second diagram of Figure 10, the residuals are reduced to one-quarter of their magnitude when the span between marks is reduced one-half, and secondly, eight fiducial marks provide local control of film distortion in the model pass-point areas so that residual-distortion affects only the individual models and is not propagated in aerotriangulation.

6. Control Identification

If there is any source of error greater than film distortion, it is control-point identification. Analytic aerotriangulation precision has made the need for premarking horizontal control points much more apparent. In fact, it now seems that even the most expert field men can seldom, if ever, find nearby objects suitable for use as substitute control-points for analytic aerotriangulations which have standard errors of from 25 to 50 microns at plate scale. Inasmuch as premarking is not always practicable, present requirements for the selection of substitute stations state that unless the object is very small and symmetrical in shape, the contrast between it and its background must be low and its reflectivity must be such that the resulting image density will occur in the middle-gray tones to minimize image spread in the emulsion. Furthermore, at least two substitute points must be established for each horizontal-control station. In spite of these precautions, *the residual errors between the pairs of adjacent substitute control-points are frequently greater than the maximum errors obtained in premarked test area aerotriangulations.* It is therefore concluded, that when maximum accuracy is desired, horizontal-control points must be premarked with symmetrical photographic targets.

Vertical Accuracy Analysis

BERNARD J. COLNER,
Highway Engineer, Bureau of Public Roads, Washington, D. C.

ABSTRACT: *This paper describes a procedure of making an analysis of vertical accuracy based on a comparison of field-surveyed cross-section elevations with photogrammetrically measured elevations. There definitely is a need for more research and data in this field. Only when and if enough supporting evidence is amassed, will photogrammetric engineers overwhelmingly convince contractors to accept photogrammetrically measured cross sections.*

ANALYSIS of vertical accuracy based on a comparison between field-surveyed and photogrammetrically-measured elevations was made after completing the compilation of a set of ten topographic maps for preliminary survey of a road, referred to as California Forest Highway 6-Beegum-Peanut. This is part of a "Report on Photogrammetric Methods of Compiling Topographic Maps and Accuracies Achieved," which was published by the Bureau of Public Roads, U. S. Department of Commerce.

Horizontal and vertical-control points were obtained with only a small amount of field surveying. Previously established stations of basic control and existing aerial photography at a scale as small as 1:50,000 were used. Vertical-control points were measured to control future stereomodel orientations based on such control and photography.

Supplemental horizontal-control points were established by radial plot assembly of slotted model-size stereotemplets using the Kelsh Plotter and specially flown (1:19,200) bridging photography. The maps were compiled at a scale of 100 feet-to-one-inch with a contour interval of five feet, utilizing 400 feet-to-one-inch scale photography taken with an $8\frac{1}{4}$-inch focal-length aerial camera.

The basic data for the analysis consisted of elevations of points measured from contours photogrammetrically measured and delineated, and elevations of field-surveyed cross-sections. These cross-section elevations were determined by hand levels and the right angles were turned by a 90° prism and by cruder methods.

The elevations of points used in the comparisons were located (1) in areas not completely obscured by trees, as evidenced by contours not dashed on the topographic maps (Tables I and II), and (2) in areas of dashed contours (Table III).

For computing construction quantities, cross-sections were measured on this project at all significant changes in ground slope,

TABLE 1

VERTICAL ACCURACY ANALYSIS

(NON-DASHED CONTOUR AREAS)

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
Difference (Feet)	*No. of Points*	*Cumulative Total*	*Cumulative (%)*	*Difference (Feet)*	*No. of Points*		*Algebraic Σ [6+7]×5*		*Numerical Σ 6+7*	*$n(e)^2$ 10×5^2*
					+	−	+	−		
+6.4	2	791	100	0.1	4	6		0.2	10	0.1
6.1	1	789	99.7	0.2	9	10		0.2	19	0.7
6.0	4	788	99.6	0.3	15	8	2.1		23	2.1
5.9	1	784	99.1	0.4	14	14			28	4.5
5.6	1	783	99.0	0.5	10	10			20	5.0
5.4	3	782	98.9	0.6	14	4	6.0		18	6.5
5.2	1	779	98.5	0.7	11	11			22	10.8
4.8	1	778	98.4	0.8	17	16			33	21.1
4.6	1	777	98.2	0.9	7	9		1.8	16	13.0
4.4	3	776	98.1	1.0	31	17	14.0		48	48.0
4.3	3	773	97.7	1.1	12	12			24	29.0
+4.0	7	770	97.3	1.2	15	8	8.4		23	47.5
3.9	1	763	96.5	1.3	14	8	7.8		22	37.2
3.8	5	762	96.3	1.4	27	16	15.4		43	84.3
3.7	1	757	95.7	1.5	7	2	7.5		9	20.3
3.6	5	756	95.6	1.6	12	12			24	61.4
3.5	6	751	94.9	1.7	14	13	1.7		27	62.2
3.4	5	745	94.2	1.8	22	12	18.0		34	110.2
3.3	2	740	93.6	1.9	9	3	11.4		12	43.3
3.2	7	738	93.3	2.0	20	13	14.0		33	132.0
3.1	4	731	92.4	2.1	10	5	10.5		15	66.2
+3.0	12	727	91.9	2.2	17	9	17.6		26	125.8
2.9	1	715	90.4	2.3	7	1	13.8		8	42.3
2.8	5	714	90.3	2.4	14	12	4.8		26	149.8
2.7	9	709	89.6	2.5	8	4	10.0		12	75.0
2.6	15	700	88.5	2.6	15	4	28.6		19	128.4
2.5	8	685	86.6	2.7	9	3	16.2		12	87.5
2.4	14	677	85.6	2.8	5	12		19.6	17	133.3
2.3	7	663	83.8	2.9	1	1			2	168.2
2.2	17	656	82.9	3.0	12	3	27.0		15	135.0
2.1	10	639	80.8	3.1	4	3	3.1		7	67.3
+2.0	20	629	79.5	3.2	7	2	16.0		9	92.3
1.9	9	609	77.0	3.3	2	5		9.9	7	76.2
1.8	22	600	75.9	3.4	5		17.0		5	57.8
1.7	14	578	73.1	3.5	6			21.0	6	73.5
1.6	12	564	71.3	3.6	5	2	10.8		7	90.7
1.5	7	552	69.8	3.7	1	1			2	27.4
1.4	27	545	68.9	3.8	5	5			10	144.0
1.3	14	518	65.4	3.9	1	2		3.9	3	45.6
1.2	15	504	63.7	4.0	7	3	16.0		10	160.0
1.1	12	489	61.6	4.1		3		12.3	3	50.4
+1.0	31	477	60.3	4.2		3		12.6	3	52.9
0.9	7	446	56.4	4.3	3	3			3	55.5
0.8	17	439	55.5	4.4	3		13.2		3	58.1
0.7	11	422	53.4	4.6	1	2		4.6	3	63.5
0.6	14	411	52.0	4.7		2		9.4	2	44.2
0.5	10	397	50.2	4.8	1	1			2	46.1
0.4	14	387	48.9	5.0		2		10.0	2	50.0
0.3	15	373	47.2	5.1		1		5.1	1	26.0
0.2	9	358	45.3	5.2	1	1			2	53.0
+0.1	4	349	44.1	5.4	3	1	10.8		4	116.6
0.0	38	345	43.6	5.6	1	1			2	62.7

(*Table 1 continued on next page*)

TABLE 1—(*Continued*)

1 Difference (Feet)	*2* No. of Points	*3* Cumulative Total	*4* Cumulative (%)	*5* Difference (Feet)	*6* No. of Points +	*7* No. of Points −	*8* Algebraic Σ [6+7]×5 +	*9* Algebraic Σ [6+7]×5 −	*10* Numerical Σ 6+7	*11* $n(e)^2$ 10×5^2
0.0		307	38.8	5.8		1		5.8	1	33.6
−0.1	6	301	38.1	5.9	1		5.9		1	34.8
0.2	10	291	36.8	6.0	4	2	12.0		6	216.0
0.3	8	283	35.8	6.1	1		6.1		1	37.2
0.4	14	269	34.0	6.4	2	1	6.4		3	122.9
0.5	10	259	32.7	6.6		1		6.6	1	43.6
0.6	4	255	32.2	6.9		1		6.9	1	47.6
0.7	11	244	30.8							
0.8	16	228	28.8	Total	791		352.1–129.9			3900.1
0.9	9	219	27.7				= +222.2			
−1.0	17	202	25.5							
1.1	12	190	24.0							
1.2	8	182	23.0							
1.3	8	174	22.0							
1.4	16	168	21.2							
1.5	2	156	19.7							
1.6	12	144	18.2							
1.7	13	131	16.6							
1.8	12	119	15.0							
1.9	3	116	14.7							
−2.0	13	103	13.0							
2.1	5	98	12.4							
2.2	9	89	11.2							
2.3	1	88	11.1							
2.4	12	76	9.6							
2.5	4	72	9.1							
2.6	4	68	8.5							
2.7	3	65	8.2							
2.8	12	53	6.7							
2.9	1	52	6.6							
−3.0	3	49	6.2							
3.1	3	46	5.8							
3.2	2	44	5.6							
3.3	5	39	4.9							
3.6	2	37	4.7							
3.7	1	36	4.6							
3.8	5	31	3.9							
3.9	2	29	3.7							
−4.0	3	26	3.3							
4.1	3	23	2.9							
4.2	3	20	2.5							
4.3	3	17	2.1							
4.6	2	13	1.9							
4.7	2	13	1.6							
4.8	1	12	1.5							
−5.0	2	10	1.3							
5.1	1	9	1.1							
5.2	1	8	1.0							
5.4	1	7	0.1							
5.6	1	6	0.1							
5.8	1	5	0.1							
6.0	2	3	0.0							
6.4	1	2	0.0							
6.6	1	1	0.0							
−6.9	1									

$$\text{Arithmetic Mean} = \frac{+222}{791} = +0.28 \text{ ft.}$$

$$\text{Standard Deviation} = \sqrt{\frac{3900.1}{791} - (0.28)^2} = \sqrt{4.93 - .0784} = 2.20 \text{ ft.}$$

From table:

1 Difference (Feet)	4 Cumulative %
+3.5	94.9
−3.5	4.8
	90.1

Given: Flight height = 3,600 feet

$$\text{c-factor} = \frac{3{,}600'}{2\times 3.5'} = 512$$

TABLE II

VERTICAL ACCURACY ANALYSIS

(NON-DASHED CONTOUR AREAS)

Map Sheet No.	No. of Cross Sections	Flight Height (Feet)	C-factor		Arithmetic Mean (Feet)		Consistent Floating	Consistent Digging	Skewed	Type of Terrain†			Tree Coverage	
									No. of Cross Sections	No. of X-Sections			No. of Cross Sections	
			Used c.i.=5′	Resultant* c.i.=7′	+	−				Steep	Moderate	Gentle	Medium	Clear to Light
1	9	4,300	860	614		.31		x	—	—	7	2	1	8
2	6	4,300	860	614		.27		x	6	1	3	2	3	3
3	11	4,300	860	614		.53		x	10	1	10	—	10	1
4	10	3,690	738	527	1.76		x		—	3	5	2	6	4
5	5	3,690	738	527	.07		x		—	3	2	—	5	—
5A	8	3,690	738	527	.87		x		—	—	5	3	3	5
6	5	4,440	888	629	.62		x		—	4	1	—	4	1
7&7A	14	3,440	688	491	.98		x		—	6	8	—	9	5

* Since 90% of points tested were accurate to within 3½ feet, the contour interval should have been 7 feet, and resultant *c*-factor would be as tabulated.

† See Figure 3.

resulting in an average centerline spacing of 25 feet for the cross-sections. A 5% sample of these cross-sections was used in making the comparisons reported here, and the average spacing of the test cross-section was about 500 feet.

The vertical accuracy analysis (differences in elevation) made for this project included 68 cross-sections tabulated in Table II and 24 cross-sections tabulated in Table III. These cross-sections represent all portions of the mapping extended across a band about 200 feet wide (100 feet each side of the highway centerline). There is an average of eleven points to each cross-section used in the comparisons.

According to Mr. L. L. Funk,* an authority on this type of study, there is a probability of less accuracy in the field surveys than in the photogrammetric compilation. Based on Mr. Funk's findings, the term "difference" was used rather than the term "error."

Areas in which point differences of 5.5 feet or more occurred were examined with particular care for gross errors by resetting the stereoscopic models. If there appeared to be any possibility of an error in the field elevation, the point was rejected. An example would be an area where ground is obviously rising uniformly in the stereomodel, but the field-measured cross-section, after agreeing with this uniformity for some of the measured points, dropped or rose abruptly.

Table I contains the details of the vertical accuracy analysis. The differences to the nearest 0.1 foot are entered in column 1 in descending order from the largest plus difference. The number of points in each difference group is entered in column 2.

The graph in Figure 1 shows the frequency of the number of points in each of the difference groups.

A cumulative total is entered in column 3 of Table I. It will be noted that the largest minus error of −6.9 feet is not entered opposite −6.9, but opposite the group above i.e. −6.6 feet. This indicates that in this case only one point has a difference of more than −6.6 feet. As will be noted at the top of column 3, the total number of points which have a difference of +6.1 feet or less is 791.

The cumulative total for each group is converted to percent and is entered in column 4. These percentages provide an easy means of determining the percentage of points within any desired range. For example, 77.5% (86.6−9.1) are in error by not more than ±2.5 feet.

The percentages shown in column 4 are plotted on arithmetic probability paper in Figure 2 to demonstrate frequency distributions. These are so plotted that for any minus error the percent indicates a difference of

TABLE III

RÉSUMÉ OF VERTICAL ACCURACY DETAIL IN DASHED-CONTOUR AREAS

Map Sheet No.	No. of Cross Sections	Flight Height (Feet)	Arithmetic Mean (Feet)	Maximum	
				+	−
1	4	4,300	4.90	2.7	10.7
2	6	4,300	3.52	6.3	9.3
3	13	4,300	0.97	9.2	10.6
5A	1	3,690	3.90		8.0

* Funk, L. L., "Terrain Data for Earthwork Quantities," *H.R.B. Bulletin* 228, 1959, p. 64.

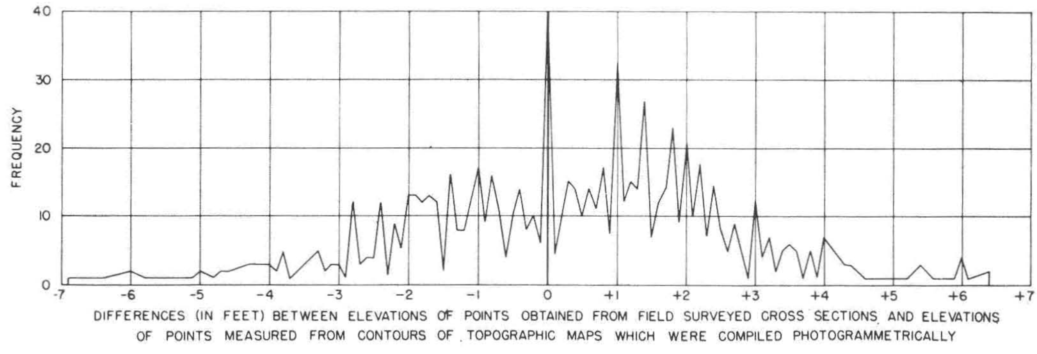

FIG. 1. Graph of frequency of points in difference groups.

"more than," and for any plus error it shows a difference of "equal to or less than." The two straight lines in this figure represent the normal law of error distribution for a five- and a seven-foot contour interval. The distribution of differences more nearly follows the seven-foot than it does the five-foot contour interval line. This points up the fact that the more realistic contour interval for the photogrammetric compilation is seven feet, *assuming the field surveyed elevations are correct.*

The difference groups, without regard to sign, are entered in descending order in column 5 of Table I. The number of plus and minus points in each group is entered in columns 6 and 7, respectively. The algebraic sum of these columns 6 and 7 for each group is multipled by the size of the error, and the result is entered in columns 8 and 9, depending on the sign. The division of the algebraic sum of the totals of columns 8 or 9 by the total number of points results in an arithmetic mean of +0.28 foot.

The numerical sum of columns 6 and 7 for each group is entered in column 10. Each numerical sum is then multiplied by the

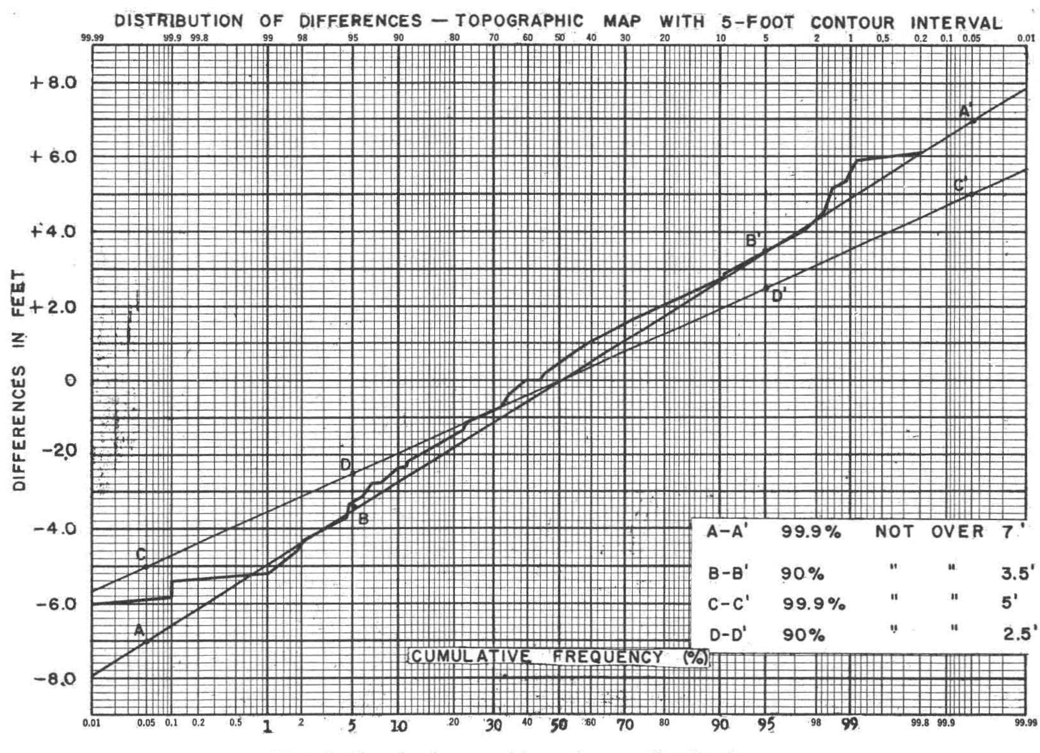

FIG. 2. Graph of normal law of error distribution.

square of the error in feet, and the result entered in column 11. If the total of this column were divided by the number of points and the square root were then extracted, the result would be the standard deviation from zero difference. In order to provide a true measure of dispersion, the standard deviation was calculated on the basis of deviation from the arithmetic mean. This was done by dividing the total of column 11 by the number of points and subtracting the square of the arithmetic mean from the result. The square root of the resulting number gives the standard deviation from the arithmetic mean of 2.20 feet.

The calculated *c*-factor for topographic mapping is determined by finding the difference range in feet which includes a total of 90% of the points. Column 4 is used to determine the 90% range. In this case ±3.5 feet, by interpolation, includes 90.1% of the points. The flight height of the aerial photography divided by twice this error range gives the calculated *c*-factor of 512. The contour interval which would have fulfilled the 90% specification requirement is 7 feet.

The number of cross-sections, the flight heights, the *c*-factors, arithmetic means, and other factors for each sheet are summarized in Table II. As illustrated by this table, practically all of the cross-sections in one portion surveyed in the field were apparently skewed when compared by elevations with cross-sections carefully measured on the map. Along the highway centerline and for the width of the cross-sections, the tree coverage varies from clear to medium and the topography varies from gentle to steep. Figure 3 illustrates the relative gradient of the items in the columns under the heading of "Types of Terrain" of Table II.

There is no evident correlation between the flight-height and the arithmetic-mean on this project. The points on 3 sheets had an overall consistent digging difference, and points on the other sheets had an overall consistent floating difference.

The vertical accuracy that can be expected in areas where the ground is obscured by tall trees, similar to this project, is illustrated by Table III. Tabulated in this table is the arithmetic mean and also the maximum plus-

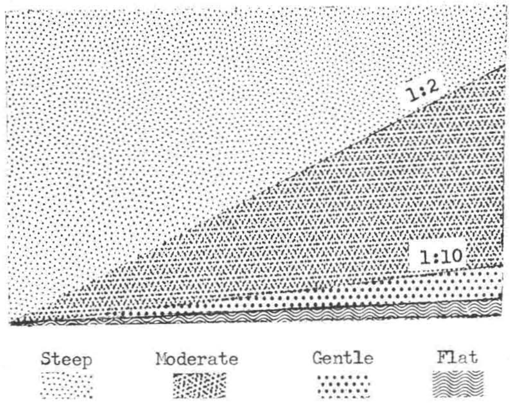

FIG. 3. Types of terrain.

and-minus differences. It is apparent there is a consistent digging difference with a maximum of −10.7 feet. The consistent digging can probably be attributed to a desire to avoid floating in these heavily wooded areas.

To summarize, the reasons for the differences between the field-measured and photogrammetrically-measured elevations can be assigned to these factors:

1. Cross-sections surveyed in the field were not normal to centerline.
2. Errors in measurements and in noting cross-section elevations and distances in field books.
3. According to the statistical approach by Mr. L. L. Funk of the California Division of Highways, grading quantities determined from photogrammetrically made surveys by his Division were more accurate than those obtained by commonly accepted field survey methods.
4. Compilation blunders, systematic, and random errors because of the stereoinstrument operator's inability to be perfect and because of inherent photogrammetric procedure inaccuracies, e.g., reliance on bridged horizontal and vertical-control, isolated tree-covered areas which obscure the ground, and use of positives (emulsion to emulsion prints on glass) instead of diapositives (emulsion through the aerial film base to the emulsion of the glass while being printed by means of a point-light source printer).