Report on Analytic Aerotriangulation‡

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ABSTRACT:** *The Coast and Geodetic Survey has developed a method of Analytic Aerotriangulation for the IBM-650 electronic computer which consists of a series of distinct steps. These steps are described comparing each with its counterpart in the more familiar instrumental method of aerotriangulation. The results of tests and the accuracies attained in production are reported, and the primary sources of random errors discussed. The results of a comparison of wide and super-wide angle aerial cameras are also reported.*

1. INTRODUCTION

THE Coast and Geodetic Survey has had an analytic system of aerotriangulation working alongside conventional first-order plotting instruments in routine production for the past year. Although the analytic method is somewhat slower with present instrumentation, it has proven to be superior in many ways. The redundancy of model pass-points, permitted by the mathematical model, eliminates the need for ever rerunning an entire strip; the permanent marking of pass-points, along with the numerical orientation, permits the extension of a strip or block at any time Furthermore, photographs of different scales, made with cameras of different focal-lengths and angular fields, can be combined into a single strip or block aerotriangulation. The additional precision of the analytic method is also an invaluable aid in the recognition and elimination of faulty control points.

The C&GS method is an adaptation, for a medium-small electronic computer, of the approach developed by Dr. Hellmut Schmid of the Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland. A full description of the method was published in the March 1962 issue of PHOTOGRAMMETRIC ENGINEERING[a] and will be contained in a forthcoming *C&GS Technical Bulletin*, which will include formula derivation. Except for a brief outline of the method as background, this paper will be devoted to the results of comparisons and tests.

[a] Vol. XXVIII, no. 1, p. 44.

2. DATA PROCUREMENT

If data procurement is begun with the glass diapositive photograph, the first step is the permanent marking of all photogrammetric points with small holes drilled through the photographic emulsion. The Wild PUG stereoscopic point-marker (Figure 1) is used for this operation. This relatively inexpensive instrument is normally used to prepare diapositives for use on conventional plotting instruments. Therefore, one might say, that for precision aerotriangulation, the analytic and instrumental methods are identical to this point.

The next and last step in data procurement is the measurement of the photo coordinates of the drilled holes. This is accomplished with the Mann monocular comparator (Figure 2) which has a reading precision of one micron and a standard error of coordinate measurement of approximately two microns. Through the use of digitizing heads and a storing pulse counter, the coordinates are automatically recorded by typewriter and punched paper tape. The cathode ray tube on the right is the display of a recently installed electronic scanner which has lessened operator fatigue and increased the productivity of the instrument.

3. DATA PROCESSING

Data processing is divided into three phases to enable economical solution with the IBM-650 electronic computer. The first phase is the correction of photo-coordinates for all known systematic errors (Figure 3). Here, in a single computer program, film distortion, lens dis-

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* Mr. Harris is Development Assistant to the Chief of the Photogrammetry Division.
** The Abstract for this paper is on page 347 of the 1962 YEARBOOK.

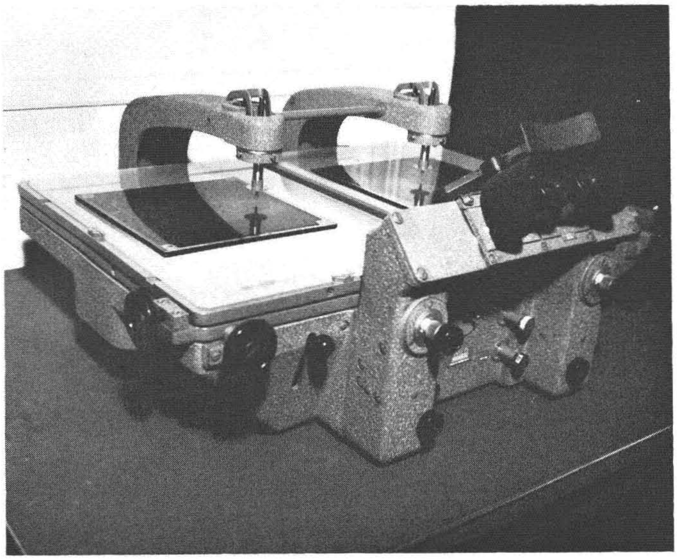

FIG. 1. The Wild PUG stereoscopic point marker.

tortion, and atmospheric refraction are compensated to obtain improved coordinates for aerotriangulation. The need for compensation of earth curvature is avoided through the use of a geocentric coordinate system for control data.

The second phase is the derivation of approximate data for the final block adjustment. It is here that the analytic method simulates the conventional instrumental method. The first step is the relative orientation of successive photographs to determine the orientation parameters and to eliminate point drilling and measurement errors. The second step,—the cantilever assembly—consists of scaling and attaching the consecutive relatively oriented models to form the strip aerotriangulation in much the same way this is done in instrumental bridging. The third step—the adjustment of the cantilever strip—is the same third-degree conformal adjustment that is used for fitting instrumental strip aerotriangulations to ground-control points.

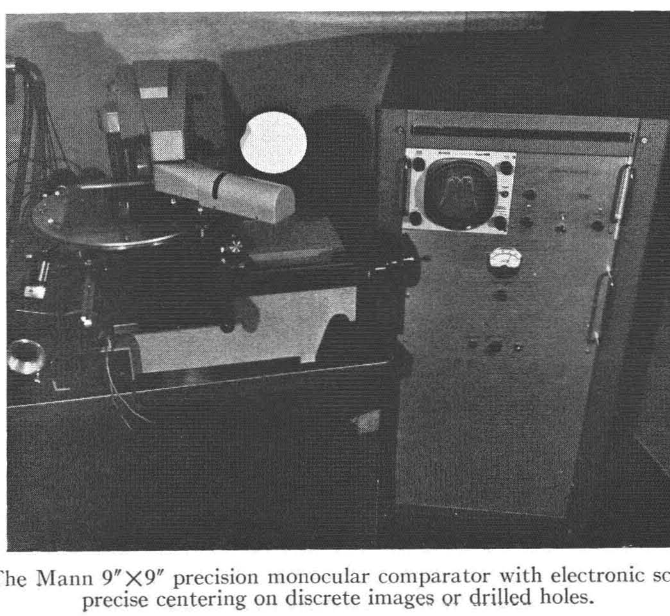

FIG. 2. The Mann 9″×9″ precision monocular comparator with electronic scanner for precise centering on discrete images or drilled holes.

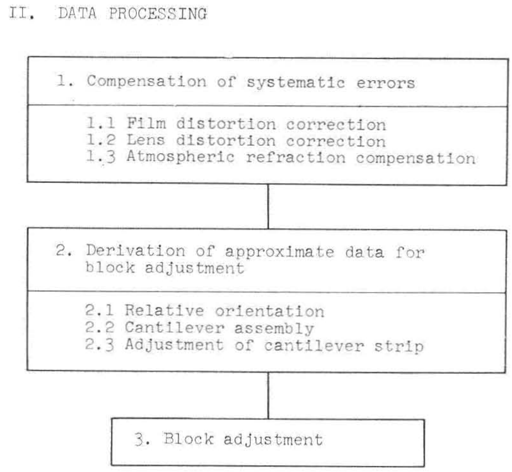

FIG. 3. Block diagram of IBM-650 Computer programs for Analytic Aerotriangulation.

It is at this point that most of our testing and all of our regular mapping aerotriangulations have been terminated with *absolute accuracies and inter-strip ties considerably better than we have ever been able to obtain using instrumental methods.*

The third and final phase—called the block-adjustment, whether it be a strip or a block of strips—is the simultaneous solution of the absolute orientation of all photographs in such a way that the sum of the squares of the observational or photo-coordinate errors is a minimum. By beginning the block-adjustment with near perfect values for the individual photograph orientation parameters, *the iterative block-adjustment problem is solved in one iteration.* It seems certain that when tests are completed on the block-adjustment, we will find that control bridging for large-scale mapping can be obtained from small-scale photography using randomly distributed ground control.

4. ACCURACY TESTS

Each computer program was tested with fictitious data as it was completed. But the most interesting part of the development of analytic aerotriangulation was the step-by-step testing with real photographs and the comparison of results with the rather well perfected instrumental method. The final comparison was made on 11 models of a strip of over-controlled 1:40,000 scale photographs (Figure 4) which was flown specially for the evaluation of the analytic method. The upper diagram shows the distribution of horizontal ground-control stations all of which were premarked with temporary photographic target panels. The six solid triangles indicate the control-points that were used for a least squares adjustment of both instrumental and analytic aerotriangulations, and the open triangles, the points that were withheld for the adjustment but included in the accuracy evaluation. The lower diagram shows the distribution of vertical points. These points, though not premarked, were established at sites where the terrain was flat. The same diapositives were used for both methods with the following results. The *rms* errors for the analytic method were 1.6 feet for horizontal position and 1.4 feet for elevation, while the instrumental method gave *rms* errors of 6.4 feet for position and 3.5 feet for elevation.

Heavy production schedules on the plotting instrument prevented a second comparison with different photographs, but the analytic method was used on another strip of photo-

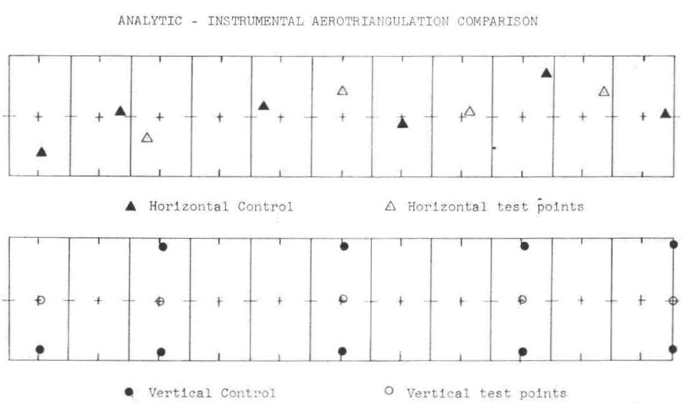

FIG. 4. Control diagram for portion of U. S. Route 11 temporary test area used for a comparison between analytic and instrumental methods.

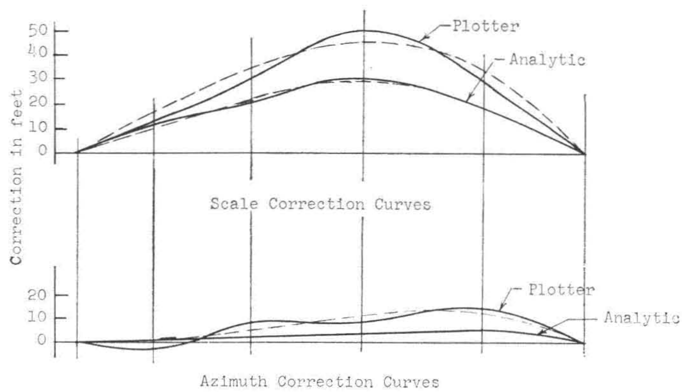

FIG. 5. Scale and azimuth correction curves for test between analytic and instrumental strip aerotriangulations on area shown in Figure 4.

graphs of the same area to compare the elevations obtained for the 69 common pass-points between the strips. The *rms* elevation difference for these points was 2.4 feet for the two independent analytic aerotriangulations, whereas, it was 6.9 and 7.4 feet between the instrumental and two analytic solutions. The maximum observed difference for the pass-points was 6.6 feet between the two analytic strips and 16 and 22 feet between the instrumental and the analytic strips.

As mentioned earlier, the C&GS method uses a third-degree conformal adjustment for the cantilever strip to obtain a better fit to ground control-points, and more correct camera orientation parameters for the final block adjustment. This mathematical spline-bending technique has been justified, or at least rationalized, by several mathematical derivations. It is hoped that the reduction of systematic and random errors along with the least squares block adjustment will eventually permit dispensing with this device. In the meantime, adjustment curves (Figure 5), such as were obtained for the analytic and instrumental aerotriangulations in the test just described, will be needed. The upper diagram shows the required scale correction for each method as a solid line. The dashed lines show the third-degree corrections that were applied by the least squares curve fitting adjustment, and the separation between the solid and dashed lines shows the residual errors after adjustment. The lower diagram shows the same curves for the azimuth of the strip. The increased precision of the analytic method is indicated by the amplitude and smoothness of the curves. Notice also that the residual errors in azimuth are smaller, even to the point of practical nonexistence. These curves are typical of our experience over the years and for this reason, we always

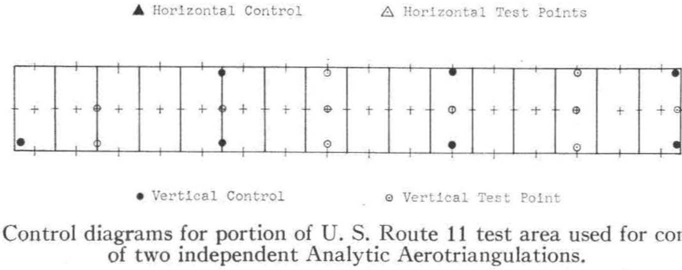

FIG. 6. Control diagrams for portion of U. S. Route 11 test area used for comparison of two independent Analytic Aerotriangulations.

try to plan flight-lines and control-surveys so that a minimum of four well-spaced control points occur in each strip aerotriangulation.

The real test of analytic aerotriangulation was made by extending the number of models from 11 to 16 (Figure 6), thereby including seven bands of vertical ground-control points so that alternate bands could be withheld to serve as mid-span and mid-strip check-points. At the same time, all but four of the horizontal-control points were withheld. This arrangement of control permitted the use of third-degree adjustment curves and gave a span of 8 models or *19 miles* between points of known position and a span of $5\frac{1}{2}$ models or *12 miles* between points of known elevation. Two separate sets of 1:40,000 scale photographs made with a six-inch camera on a low distortion polyester aerial film were used on this test, to determine the repeatability of results and to compare the positions and elevations of hundreds of pass-points obtained by two independent aerotriangulations. The *rms* horizontal error, based on all 15 control points, was 3.2 feet, which is 24 microns at plate-scale for one strip, and 4.4 feet or 33 microns for the other. The corresponding vertical errors were 1.6 and 3.1 feet. Expressing these accuracies as fractions of the flight altitude, the *standard error of position for 8 model spans was one part in five thousand of the flight altitude*, and the *standard error of elevation for 6 model spans was one part in eight thousand of the flight altitude.*

From the cartographer's point-of-view, the accuracy of these aerotriangulations is equivalent to having all model pass-points located and leveled by high-order field surveys. That is, with any but the most precise stereoscopic plotting instruments, working at scales up to five times the scale of the photography, the resulting improvement of relative and absolute orientation could not be set on an instrument, and even if it could, the additional accuracy would be lost in drafting. It was surprising to find that prior to the non-linear adjustment, the accumulated scale-error at the center of the 38 mile strip, amounted to 40 feet or 300 microns at plate-scale, when the standard y-parallax within the models was only 5 microns and the standard error of tie between models was only 16 microns. It is expected that the use of a block adjustment will increase the y-parallaxes and reduce intermodel tie and accumulated scale-errors.

In addition to the absolute accuracy evaluation of these two strips, all pass-points were inter-transferred between the two sets of photographs so that the agreement between independent aerotriangulations could be determined for points along the edges of the strip. The *rms* differences in pass point position elevation determination between the two aerotriangulations were 3.5 feet and 3.8 feet respectively. The maximum observed differences were 8 feet in position and 12 feet in elevation. We are now anxiously awaiting the block adjustment of these two strips to see how much more improvement can be made with our present knowledge of systematic errors such as lens and film distortion.

In another series of tests, our permanently targeted Ohio test area (Figure 7) was photo-

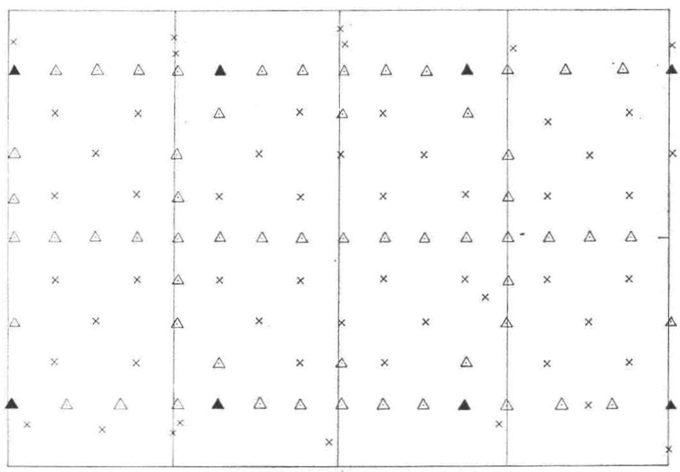

FIG. 7. Control diagram for portion of the C&GS Ohio Camera Calibration Area used for comparisons of wide and super-wide-angle cameras and several aerial films. The triangles show the distribution of permanently marked control points and the crosses show points of known elevation.

FILM - CAMERA COMBINATIONS

	RC-8 Camera Polyester base	RC-9 Camera Polyester base	RC-9 Camera Topo base
RMS residual Y-parallax			
72 Relative orientation points	4.8 microns	7.6 microns	7.5 microns
300 Photogrammetric points	6.3	7.3	7.6
RMS intermodel tie residuals			
36 Horizontal position	7.2	11.5	6.7
36 Elevation	12.9	7.8	10.4
RMS errors after adjustment			
Horizontal control points	16.3(1.3 ft)	15.4(1.2 ft)	18.2(1.4 ft)
Vertical control points	15.3(1.2 ft)	18.2(1.4 ft)	37.7(2.9 ft)
RMS errors after block adjustment	?	?	?

FIG. 8. Tabulated results of Ohio Camera Calibration tests of two cameras and two film bases.

graphed with several stable-base aerial films and with both wide-angle and the super-wide-angle cameras. By selecting a photographic scale of 1:24,000, it was possible to make a short strip of five photographs on which the model pass-point areas contained premarked horizontal and vertical-control points. The stereomodel limits shown by the four quadrangles are representative of all strips used for camera and film comparisons. The 66 triangles show the distribution of premarked geodetic points of known position and elevation, and the crosses show unmarked areas where the terrain elevation is known. The eight solid triangles indicate the three-dimensional control that was used for the adjustment of the strip.

Data reduction is not complete on this airborne comparison of camera-film combination, but results to date (Figure 8) indicate that both film and lens distortions are now well enough determined and compensated to enable the use of any combination without fear of serious deterioration of accuracy. The first two columns show the step-by-step comparison of the two cameras using the same film, and the last two columns, the comparison of two types of film bases in the same camera. The only significant difference in the size of y-parallax residuals was the low 4.8 microns obtained in relative orientation with the wide-angle camera. This figure, which was obtained from clusters of four points at each of the pass-point areas, increased to 6.3 microns when all 300 points in the strip were included.

The difference in the base-height ratios of the cameras explains the reversal of position and elevation precisions indicated by the intermodel ties, except for the topographic film base in the RC-9 super-wide-angle camera which gave essentially the same results as the polyester base in the wide-angle camera. This will probably be explainable when data reduction is complete on the wide-angle camera with topographic base film. The only significantly different value in the final adjusted ground position and elevation errors is the 37.7 microns or 2.9 feet for vertical points in the super-wide-angle camera-topo base combination, which is twice that of any other combination. This is directly related to the intermodel tie discrepancy above it, and will also probably be resolved when data reduction is complete. We are very anxious to see the results of the block adjustment on this test series because the residual plate coordinate errors will point out any residual systematic distortions that may exist.

5. FILM DISTORTION STUDIES

While pleased with the accuracies already attained with the analytic method, we are now more determined than ever to reduce residual systematic errors of the photograph. The largest of these is probably the residual film-distortion that remains after a mathematical restoration of the fiducial marks. An aversion to the use of glass negatives or pressure-plate reseaus in the aerial camera has led to an investigation of the metric characteristics of the aerial film after processing, drying and storage. For this investigation, a calibrated grid plate was contact printed on several types of film along with aerial photography so that the grid exposures would be treated as ordinary aerial photography. Incidentally, through the use of these grid

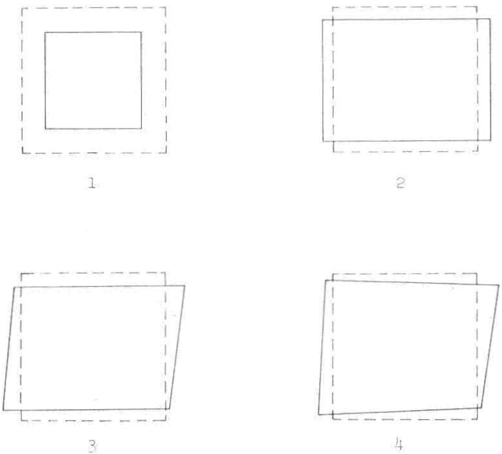

FIG. 9. Diagrams of film distortion illustrating: (1) overall shrinkage, (2) differential shrinkage, (3) angular distortion, (4) random distortion.

exposures, one of our older developing machines was found to be contributing measurably to film distortion. At regular time intervals, the grid exposures are printed on glass plates and measured with the precision comparator. The resulting plate-coordinates are first treated with a regular scale change to determine the residual-distortion which exists in the model formed in the stereoplotter during map compilation; then they are treated with the analytic non-affine transformation to determine the residuals that are propagated through the strip or block.

The first diagram in Figure 9 illustrates regular shrinkage which is readily compensated by ratio printing of principal-distance adjustment on the plotting instrument. This results in an averaging of the differential distortion as shown in the second diagram. A closer look at the residuals at this point reveals that the original square figure has not become a true rectangle, but is more like a parallelogram as shown in Figure 3. Finally, a very close examination shows the parallelogram to be actually a quadrilateral something like diagram 4. This is the figure that is compensated in the analytic distortion compensation program which restores the fiducial marks to their calibrated positions.

Through the use of the grid exposures, it was possible to investigate the effectiveness of the analytic compensation at points between the fiducial marks. As one might expect, the residual distortion, although quite small, was non-linear. This type of distortion is shown in a general way in the first diagram of Figure 10. Although analysis has not progressed sufficiently to draw final conclusions regarding the adequacy of film negatives for aerotriangulation, or the required density of film distortion control points, it is apparent that films having smaller regular shrinkage do not necessarily have smaller residual distortion after mathematical distortion compensation. It is also apparent that the center-point defined by the intersection of lines connecting the fiducial marks is determined with only a little more precision than the mid-points of the sides. Nevertheless, it is recommended that aerial cartographic cameras used for analytic photogrammetry be equipped with at least eight fiducial marks located at the four corners and the mid-points of the sides. Furthermore, these marks should reproduce on the negative with sufficient legibility to enable determination of their plate coordinates within three microns. The reason for eight marks is twofold; first, as shown in the

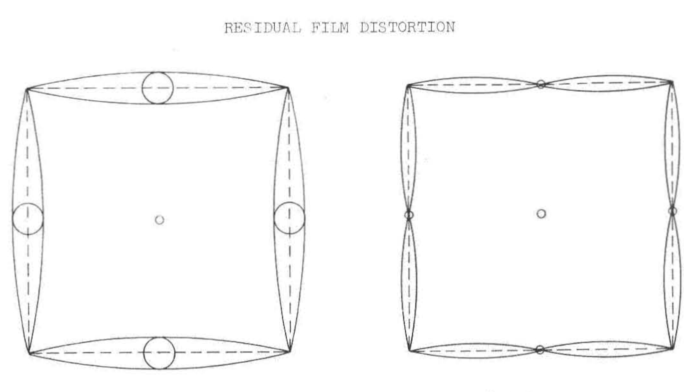

FIG. 10. Diagrams showing residual film distortions after numerical compensation using four and eight camera fiducial marks.

second diagram of Figure 10, the residuals are reduced to one-quarter of their magnitude when the span between marks is reduced one-half, and secondly, eight fiducial marks provide local control of film distortion in the model pass-point areas so that residual-distortion affects only the individual models and is not propagated in aerotriangulation.

6. Control Identification

If there is any source of error greater than film distortion, it is control-point identification. Analytic aerotriangulation precision has made the need for premarking horizontal control points much more apparent. In fact, it now seems that even the most expert field men can seldom, if ever, find nearby objects suitable for use as substitute control-points for analytic aerotriangulations which have standard errors of from 25 to 50 microns at plate scale. Inasmuch as premarking is not always practicable, present requirements for the selection of substitute stations state that unless the object is very small and symmetrical in shape, the contrast between it and its background must be low and its reflectivity must be such that the resulting image density will occur in the middle-gray tones to minimize image spread in the emulsion. Furthermore, at least two substitute points must be established for each horizontal-control station. In spite of these precautions, *the residual errors between the pairs of adjacent substitute control-points are frequently greater than the maximum errors obtained in premarked test area aerotriangulations.* It is therefore concluded, that when maximum accuracy is desired, horizontal-control points must be premarked with symmetrical photographic targets.

Vertical Accuracy Analysis

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ABSTRACT: *This paper describes a procedure of making an analysis of vertical accuracy based on a comparison of field-surveyed cross-section elevations with photogrammetrically measured elevations. There definitely is a need for more research and data in this field. Only when and if enough supporting evidence is amassed, will photogrammetric engineers overwhelmingly convince contractors to accept photogrammetrically measured cross sections.*

ANALYSIS of vertical accuracy based on a comparison between field-surveyed and photogrammetrically-measured elevations was made after completing the compilation of a set of ten topographic maps for preliminary survey of a road, referred to as California Forest Highway 6-Beegum-Peanut. This is part of a "Report on Photogrammetric Methods of Compiling Topographic Maps and Accuracies Achieved," which was published by the Bureau of Public Roads, U. S. Department of Commerce.

Horizontal and vertical-control points were obtained with only a small amount of field surveying. Previously established stations of basic control and existing aerial photography at a scale as small as 1:50,000 were used. Vertical-control points were measured to control future stereomodel orientations based on such control and photography.

Supplemental horizontal-control points were established by radial plot assembly of slotted model-size stereotemplets using the Kelsh Plotter and specially flown (1:19,200) bridging photography. The maps were compiled at a scale of 100 feet-to-one-inch with a contour interval of five feet, utilizing 400 feet-to-one-inch scale photography taken with an $8\frac{1}{4}$-inch focal-length aerial camera.

The basic data for the analysis consisted of elevations of points measured from contours photogrammetrically measured and delineated, and elevations of field-surveyed cross-sections. These cross-section elevations were determined by hand levels and the right angles were turned by a 90° prism and by cruder methods.

The elevations of points used in the comparisons were located (1) in areas not completely obscured by trees, as evidenced by contours not dashed on the topographic maps (Tables I and II), and (2) in areas of dashed contours (Table III).

For computing construction quantities, cross-sections were measured on this project at all significant changes in ground slope,