*A Two Stage Rectification System**

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Multi-stage rectification has been practised almost as long as rectifying instruments for photographic transformations of photographs or other graphical materials have been in existence. This process is presently being used with automatic focusing rectifiers. Although results obtained are satisfactory for all practical purposes the process is complicated, time consuming and sometimes wasteful.

Single-stage transformations are generally superior to the repetitive processes, but, unfortunately, the applicable angle of tilt of the aerial photographs is limited by the mechanical range of the equipment or by optical conditions involved. The ratio of the rectifier focal-length over the aerial camera focal-length determines the magnitude of applicable easel tilt. If this ratio is greater than one, the easel tilt is greater than the aerial tilt. Whenever the easel tilt approaches 90° the entire process breaks down.

Multi-stage rectification overcomes this failure by performing the transformation in several discrete steps. This creates a very healthy condition since the easel-tilt is reduced practically in proportion to the number of stages used in the transformation, the angles of incidence of the image forming rays upon the photosensitive emulsion are greatly moderated, and the chances for better photographic quality of the end product are improved. Unfortunately, much of this improvement is lost again through multiple photographic processing necessary to arrive at the final results.

The new concept presented in this paper is based on a two-stage solution but it eliminates the image deteriorating effects of multiple photographic processing, and reduces the production time from hours to minutes.

The theoretical foundation of the new concept may be found in the excellent treatise on the subject of rectification by Robert Altenhofen which has been published in the MANUAL OF PHOTOGRAMMETRY, pages 449–501. The formulae given by Mr. Altenhofen are derived for the general solution of rectification using n-stages and any desired isoline magnification m of the end product. When these formulae are applied to two-stage rectification and reduced to unit magnification of the isometric scale they assume very simple forms and require only a small amount of computation. They are the basis upon which a new instrument has been built. It has, however, been found advantageous to amend them by a few additional formulae which permit the use of the new instrument as a single-stage rectifier also. In this case its first stage components, i.e. negative stage, lens and first easel, may be oriented in accordance with the tilting lens rectification principle. The course of computations is then as follows (Figure 1).

The geometrical diagram of a two-stage rectifier designed for unit isoline magnification, which is able to perform the two step process in a single operation, is shown in Figure 2. It is composed of two sets of rectifying units mounted in series, each having an object plane, a projection lens and an easel. Their centers and axes of rotation are aligned along the principal axis of their respective stages. Since they are arranged in series, the

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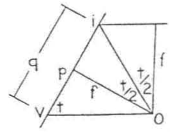

$\overline{pi} = f \cdot \tan\frac{t}{2}$; $\overline{VP} = f \cdot \cot t$

$q = f\left(\tan\frac{t}{2} + \cot t\right) = f \cdot \operatorname{cosec} t$

q = perspective index

K = given

	SINGLE STAGE		MULTIPLE STAGE (n)	
	$m \gtrless 1$	$m = 1$	$n > 1 \quad m \gtrless 1$	$n = 2, \ m = 1$
NON-TILTING LENS RECTIFICATION			$Q = (1+n)\,q$	$Q = 2q$
	$\sin\beta = \frac{F_R}{q}$	$\sin\beta = \frac{F_R}{q}$	$\sin\beta = \frac{F_R}{Q}$	$\sin\beta = \frac{F_R}{2q}$
	$\sin\alpha = \frac{\sin\beta}{m}$	$\alpha = \beta$	$\sin\alpha = \frac{\sin\beta}{m}$	$\alpha = \beta$
	$d_i = \left(1 - \frac{\cos\beta}{\cos\alpha}\right)q$	$d_i = 0$	$d_i = \left(1 - \frac{\cos\beta}{\cos\alpha}\right)Q$	$d_i = 0$
	$Y = d_i - f \cdot \tan\frac{t}{2}$	$Y = -f \cdot \tan\frac{t}{2}$	$Y = d_i - f \cdot \tan\frac{t}{2}$	$Y = -f \cdot \tan\frac{t}{2}$
	$m' = \frac{\cos\alpha}{\cos\beta} \cdot m$	$m' = m = 1$	$m' = \frac{\cos\alpha}{\cos\beta} \cdot m$	$m' = m = 1$
	$B = F_R\,(1 + m')$	$B = A = 2\,F_R$	$B = F_R\,(1 + m')$	$B = A = 2\,F_R$
	$A = \frac{B}{m'}$		$A = \frac{B}{m'}$	Dihedral Angle $\Theta = 2\beta$
TILTING LENS RECTIFICATION	$\tan\gamma = \frac{Y \cdot \cos\alpha}{A + Y \cdot \sin\alpha}$			
	$\alpha' = \alpha + \gamma$			
	$\beta' = \beta - \gamma$			
	$A' = A \cdot \frac{\cos\alpha}{\cos\alpha'}$			
	$B' = B \cdot \frac{\cos\beta}{\cos\beta'}$			
	$Y = 0$			

FIG. 1. Rectification formulae.

easel plane of the first stage assumes the function of the object plane of the second stage. The conjugate distances between object plane lens and image plane of each stage equal $2F$ of their respective lens.

The isocenter of the aerial photograph is placed on the principal axis of the first stage by offsetting its principal point by the computed y-displacement. The isometric parallel, i.e. the line passing through the isocenter which is parallel to the photograph horizon is made collinear with the axis of rotation of the film stage by swing rotation K of the photograph about its principal point. The computed angles α and β for the rotation from perpendicularity of film stage and the easel, are applied to both components of the first stage. The Scheimpflug condition, which calls for the intersection of the object, lens, and image planes in one common line is, thereby, met and assures the over-all critical focus of the image projected onto the easel.

Exactly the same set-up applies to the second stage unit calling for the same tilt-angle β of its object plane (the first easel) and of its second easel. Since both angles β are set up as departures from perpendicularity to the the principal axis of the second unit, the Scheimpflug condition is established for the second stage.

The angular relationship of the principal axes of the first and the second stage units resolves to be 2β. To establish mechanically

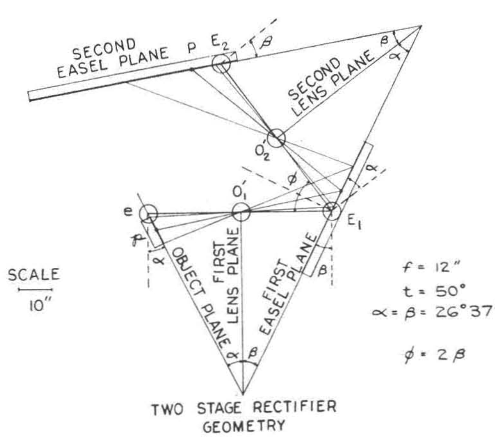

FIG. 2

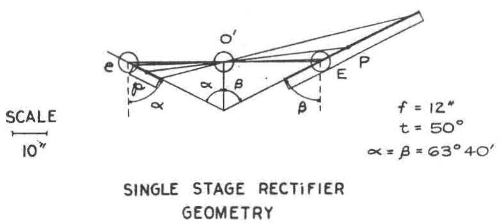

FIG. 3

this relationship for any given value of β, a pivoting motion of one of the two units about the point of intersection E_1 of their principal axes must be provided. Since the isometric parallel of the properly aligned aerial photograph coincides with the axis of rotation of the film stage, and subsequently is projected upon the axis of rotation of the first easel, it will also be projected upon the axis of the second easel irrespective of the magnitude of the dihedral angle 2β. Since, furthermore, both stages work at 1:1 magnification the isometric scale of the photograph to be rectified is maintained throughout the process.

For the benefit of a comparison of the alignment geometry between two-stage and single-stage rectification, Figure 2 represents the transformation of an $f=12''$ photograph of $t=50°$ aerial tilt. The setting angles for all components are $\alpha=\beta=26°37'$. Figure 3 shows the same case for the single-stage process which yields $\alpha=\beta=63°40'$. The sharp increase of the angles of incidence upon the photosensitive material on the respective easels is quite prominent.

The translation of the two-stage rectifier geometry into hardware is shown in Figure 4. On a heavy, three-point supported base casting are mounted the film stage (left background), the first lens with bellows connected to the film stage, the left easel facing the lens, the second lens (hidden behind the right edge of the first easel) with light shields attached which protect the area of the second stage from stray light of the first, and the second easel which is quite unsymmetrical to its axis of rotation, at the right corner of the base. The front or operating side of the instrument shown in Figure 5 presents the first stage components, the film-stage and the first lens, as mounted on a bench-type beam which is pivotable about the axis of the first easel (shown in profile on the right). Visible is also the second-stage lens, but the second easel is hidden behind the black curtain.

In apparent disagreement with the geometrical layout of Figure 2 the bench-like beam has machined ways along which the two carriages which support the film-stage and the first lens may be moved. Obviously, the magnification ratio of the first-stage unit can be changed, although in two-stage operation it will always be 1:1. This provision is made to make possible the use of the instrument in single-stage procedure when low tilt photography, i.e. with aerial tilt angles less than 45°, shall be rectified. The magnification may then be changed between the limits $m=0.75$ and 2.2. Verniers and a hand wheel are installed with which the film-stage and the lens can be set along separate conjugate scales to precomputed readings.

In single-stage application the tilted lens geometry (i.e. rotation of the film-stage, the lens and the easel to establish the Scheimpflug condition) has certain advantages over the non-tilting pattern of alignment. The y-displacement of the photo principal point remains at zero and the image trapezoid is placed more favorably upon the easel area. The lens is, therefore, made rotatable on its post and the required degree of rotation is applied by circle and vernier. Similar provisions are made at the film stage and both easels. The foot of the first easel has an additional divided circle concentric with easel circle which serves to set the angle between the two principal axes, the so-called dihedral angle. The setting is achieved by a reversible electric motor which pivots the beam of the first stage and is operated by a toggle switch.

The film-stage contains the conventional installations, i.e. film spool holders for uncut roll film and glass stage and pressure plates with $9''\times 9''$ free apertures. These parts are mounted on a turn table for setting the swing angle on a 360° divided circle. All components including the light source are encased in a ventilated housing supported on a rectangular frame along which they can be translated horizontally as required by the computed y-displacement of the photo principal point.

A great deal of effort in the development of this instrument went into research and the design of the illumination system. It is quite obvious that extraordinary means had to be employed in order to project an aerial photograph through one lens on a screen (first easel) and immediately reproject its image through a second lens onto a second screen (or easel). This requires a very powerful light source. Under the extreme conditions of aerial tilt which this instrument must handle, the object and image planes of both rectification stages are strongly oblique and cause the well known and very inconvenient drop of

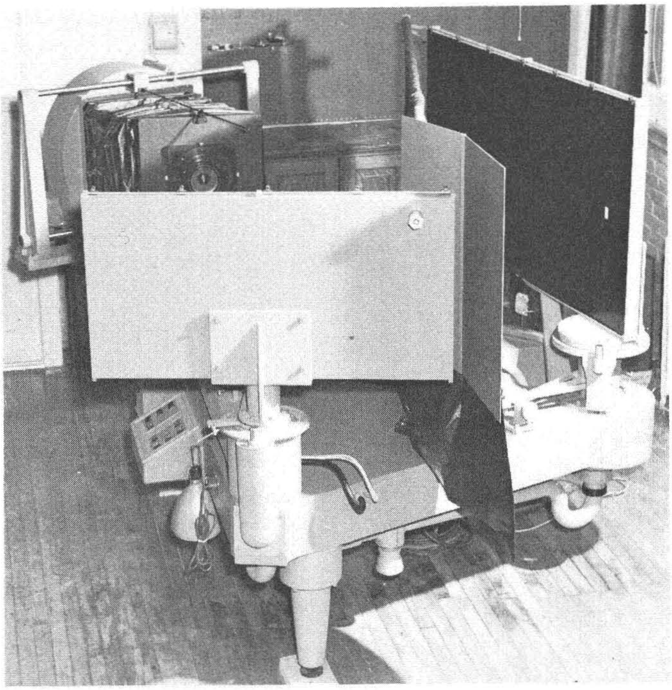

FIG. 4. Two stage rectifier, sideview.

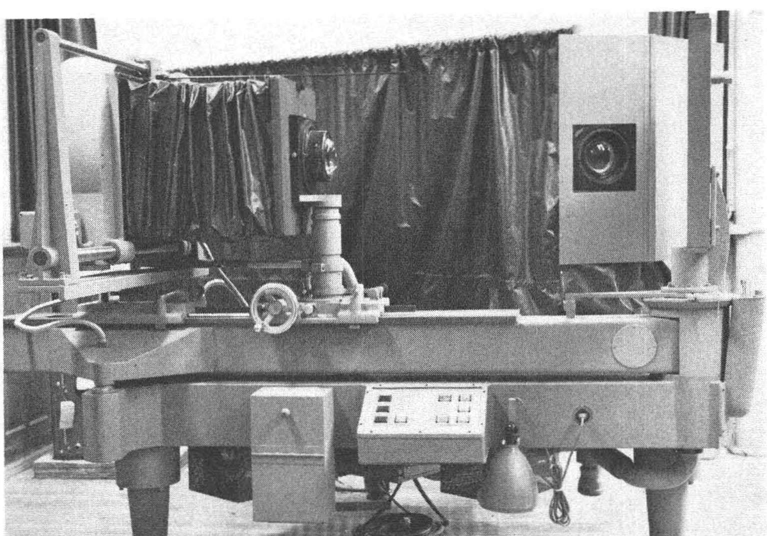

FIG. 5. Two stage rectifier, frontview.

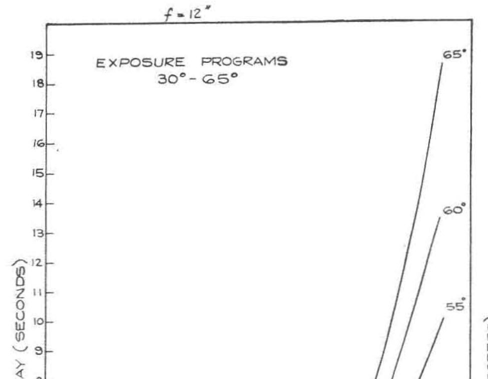

FIG. 6. Punched cards exposure program.

image brightness from the foreground region to the background zone of the terrain image. This situation calls for progressively longer exposure of the photographic material with increasing nadir distance of the image detail on the easel. The progressive fading of light intensity is more strongly pronounced on the second stage easel. Any attempt to employ conventional hand dodging technique is doomed to failure due to the large physical dimensions of the rectified print (particularly in high oblique photography), and to the low level of illumination encountered at the extreme ends of the rectification trapezoid.

The problem of obtaining uniform exposure of the entire rectified area was successfully solved by a mobile light source of linear shape which travels during the exposure process across the film format with variable timing. The lamp is a 16,000 Volt, 1,000 Watt G.E. Xenon arc tube of approximately 12″ arc length, vertically mounted in the focal line of an elliptical channel reflector and moved along a threaded shaft by an electric, low voltage, high torque motor. The lamp movement is not continuous. It is composed of about 70 steps of $\frac{1}{4}''$ length each. The variable rate of progression is obtained by controlled delays between the individual steps. During the stepping motion a narrow band of intense light illuminates a vertical section of the film and prints its image content on the photographic material. In single-stage operation this material is placed on the first easel. In two-stage operation the first easel is dressed with a grainless white diffuse reflecting film. The photographic material (film of the commercial ortho emulsion type) is held on the second easel.

The lamp passage is electronically controlled by a punched card which contains a lamp advance program. This card is mounted on the cylinder of an I.B.M. card reader. The punched holes along 10 horizontal rows of numbers (Figure 6) command the electronic circuitry to build up delay times, ranging from 1/10 sec. (0-row) in geometrical progression to 48 seconds (numbers 9-row). The resultant delay time between each step of lamp advance is the sum of the delays represented by the punched holes found in the vertical column of each of the 70 steps provided in the program.

The stepping motion of the lamp and the card reader cylinder are synchronized to achieve a programmed scanning movement across the film format which begins usually with a very rapid advance of the lamp over the foreground region of terrain imagery. It gradually slows down as the vertical sum of the programmed delay intervals increases. In the example given for two-stage rectification of an $f=12''$ photograph of 60° tilt, the total exposure duration is 110 seconds. The illustration shows the remarkable flexibility of this mode of exposure automation. It permits operation (within limits) with various lens apertures and subdivision of the total exposure into several passages of the light source across the film format.

The preparation of exposure programs is done graphically (Figure 6). A family of curves with tilt increments of 5° for a given aerial camera focal-length (e.g. $f=12''$) is constructed with reference to the steps of

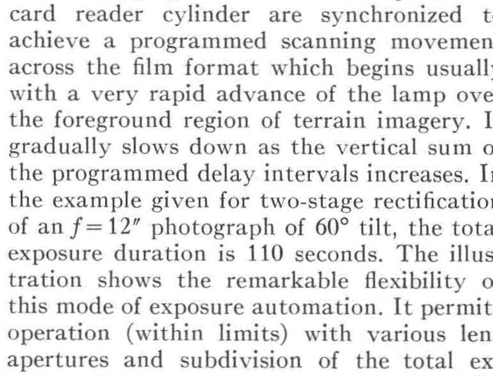

FIG. 7. Card reader, control panel.

lamp advance (abscissa) and time delay in seconds (ordinate). For any given intermediate tilt the exposure program can be interpolated with sufficient accuracy. Experience will show that these programs can readily be modified whenever the density distribution in the aerial negative deviates from uniformity, and therefore requires a different rate of exposure increase in foreground or background zones of the terrain images.

In lower tilt photography ($t<20°$) the program curves straighten out and have a small slope angle. The total exposure time decreases rapidly and may be as short as five seconds for near-vertical photography, while high tilt photography at 75° may require up to eight minutes.

The entire procedure of rectification consists of relatively few manual steps of aligning the photograph, film-stage and easels by precomputed data, selecting the punched program card, inserting it in the card reader (Figure 7), operating a few switches on the control panel, energizing the powerful vacuum unit which holds the screen and photomaterial flat on the easels and cools the Xenon arc lamp, and finally pushing the program button which lights the lamp and sets the exposure program in motion. At the end of the cycle, the lamp is turned off automatically and returns to its starting position, ready for the next run. The final product is a positive or negative transparency, or a paper print of completely uniform exposure and, naturally, of high geometric fidelity.

The project of building a series of instruments of this kind was sponsored by the United States Government. They will be used by several Government Services as a means of rapid rectification by precomputed orientation data of large quantities of oblique exposures ranging from verticals to 76° obliquity.

*Contrast Control for Diapositives**

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ABSTRACT: *The objective of contrast control or dodging in diapositive printing is to permit a transfer of photographic images in which the contrast between minute contiguous images is retained and, at the same time, the maximum and minimum densities of the diapositive are limited.*

Contrast control in diapositive printing improves the accuracy of stereoscopic pointing in projection-type plotting equipment. The improvement is greatest for aerial negatives having extreme density ranges. Diapositives for four representative stereoscopic models were prepared in a 153/55 ratio-printer using an infrared quenching-type contrast control, an electronic feedback-type control, and using no control. The standard deviation of height readings expressed as a fraction of the flight height was 1/14,360 for the infrared-type control, 1/12,670 for the electronic control, and 1/11,140 for no control.

OBJECTIVE OF CONTRAST CONTROL

AT PRESENT (1961) almost all topographic maps are compiled from aerial photographs through the use of stereoscopic plotting instruments. Most of these instruments utilize glass-plate diapositives prepared either at negative scale or a reduced scale. Insofar as image-quality serves stereoplotting efficiency, it is desirable to transfer from the negative to the diapositive, as completely as possible, all of the imagery appearing on the original negative. To achieve such a transfer, some means of controlling image-contrast is necessary, particularly for aerial negatives having extremely large density ranges.

The objective of contrast-control or dodging in diapositive preparation is to permit a transfer of photographic images in which the

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