S_0' of image coordinate measurements for nine different radii in ten photographs (Hallert, 1960, table 12, p. 41). In Diagram 1 the averages of the standard errors are graphically demonstrated for different radii. Evidently there is a nearly linear increase of the standard errors with the radii. The lower curve of Diagram 1 demonstrates the standard errors of unit weight of image coordinates

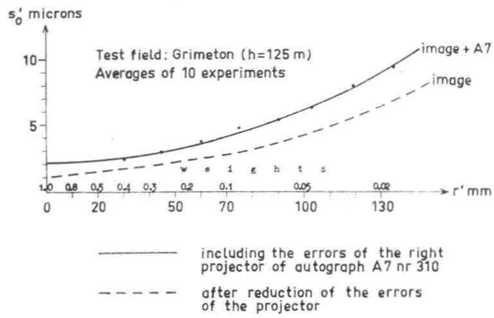

DIAGRAM 1. Standard errors of unit weight of image coordinates for different radii. Aviogon $c=152$ mm.; film, high tower tests (Grimeton).

after correction for the standard errors of the measuring device (the right projector of the autograph A7 *nr* 310). For the principal point the standard error of unit weight is found to be approximately one micron. With some minor approximation the equation of the curve can be expressed as

$$s_0' = 1 + 0.008r' + 0.00028r'^2$$

where s_0' in microns for r' in millimeters. The weights can then be determined as inverse proportional to the squares of the s_0'. In Diagram 1 the weights are indicated for certain radii. There is evidently a considerable weight variation.

WEIGHT TESTS FROM THE AIR

Some results of investigations of aerial photographs over the Öland test area have been demonstrated by the author (Hallert, 1960, point 5, p. 49). In this previous writing (Hallert 1960, Diagram 27 and Table 18, page 52) the radial distortion and the standard errors of unit weight of the image coordinates are demonstrated for different radii. In the same manner a comparatively great number of photographs has been treated. The results of the test measurements in three aerial photographs, taken under similar conditions as those mentioned above, are demonstrated herein in Diagrams 4–12. The averages of the standard errors of unit weight have been computed from the five photographs and are demonstrated in Diagram 2 for the corresponding radii. The corresponding curve has further been drawn by estimation. The influence of the errors from the measuring instrument (autograph A7 *nr* 310, the right pro-

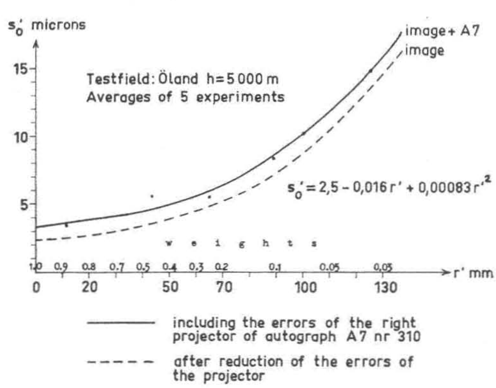

DIAGRAM 2. Standard errors of unit weight of image coordinates for different radii. Aviogon $c=152$ mm.; film, test from the air (Öland).

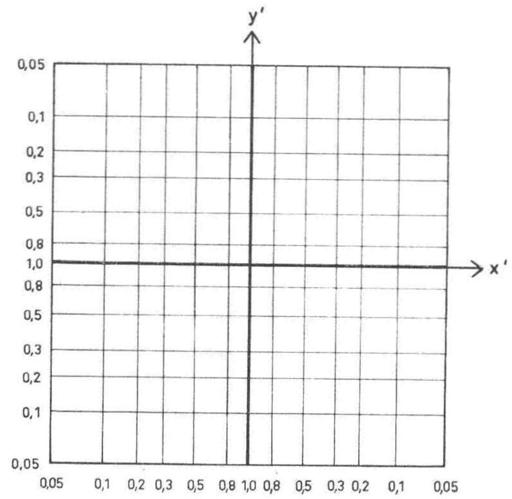

DIAGRAM 3a. Distribution of the weights of image coordinates according to averages from the Grimeton and Öland tests. Wide angle cameras, $c=152$ mm.; film. Mainly radial sources of error. This assumption was made in the adjustment of the relative orientation.

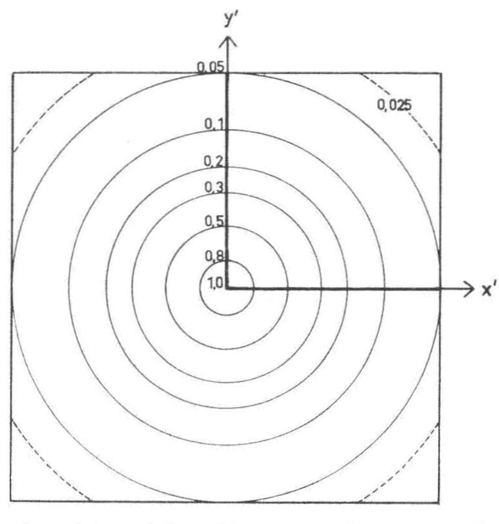

DIAGRAM 3b. Distribution of the weights of image coordinates according to averages from the Grimeton and Öland tests. Radial and tangential sources of errors.

jector) has also been reduced and the resulting curve is also demonstrated. The equation of the curve is approximately

$$s_0' = 2.5 - 0.016r' + 0.00083r'^2.$$

The weights, computed as inverse proportional to the squares of the standard errors (variances) and referred to unit weight in the principal point, are also demonstrated in Diagram 2. The weight distribution is evidently approximately the same for the photographs from the tower and from the air. But the standard errors are considerably larger for the photographs from the air. This is probably at least partly due to the influence of irregular refraction. The weight distribution (average) of image coordinates is demonstrated in Diagram 3a and 3b. See also Diagrams 4 to 12.

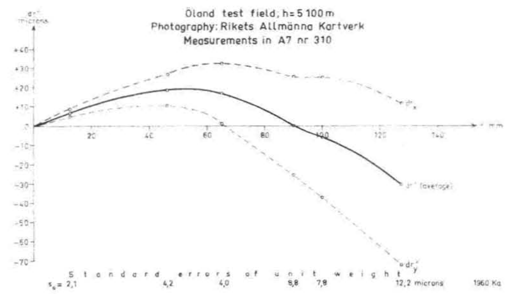

DIAGRAM 4. Radial distortion. Aviogon 29; Hec 5901 à 08; $c=152{,}28$ mm.

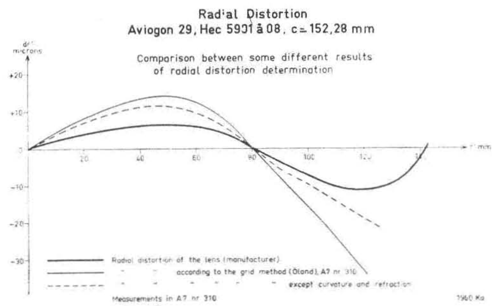

DIAGRAM 5. Radial distortion. Aviogon 29; Hec 5901 à 08; $c=152{,}28$ mm.

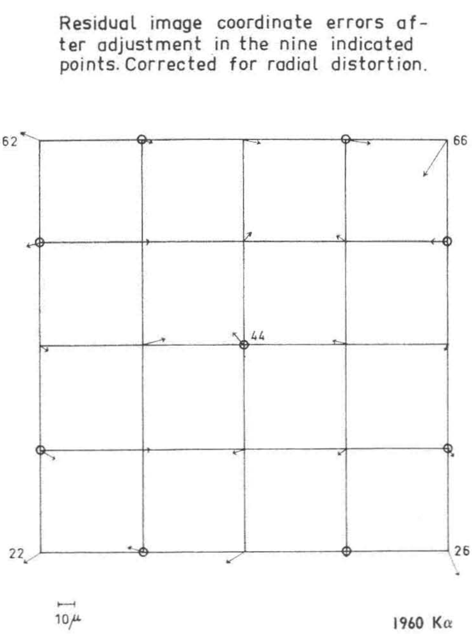

DIAGRAM 6. Aviogon 29; Hec 5901 à 08; $c=152{,}28$ mm.

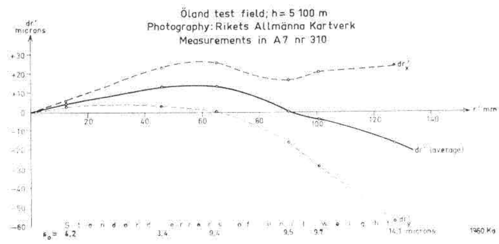

DIAGRAM 7. Radial distortion. Aviogon 38; Hec 5901 q 15; $c = 152{,}45$ mm.

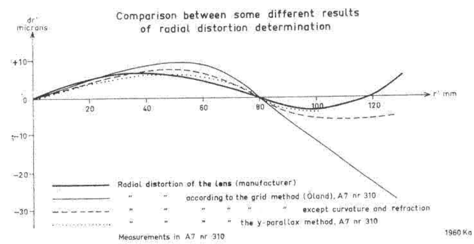

DIAGRAM 8. Radial distortion. Aviogon 38; Hec 5901 9 15; $c = 152{,}45$ mm.

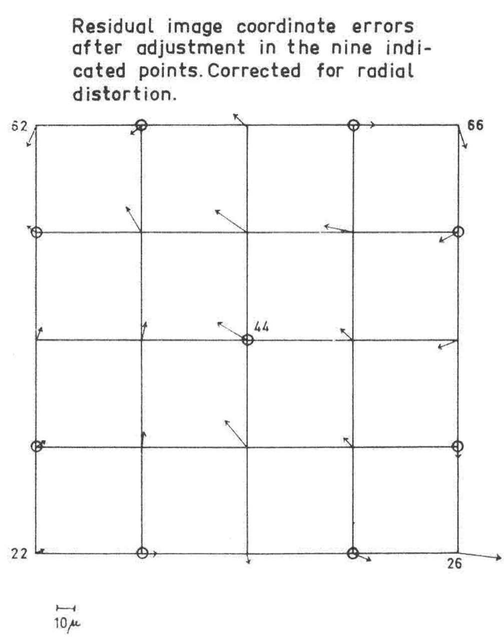

DIAGRAM 9. Aviogon 38; Hec 5901 q 15; $c = 152{,}45$ mm.

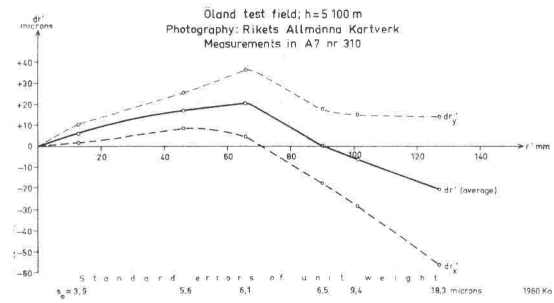

DIAGRAM 10. Radial distortion. Aviogon 41; Hec 5902 b 05; $c = 152{,}35$ mm.

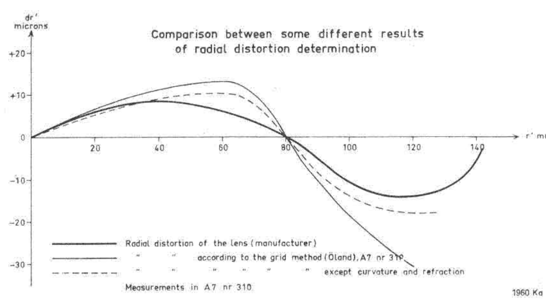

DIAGRAM 11. Radial distortion. Aviogon 41; Hec 5902 b 05; $c = 152{,}35$ mm.

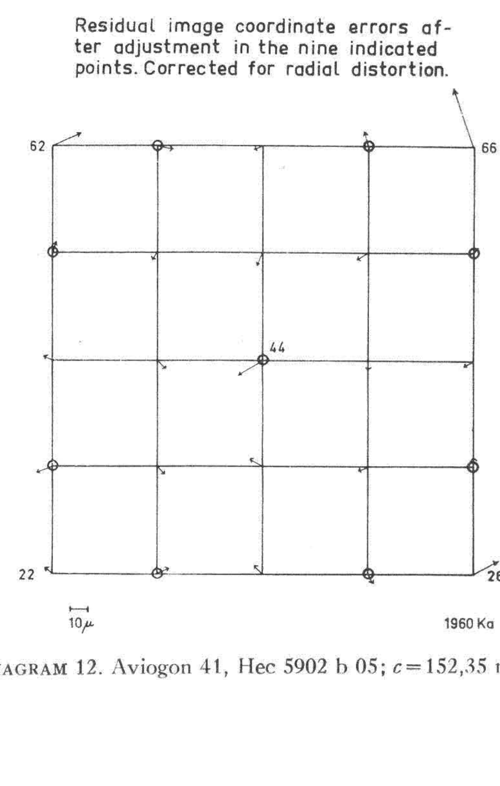

DIAGRAM 12. Aviogon 41, Hec 5902 b 05; $c = 152{,}35$ mm.

SUMMARY

In the investigated cases a considerable weight variation of the image coordinates has been found. It seems most important that similar investigations be performed under other outer circumstances in order to check the results above demonstrated. The consequences of a weight variation of image coordinates are evident. In particular in analytical photogrammetric procedures where image coordinates are directly measured, e.g. in stereocomparators, attention should be paid to the weight variation. Also in ordinary stereoscopic restitution attention should be paid to the weight variation, for instance at the relative orientation. The distribution of residual y-parallaxes must be dependent upon the weights.

Since y-parallaxes are *differences* between image coordinates (frequently projected coordinates) the standard errors of the y-parallaxes would theoretically amount to the values of the standard errors of the y-coordinates multiplied by $\sqrt{2}$. In practice, however, these relations become influenced by the correlation between the image coordinates in adjacent photographs. Consequently, a direct determination of the standard error of y-parallax-measurements from a numerical adjustment of measured y-parallaxes is to be expected to give a lower value than the standard error of the y-coordinates multiplied by $\sqrt{2}$. Practical tests have also proved this assumption. The standard errors of y-parallax measurements usually prove to be of approximately the same magnitude as the standard error of the corresponding y-coordinates. This fact has been used in the formulas for the error propagation from the fundamental operations to the final coordinates.

ADJUSTMENT OF THE RELATIVE ORIENTATION

As an example of the application of the weight distribution we will here demonstrate the formulas for the numerical adjustment of the relative orientation after y-parallax measurements in six and nine points. The standard error of unit weight is also assumed to be determined from 15 points. The computations have been performed by Mr. P. Kaasila, civil engineer, at the Division of Photogrammetry, Royal Institute of Technology, Stockholm. We use the following well known general working correction equation:

$$v = -\,dby_2 - (x-b)d\kappa_2 + \frac{y}{h}\,dbz_2 - \frac{(x-b)y}{h}\,d\phi_2 + \left(1 + \frac{y^2}{h^2}\right)hd\omega_2 - p_y$$

Concerning signs etc. and notations see Figures 1 and 2.

Equation

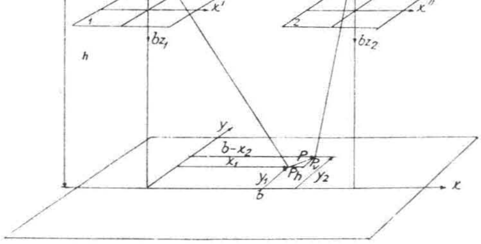

FIG. 1

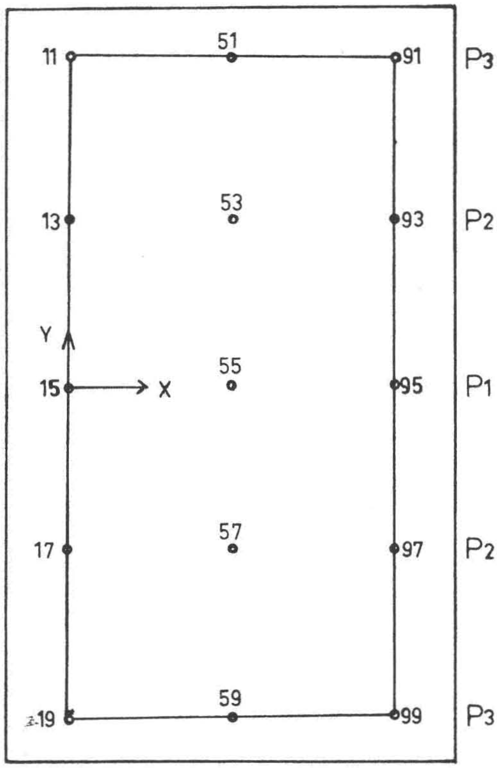

FIG. 2

Weighted working correction

$$\sqrt{P}\, v = -\sqrt{P}\, dby_2 - \sqrt{P}\,(x-b)d\kappa_2 + \sqrt{P}\,\frac{y}{h}\, dbz_2 - \sqrt{P}\,\frac{(x-b)y}{h}\, d\phi_2$$
$$+ \sqrt{P}\left(1 + \frac{y^2}{h^2}\right) h d\omega_2 - \sqrt{P}\, p_y$$

where P is the weight and p_y the measured y-parallax, defined as $y_2 - y_1$. For the six orientation points 15, 95, 11, 91, 19, 99 and their weights according to Figure 2 the corrections from the normal equations are

$$dby_2 = \frac{1}{2(P_1 + 2P_3)} \{-2P_1 p_{95} + P_3(-2p_{15} - 2p_{95} + p_{11} - p_{91} + p_{19} - p_{99})\}$$
$$+ \frac{h^2}{4d^2}(-2p_{15} - 2p_{95} + p_{11} + p_{91} + p_{19} + p_{99})$$

$$d\kappa_2 = \frac{P_1(p_{15} - P_{95}) + P_3(p_{11} - p_{91} + p_{19} - p_{99})}{b(P_1 + 2P_3)}$$

$$dbz_2 = \frac{h(p_{91} - p_{99})}{2d}$$

$$d\phi_2 = \frac{h(p_{11} - p_{91} - p_{19} + p_{99})}{2bd}$$

$$d\omega_2 = \frac{h(-2p_{15} - 2p_{95} + p_{11} + p_{91} + p_{19} + p_{99})}{4d^2}$$

The weight- and correlation numbers are

$$Q_{by_2by_2} = \frac{P_1 + P_3}{P_1(P_1 + 2P_3)} + \frac{(P_1 + 2P_3)h^4}{4P_1P_3d^4} + \frac{h^2}{P_1d^2}$$

$$Q_{\kappa_2\kappa_2} = \frac{2}{(P_1 + 2P_3)b^2}$$

$$Q_{bz_2bz_2} = \frac{h^2}{2P_3d^2}$$

$$Q_{\phi_2\phi_2} = \frac{h^2}{P_3b^2d^2}$$

$$Q_{\omega_2\omega_2} = \frac{(P_1 + 2P_3)h^2}{4P_1P_3d^4}$$

$$Q_{by_2\kappa_2} = \frac{1}{(P_1 + 2P_3)b}$$

$$Q_{by_2\omega_2} = \frac{(P_1 + 2P_3)h^3 + 2P_3hd^2}{4P_1P_3d^4}$$

$$Q_{bz_2\phi_2} = -\frac{h^2}{2P_3bd^2}$$

The square sum $[Pvv]$ and the standard error of unit weight are

$$[Pvv]_6 = \frac{P_1P_3}{4(P_1 + 2P_3)}(-2p_{15} + 2p_{95} + p_{11} - p_{91} + p_{19} - p_{99})^2$$

$$s_{06} = \sqrt{[Pvv]}$$

For the nine orientation points (15, 55, 95, 11, 51, 91, 19, 59, and 99) and the corresponding weights, see Figure 2, we find

$$dby_2 = \frac{1}{6(P_1 + 2P_3)}\{-P_1(-p_{15} + 2p_{55} + 5p_{95}) + P_3(-4p_{15} - 4p_{55} - 4p_{95} + 3p_{11} - 3p_{91} + 3p_{19} - 3p_{99})\} + \frac{h^2}{6d^2}(-2p_{15} - 2p_{55} - 2p_{95} + p_{11} + p_{51} + p_{91} + p_{19} + p_{59} + p_{99})$$

$$d\kappa_2 = \frac{P_1(p_{15} - p_{95}) + P_3(p_{11} - p_{91} + p_{19} - p_{99})}{(P_1 + 2P_3)b}$$

$$db z_2 = \frac{h(-p_{11} + 2p_{51} + 5p_{91} + p_{19} - 2p_{59} - 5p_{99})}{12d}$$

$$d\phi_2 = \frac{h(p_{11} - p_{91} - p_{19} + p_{99})}{2bd}$$

$$d\omega_2 = \frac{h(-2p_{15} - 2p_{55} - 2p_{95} + p_{11} + p_{51} + p_{91} + p_{19} + p_{59} + p_{99})}{6d^2}$$

$$Q_{by_2by_2} = \frac{5P_1 + 4P_3}{6P_1(P_1 + 2P_3)} + \frac{(P_1 + 2P_3)h^4}{6P_1P_3d^4} + \frac{2h^2}{3P_1d^2}$$

$$Q_{\kappa_2\kappa_2} = \frac{2}{(P_1 + 2P_3)b^2}$$

$$Q_{bz_2bz_2} = \frac{5h^2}{12P_3d^2}$$

$$Q_{\phi_2\phi_2} = \frac{h^2}{P_3b^2d^2}$$

$$Q_{\omega_2\omega_2} = \frac{(P_1 + 2P_3)h^2}{6P_1P_3d^4}$$

$$Q_{by_2\kappa_2} = \frac{1}{(P_1 + 2P_3)b}$$

$$Q_{by_2\omega_2} = \frac{(P_1 + 2P_3)h^2 + 2P_3hd^2}{6P_1P_3d^4}$$

$$Q_{bz_2\phi_2} = \frac{h^2}{2P_3bd^2}$$

The square sum $[Pvv]$ and the standard error of unit weight are

$$[Pvv]_9 = \frac{P_1P_3}{4(P_1 + 2P_3)}(-2p_{15} + 2p_{95} + p_{11} - p_{91} + p_{19} - p_{99})^2$$
$$+ \frac{1}{6}\{P_1(p_{15} - 2p_{55} + p_{95})^2 + P_3(p_{11} - 2p_{51} + p_{91})^2 + P_3(p_{19} - 2p_{59} + p_{99})^2\}$$

$$s_{09} = \frac{1}{2}\sqrt{[Pvv]}$$

For all 15 points the following expressions are obtained for $[Pvv]$ and s_0

$$[Pvv]_{15} = \frac{P_1P_3}{4(P_1 + 2P_3)}(-2p_{15} + 2p_{95} + p_{11} - p_{91} + p_{19} - p_{99})^2$$
$$+ \frac{1}{6}\{P_1(p_{15} - 2p_{55} + p_{95})^2 + P_3(p_{11} - 2p_{51} + p_{91})^2 + P_3(p_{19} - 2p_{59} + p_{99})^2$$

$$+ P_2(p_{13} - 2p_{53} + p_{93})^2 + P_2(p_{17} - 2p_{57} + p_{97})^2\}$$

$$+ \frac{P_1P_2P_3}{6(P_1P_2 + 16P_1P_3 + 18P_2P_3)}(-p_{11} + 4p_{13} - 6p_{15} + 4p_{17} - p_{19} - p_{51}$$

$$+ 4p_{53} - 6p_{55} + 4p_{57} - p_{59} - p_{91} + 4p_{93} - 6p_{95} + 4p_{97} - p_{99})^2$$

$$+ \frac{P_1P_2P_3}{4(4P_1P_2 + 9P_1P_3 + 2P_2P_3)}(2p_{11} - 3p_{13} + 2p_{15} - 3p_{17} + 2p_{19} - 2p_{91}$$

$$+ 3p_{93} - 2p_{95} + 3p_{97} - 2p_{99})^2 + \frac{P_2P_3}{6(P_2 + 4P_3)}(p_{11} - 2p_{13} + 2p_{17} - p_{19}$$

$$+ p_{51} - 2p_{53} + 2p_{57} - p_{59} + p_{91} - 2p_{93} + 2p_{97} - p_{99})^2$$

$$+ \frac{P_2P_3}{4(P_2 + 4P_3)}(p_{11} - 2p_{13} + 2p_{17} - p_{19} - p_{91} + 2p_{93} - 2p_{97} + p_{99})^2$$

$$s_{015} = \sqrt{\frac{[Pvv]}{10}}$$

In summary the weight P_1 refers to the points 15, 55, and 95; the weight P_2 refers to the points 13, 53, 93, 17, 57, and 97 and the weight P_3 refers to the points 11, 51, 91, 19, 59, and 99.

Finally, in a similar manner all functions of image coordinates can be treated with respect to the weights.

Reference

Hallert, B., "Results of Practical Investigations into the Accuracy of Aerial and Terrestrial Photographs." *Svensk Lantmäteritidskrift* 1960: 3 (Congress Number). November 1960.

*Vertical Aerial Triangulation Block Adjustments**†

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(*Abstract is on next page*)

The U. S. Army Map Service recently developed a method of block adjusting horizontal aerial triangulation data mathematically, using a high-speed electronic computer (UNIVAC). It has used this method successfully on several map production projects. The block adjustment of vertical aerial triangulation data, using similar techniques, remains an unachieved, although very desirable goal.

The block adjustment technique, as a tool of the photogrammetrist, is relatively new, and photogrammetric literature generally has little information concerning this important subject.

The method of vertical block adjustment presently used at the Army Map Service was first used by the late Charles W. Price and is essentially a modification of a method outlined in a 1956 report by the Mapping and

* Presented at the Society's 27th Annual Meeting, The Shoreham Hotel, Washington, D. C., March 19–22, 1961.

† The information contained herein does not necessarily represent the official views of the Corps of Engineers or the Department of the Army.