$$+ P_2(p_{13} - 2p_{53} + p_{93})^2 + P_2(p_{17} - 2p_{57} + p_{97})^2\}$$

$$+ \frac{P_1P_2P_3}{6(P_1P_2 + 16P_1P_3 + 18P_2P_3)}(-p_{11} + 4p_{13} - 6p_{15} + 4p_{17} - p_{19} - p_{51}$$

$$+ 4p_{53} - 6p_{55} + 4p_{57} - p_{59} - p_{91} + 4p_{93} - 6p_{95} + 4p_{97} - p_{99})^2$$

$$+ \frac{P_1P_2P_3}{4(4P_1P_2 + 9P_1P_3 + 2P_2P_3)}(2p_{11} - 3p_{13} + 2p_{15} - 3p_{17} + 2p_{19} - 2p_{91}$$

$$+ 3p_{93} - 2p_{95} + 3p_{97} - 2p_{99})^2 + \frac{P_2P_3}{6(P_2 + 4P_3)}(p_{11} - 2p_{13} + 2p_{17} - p_{19}$$

$$+ p_{51} - 2p_{53} + 2p_{57} - p_{59} + p_{91} - 2p_{93} + 2p_{97} - p_{99})^2$$

$$+ \frac{P_2P_3}{4(P_2 + 4P_3)}(p_{11} - 2p_{13} + 2p_{17} - p_{19} - p_{91} + 2p_{93} - 2p_{97} + p_{99})^2$$

$$s_{015} = \sqrt{\frac{[Pvv]}{10}}$$

In summary the weight P_1 refers to the points 15, 55, and 95; the weight P_2 refers to the points 13, 53, 93, 17, 57, and 97 and the weight P_3 refers to the points 11, 51, 91, 19, 59, and 99.

Finally, in a similar manner all functions of image coordinates can be treated with respect to the weights.

Reference

Hallert, B., "Results of Practical Investigations into the Accuracy of Aerial and Terrestrial Photographs." *Svensk Lantmäteritidskrift* 1960: 3 (Congress Number). November 1960.

*Vertical Aerial Triangulation Block Adjustments**†

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(*Abstract is on next page*)

THE U. S. Army Map Service recently developed a method of block adjusting horizontal aerial triangulation data mathematically, using a high-speed electronic computer (UNIVAC). It has used this method successfully on several map production projects. The block adjustment of vertical aerial triangulation data, using similar techniques, remains an unachieved, although very desirable goal.

The block adjustment technique, as a tool of the photogrammetrist, is relatively new, and photogrammetric literature generally has little information concerning this important subject.

The method of vertical block adjustment presently used at the Army Map Service was first used by the late Charles W. Price and is essentially a modification of a method outlined in a 1956 report by the Mapping and

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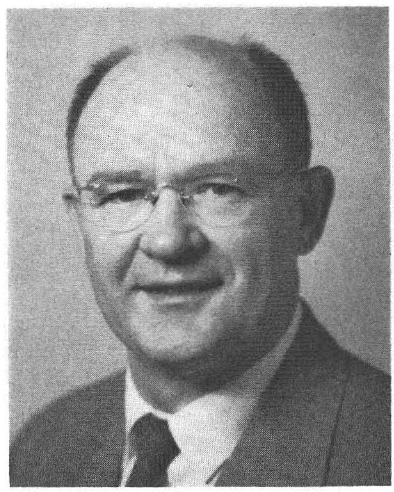
† The information contained herein does not necessarily represent the official views of the Corps of Engineers or the Department of the Army.

Charting Research Laboratory of the Ohio State University Research Foundation.[1] Prior to this, a method involving iso-error contours was used. In either case, the adjuster is allowed no margin for errors since both methods require a fundamental assumption that all control used in the solution is reliable in all respects.

At present, vertical block adjustments are infrequently used at Army Map Service and are totally graphical in nature. Generally, such methods are used in production only in localized areas where adequate vertical ground-control is lacking, where logical stream or terrain gradients are indeterminable, and where nothing else seems to work. Categorically, vertical block adjustments have been used, not as basic, but as supplementary, adjustment techniques. To borrow a colloquialism, they have been little more than "gimmicks" which redistribute, rather than remove, errors which already exist in the original strip solutions.

One of the requirements of such graphical block adjustments is that all strips within the block must first be independently adjusted to the existing vertical-control by accepted photogrammetric methods. If the elevation differences of pass-points common to adjoining strips are within the tolerances for the project, these differences are meaned, and no further adjustment beyond meaning is performed. If, however, those elevation differences generally exceed the tolerance, they may be reduced by resorting to a vertical block adjustment.

Characteristically, the pass-point differences are greater in magnitude between the control bands. A thorough analysis is made to remove differences caused by bad control or operator errors, but the remaining differences are more difficult to analyze. More specifically, are these differences caused by datum errors or by lateral tilt errors in the photogrammetric bridge, or by a combination of the two? How do the various types of errors in adjoining strips contribute to the total differences? If it were possible to analyze the pass-point differences and determine the error components for each strip, the problem of removing the differences could easily be solved.



FRANK W. MASEK

ABSTRACT: *The author describes the method used at the U. S. Army Map Service in the block adjustment of vertical aerial triangulation data for a 13,000 square mile area of a production project. An analysis of the final results for this project showed that the elevation difference of common pass-points obtained by the strip adjustment method were reduced 27.3 per cent by a graphical-mathematical readjustment of the block.*

The author proposes a method of vertical block adjustment designed to be used with electronic computers. This method, which at present is untested, involves the establishment of a unified instrument coordinate system for a block of strips and the fitting of this coordinate block data to the earth's surface. A minimum of five vertical control points, ideally located within the block, should be sufficient for the adjustment of the block data. Comprehensive testing of the method will ultimately determine the mathematical finesse required to achieve the desired results.

Figure 1 shows a lateral cross-section of a hypothetical area covered by seven strips, all of which were previously adjusted to the available vertical-control. Only strips 1 and 7 are considered satisfactorily adjusted to the proper datum and free of tilt within the tolerances for the project. Strips 2 to 6, inclusive, require further adjustment. The upper and lower portions of Figure 1 indicate only two of an infinite number of possible

[1] "Aerial Triangulation by Least Squares." Third Interim Technical Report, January–March, 1956, pages 117–145.

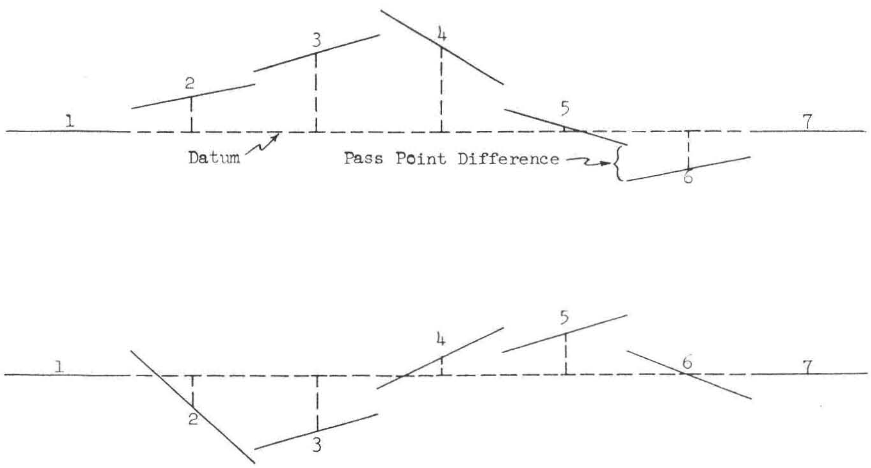


FIG. 1. Datum and tilt error possibilities in a typical block of strips.

combinations of datum and lateral tilt errors which could account for the differences shown between strips. Obviously, any method of readjustment which is based on the differences between strips and which assumes the causative factors, will have difficulty removing all the differences.

Reasonable success, however, has been acheived with the present Army Map Service method of vertical block adjustment. In this method, vertical-control is required only along the perimeter of the block. The interior strips are, therefore, uncontrolled except at the ends.

The vertical datum of each strip in the block is readjusted in the direction of the mean vertical datum of the block at preselected abscissae along the flight line of each strip by pro-rating the strip and block datum differences as a function of the distance of each strip from the middle strip of the block. Consequently, the percentage change will be greatest at the middle of the block. One-half ($\frac{1}{2}$) the adjusted pass-point differences are then used in an auxiliary graphical readjustment of the pass-points along the edges of each strip according to the best mean fit of a flexible spline to the plotted values. Any remaining differences are then meaned.

This method was used on a 13,000 square mile area of a production project, which was completely devoid of vertical-control, except along the perimeter as shown in Figure 2. Fourteen (14) east-west flights, flown at 30,000 feet above sea level, and averaging 43 models each, or a total of 602 models, were involved in the block adjustment. For this project a 50-meter contour interval with a vertical error tolerance in aerial triangulation of 10 meters on control, and a difference tolerance of 20 meters on common pass-points, had been established. Six (6) north-south cross bands, each two (2) models wide, the centers of which were approximately six (6) models apart, were used to obtain the data for the vertical block adjustment. These cross bands, *A* through *F*, are not aerial photographic missions but are narrow bands of terrain along which the elevation differences between adjoining flights were computed.

The entire project was first adjusted to the available geodetic control. The shaded area in Figure 2 was not block adjusted because the photo ties between strips were within tolerance after the initial strip adjustment. The area bounded by the dashed line in Figure 2 was block adjusted.

After the normal strip adjustment, the pass-point differences were readjusted mathematically along the cross bands to obtain the plots for the graphical correction curves along the flight line. The best mean fit of a flexible spline to these plotted values determined the corrections to be made to the pass-point elevations.

Figure 3 shows the results of the block adjustment of this area. For example, in the first column when strip 2 was compared with strip 1, after the usual strip adjustment, strip 2 was 30.0 meters higher than strip 1, as indicated by the Δ_z difference shown in cross-band *A*. Correspondingly, after the block adjustment, strip 2 was 24.6 meters higher than strip 1, as shown by the Δ_z' value in cross-band *A*. The maximum difference of 73.9 meters (cross-band *C*) was reduced to 44.1 meters. The standard error of the pass-point differences prior to the block adjustment was 19.3 meters. This was reduced by this adjustment to 14.1 meters, a reduction of

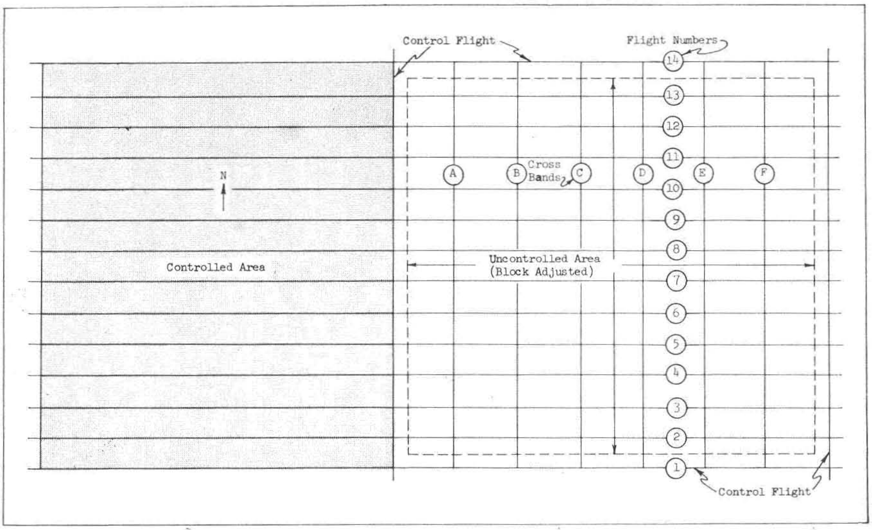


FIG. 2. Diagram of production project showing block adjusted area.

27.3 per cent in the over-all pass-point differences for the area. Prior to the block adjustment, 16.7 per cent of the pass-point differences exceeded the prescribed tolerance of 20 meters. After this adjustment, only 11.5 per cent of the differences were larger than the tolerance. In areas where large differences remained, the error tolerance during the compilation phase of the mapping was necessarily reduced from the norm.

A test of the ITC-Jerie method of vertical block adjustment was recently carried out in Austria with RC-5 photography taken at an altitude of 3,430 meters.[2] An area of 8 strips averaging 20 models per strip was block adjusted, using control spaced at 5, 7, and 10-model intervals. Using the ITC-Jerie Analogue Computer in conjunction with the IBM-650, the Austrians achieved absolute vertical accuracies of 1/4000th of the flight altitude. This is certainly a commendable achievement.

A somewhat different method of vertical block adjustment is one requiring the use of 60–65 per cent side-lap, precision aerial photography. The method, which at present remains untested, involves, in addition, the establishment of a unified instrument coordinate system for a block of strips and the fitting of this coordinate block data to the earth's surface.

[2] Waldhausal, P., *Photogrammetria*, XVI, 1959–1960, No. 1, pp. 29–37.

Strip Nos.	Cross Band Designations											
	A		B		C		D		E		F	
	Δz	$\Delta z'$	Δz	$\Delta z'$	Δz	$\Delta z'$	Δz	$\Delta z'$	Δz	$\Delta z'$	Δz	$\Delta z'$
2-1	+ 30.0	+ 24.6	- 7.3	- 18.9	+ 15.2	+ 0.6	+ 25.7	+ 23.0	- 51.0	- 28.5	- 11.9	- 4.9
3-2	- 3.5	+ 10.9	- 10.5	+ 3.8	- 19.3	- 13.0	- 14.0	- 11.6	+ 20.0	+ 33.5	- 11.9	- 25.0
4-3	- 3.5	+ 5.9	- 1.0	+ 11.9	+ 27.2	- 4.4	- 21.2	- 32.5	+ 72.0	+ 41.7	- 11.9	- 32.1
5-4	+ 11.0	+ 19.7	+ 4.0	+ 14.5	- 73.9	- 44.1	+ 0.8	+ 19.2	- 4.4	- 3.3	- 11.9	- 5.3
6-5	+ 29.0	+ 14.4	- 19.0	- 9.0	+ 22.0	+ 6.9	+ 22.0	- 6.9	- 6.7	- 18.2	- 11.9	- 0.4
7-6	+ 12.0	- 9.5	+ 55.0	+ 16.8	+ 16.2	+ 2.8	+ 6.4	- 14.5	- 0.9	- 7.1	+ 6.4	+ 13.4
8-7	- 10.7	- 4.5	+ 7.0	- 1.2	+ 14.6	+ 9.6	- 3.6	- 11.9	+ 1.7	- 2.2	- 2.0	+ 9.0
9-8	+ 19.0	+ 8.5	- 6.0	- 4.7	- 26.6	- 2.1	+ 4.2	+ 1.4	+ 0.4	- 0.8	+ 7.0	+ 9.0
10-9	- 2.9	- 8.2	+ 5.5	+ 0.7	- 2.9	+ 13.4	- 37.9	- 18.7	+ 3.5	+ 1.6	- 19.5	- 3.5
11-10	+ 19.3	+ 4.2	- 8.5	- 11.6	+ 0.8	+ 7.1	- 9.5	+ 3.0	+ 0.3	- 8.8	- 1.5	+ 7.0
12-11	- 5.6	- 11.8	+ 2.0	- 2.7	- 8.6	- 2.8	+ 10.0	+ 11.0	- 2.3	- 10.9	+ 1.0	+ 4.7
13-12	+ 5.6	+ 0.6	+ 3.7	+ 1.7	- 8.6	- 4.3	- 3.7	+ 0.4	- 11.9	- 10.2	+ 2.0	+ 7.0
14-13	+ 16.6	+ 9.6	+ 0.8	- 8.2	+ 5.2	+ 3.0	- 15.8	- 11.8	+ 1.3	+ 4.0	- 1.5	+ 1.5
RMSE	15.8	11.9	17.0	10.1	25.8	14.0	16.9	15.5	25.4	18.4	9.6	13.0

Note: Δz = Pass Point Differences Before Block Adjustment - - $\Delta z'$ = Block Adjusted Pass Point Differences.

FIG. 3. Vertical block adjustment pass point differences (in meters) before and after block adjustment.

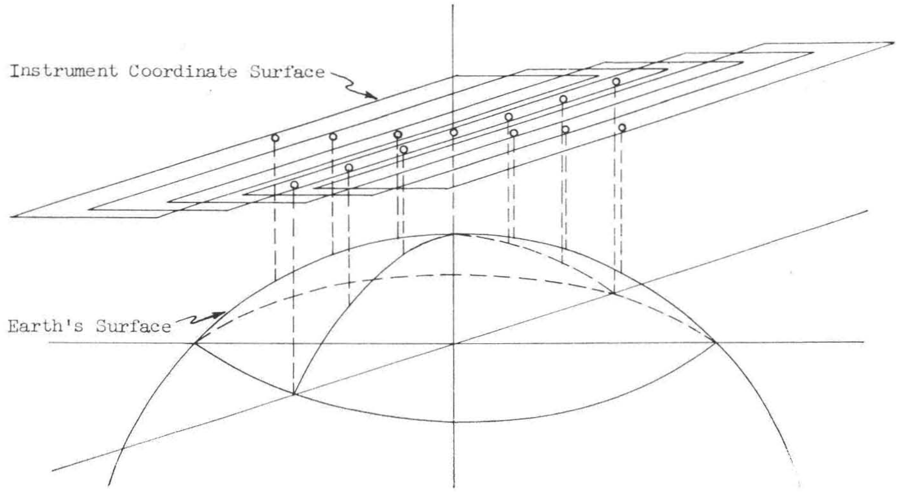


FIG. 4. Relationship between the earth's surface and the instrument coordinate surface of a block of strips.

A preliminary horizontal and vertical strip-to-strip transformation adjustment would be made to a single strip selected as the base. Because of the increased side-lap the relative differences of x-tilt, y-tilt, and scale between the several strips could be minimized. This would overcome the difficulties experienced with the previously described Army Map Service method. The result would be a common block of instrument coordinates which could then be adjusted to the available ground-control.

Figure 4 shows the relationship between the instrument coordinate surface of a block of strips and the earth's surface. A sample row of pass-points is shown along both the longitudinal and the transverse axes of the block. The vertical dashed lines from these points to the earth's surface indicate the errors between the instrument coordinate system and the geoid. The available control in the area need not be dense but should be well distributed. A minimum of five (5) good vertical-control points, one in each of the corners plus one in the center of the block, would probably suffice for such a rectangular area.

It is difficult to visualize graphical adjustment by this method. Adjustment by high-speed electronic computers would require first of all the establishment of the correct geometrical relationship between the photogrammetric coordinates and the geoid. The error surface generated would approach the equation for a surface of revolution, the general nature of which can best be determined by a series of tests.

One of the disadvantages of such a method would be the increased effort required on the part of the triangulation personnel. The increased number of strips triangulated, however, whould be more than offset by the resulting increase in accuracy. Another possible disadvantage is that the compilation phase of a project would be delayed until sizeable portions of the project were triangulated and adjusted. In the preparation of maps by mass-production techniques patience sometimes wears thin, but here again increased accuracy should be the paramount consideration.

Another consideration, appropriate at this point, would be the size of the block that could be effectively handled. The numerical and storage capacities of electronic computers vary considerably. The size of the block, therefore, would depend on the type of computer available and the accuracy desired.

The primary advantage of this method is that less ground-control than presently required would be needed to control a given area. The elimination of erroneous control points would be easier and the analysis of photogrammetric errors would be facilitated. Increased accuracy should result, since in most cases a triple check of the pass-point determinations would be made. A program of comprehensive testing will likewise indicate the mathematical finesse required to achieve the desired results.

The over-all improvement of vertical block adjustment techniques, over those of the strip, should be significant. The specific benefits of this proposed method, however, can only be determined from the results of a series of comprehensive tests.