and the established point and then rotated to attain coincidence of the azimuths. In a similar manner the vertical-control was adjusted to true datum by means of one vertical-control point.

Positional differences between true and established points and vertical differences between true and established elevations were computed from which the horizontal Circle of Probable Error and the vertical Standard Deviation of errors were computed. The size of the established network was approximately 765 square miles and contained 206 established points. Evaluation of the 206 points resulted in a horizontal CPE of 35.5 feet and a Standard Deviation of vertical errors of 16.5 feet.

This test and test results proved without doubt the capability and practicability of the system. This first attempt was a "bread board model" in which several of the components used were not designed for this application. From experience and information gained in this test, it is obvious that the system can be improved considerably in both equipment and techniques which will probably improve accuracy.

Many applications of this system are foreseeable in both military and domestic mapping operations. In the military sense, a particular advantage is that maps of known scale can be quickly compiled and made available for use prior to occupation or extension of control to the area. These maps can be oriented for limited use by the user until ground control is established and the maps adjusted to true datum. Typical applications, besides normal mapping missions, in which this system would be particularly advantageous include the mapping of areas of difficult accessibility including islands and those areas under recent atomic bombardment, uncontrolled areas, and/or areas in which military operations are anticipated and maps are quickly and urgently needed. Other applications in which this system would be adaptable are the study and measurement of clouds and ocean waves.

*The Optimization of Photographic Systems**

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ABSTRACT: *A quantity which is useful for optimizing photographic systems is the signal-to-noise ratio. This ratio is expressed as a function of the photographic parameters by means of Selwyn's equation Several examples of optimizing the signal-to-noise ratio are given. These result in the derivation of several well-known concepts in photography. In the case of exposure this procedure leads to the maximum resolving power criterion. The significance of resolving power is clarified. The magnification of the final image is an integral part of the system. A simple derivation of Selwyn's equation is given.*

INTRODUCTION

PHOTOGRAPHY is gradually changing from an art into a science. The traditional approach to photographic problems has been by trial and error. For example, if obtaining an optimum photographic exposure were particularly important, one photograph would be taken at the estimated proper exposure, a second taken with less exposure and a third taken with more exposure. One of these pictures probably would have an exposure close to the optimum.

Although the trial-and-error solution is still useful for many photographic problems, it is unsuitable for the general problem of the design of photographic systems. For example, some photographic problems involve the design of the vehicle which carries the camera. The vehicle and camera as a whole must be designed for optimum performance. Too many variables are involved to use trial-and-error methods. Moreover, because of the considerable expense involved, it is important that the best possible photographic per-

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formance be achieved at every exposure.

The systematic approach would be to select a figure of merit which represents the performance of the photographic system, and then calculate the design which optimizes this figure of merit. However, the difficulty with this method has been the problem of selecting the suitable figure of merit and expressing it mathematically.

This paper discusses the use of the signal-to-noise ratio as a figure of merit suitable for optimization calculations. The signal is described by the density differences on the film impressed by the incident signal and the noise is related to the granularity of the emulsion. Selwyn's equation gives the signal-to-noise ratio as a function of the usual photographic variables, thus enabling the signal-to-noise ratio to be optimized.

A common figure of merit is the system resolving-power or resolution. The resolution may be calculated from the signal-to-noise ratio using Selwyn's equation. It will be shown that optimizing the signal-to-noise ratio is identical with optimizing the resolution, that is, they are related concepts.

The emphasis here is on aerial photographic systems, and optimization is with respect to the rendition of fine detail of these systems.

1. The Derivation of Selwyn's Equation for Resolving Power

An equation which connects the signal-to-noise ratio with the usual photographic parameters is Selwyn's equation.[1–4]

$$\gamma C \tau(R) = \kappa G R \tag{1}$$

Here γ is the slope of the characteristic curve of the film, C is the contrast of the object, $\tau(R)$ is the modulation transfer function as a function of the spatial frequency, R is the resolving power in lines-per-millimeter, G is the Selwyn granularity[5] and κ is the minimum signal-to-noise ratio necessary for resolution.

This equation has received little recognition, in spite of the renown of its originator, primarly because it has rarely been mentioned in the American literature.

Selwyn justified his equation by elaborate calculations. However, a derivation is given in this paper that is simpler and more intuitive. This equation can be introduced by three basic assumptions:

1. There is a certain minimum signal-to-noise ratio κ which is necessary for visual resolution.
2. The signal is the density difference ΔD on the film due to an incremental log-exposure $\Delta E/E$.
3. The noise is proportional to the quantity σ, which is the standard deviation of density when the film is scanned by an aperture of area A. The noise is then the probable deviation in the density of the area A from the correct value.

The equation for the straight line portion of the characteristic curve of a film is

$$D = \gamma \log E + \text{const.} \tag{2}$$

D is the density of the film and E the exposure. This equation also applies to any portion of the toe which is small enough to be considered straight, if γ is taken as the slope.

The differential of D from equation (2) provides an expression for an output from a simple signal input $\Delta E/E$.

$$\Delta D = (\log e)\gamma \frac{\Delta E}{E} \tag{3}$$

The (log e) occurs since the conversion to natural logarithms must be made before the differentiation is performed.

The exposure of the film is proportional to the luminance B of the object for large areas. For small areas the system transfer function $\tau(R)$ attenuates the signal. R is the spatial frequency, for example, the number of lines-per-millimeter of the image of a resolution chart. The $\tau(R)$ is strictly the "sine wave" response, i.e., the response of the system to sine-wave inputs of varying spatial frequency. Much of what follows would also apply to the "square wave" response but to a lower accuracy, the approximation being made that in the case of a square wave only the fundamental frequency component is significant at the resolution limit. This is often a suitable approximation for practical systems. Then,

$$\frac{\Delta E}{E} = \frac{\Delta B}{B}\tau(R) = C\tau(R) \tag{4}$$

C is the object contrast. For a resolution chart with average luminance B_a and maximum luminance $B_{\max}$

$$C = \frac{B_{\max} - B_a}{B_a} \tag{5}$$

Alternatively, C may be regarded as the contrast between the object and the background. It is merely necessary to be consistent.

Substitution of equation (4) into (3) gives

$$\Delta D = (\log e)\gamma C \tau(R) \tag{6}$$

Since $\tau(R)$ is usually small at the resolution limit, so is $\Delta E/E$. Thus the differential expression is usually valid, even when γ is a slowly varying function in the density-exposure relation (eq. 2). Moreover the frequency of occurrence of low-contrast detail in an object scene is much greater than for high-contrast detail,[6] hence equation (6) is of general interest.

Using ΔD as the signal, σ as the noise and κ as the signal-to-noise ratio,

$$\frac{(\log e)\gamma C\tau(R)}{\sigma} = \kappa \tag{7}$$

Absorbing (log e) into κ, and substituting the Selwyn granularity:

$$\gamma C\tau(R) = \kappa \frac{G}{\sqrt{A}} \tag{8}$$

Selwyn,[5] and more recently, Higgins and Stultz,[7] have shown that G is independent of the area for scanning aperture dimensions considerably larger than the grain size and constant for a particular film, development and exposure. The choice of the area A is determined by the application. It is a characteristic of the geometry of observation. When A has been defined, then κ is fixed and may be determined empirically for a particular pattern. In the case of a bar chart with constant length-to-width ratio and continually decreasing line-width, A may be taken as the area of a black line plus a white line. In the case of a three-bar resolution pattern the area of the pattern may be used.

An interesting form of equation (8) is obtained by clearing of fractions.

$$\gamma C\tau(R)A = \kappa G\sqrt{A} \tag{9}$$

Expressed in this form the coefficient of A on the left side is the signal density on the film. The signal is proportional to the area, that is, to the number of developed grains. On the right side the noise increases approximately as the square root of the number of developed grains. This is exactly what one would expect where noise results from the random fluctuations of the number of grains. The eye determines the average density. Mathematically the determination of an average is essentially a summation. Thus the performance of the eye is equivalent to counting the grains. Actually the grains vary in size but the probable size distribution is the same over a uniformly exposed region. Taking into account the size distribution would not essentially change these conclusions.

The spatial frequency R of standard resolution targets with constant length-to-width ratios is inversely proportional to the square root of the area. Throwing the proportionality constant into κ,

$$R = \frac{1}{\sqrt{A}} \tag{10}$$

Equation (10) might be substituted directly into equation (7) to give Selwyn's equation (1). However, it is of interest to consider first the effects of the properties of the eye. A suggested modification of equation (8) to take account of the eye is:

$$\gamma C\tau(R)\gamma_e\tau_e(R/M) = \kappa\sqrt{\left(\frac{G}{\sqrt{A}}\gamma_e\right)^2 + \left(\frac{N_eR}{M}\right)^2} \tag{11}$$

The γ_e is an amplification factor of the eye which corresponds to the role of γ for the film. Its effect upon the signal and the pseudo-signal of the film noise are the same. $\tau_e(R/M)$ is the transfer function of the eye. M is the magnification of the optics with which the film is viewed. R/M is the frequency on the retina of the eye except for a proportionality constant. The product of $\tau(R)$ and $\tau_e(R/M)$ is the system transfer function for which the eye is part of the system.

The eye looks at the granularity with an aperture of effective area A. N_eR/M is the noise of the eye. N_e is assumed to be analogous to the Selwyn granularity. R/M is inversely proportional to the square root of the area of the pattern on the eye and therefore is analogous to $1/\sqrt{A}$ for the film noise. It is assumed that both sources of noise are independent and can be added as the square root of the sum of the squares. This is the method by which root-mean-square deviations are added.

In order to optimize the observed resolution it is necessary to use the optimum magnification. At this magnification $\tau_e(R/M)$ is large and near its maximum value. Selwyn has shown that the optimum magnification is 0.8 times the number of lines per millimeter being observed.

$$M = 0.8R \tag{12}$$

This means that for a resolution target with geometrically similar patterns of different size the geometry of viewing the pattern is independent of the resolution. The effective solid angle viewed from the eye and subtending the film area A is always the same. Thus, the effect of the eye is held constant in spite of the pattern size being observed.

Combining equations (10), (11) and (12) gives

$$\gamma C_T(R) = \frac{\kappa}{\gamma_e \tau_e} \sqrt{(GR\gamma_e)^2 + (1.25N_e)^2} \quad (13)$$

If G is sufficiently large then the noise of the eye may be neglected. τ_e, which is now a constant, may then be absorbed into κ, giving Selwyn's equation (1). If κ is the minimum signal-to-noise ratio necessary for resolution, then the spatial frequency R is the resolving power.

Selwyn[3] has emphasized that resolution is a statistical quantity, because the noise is a statistical quantity. The actual deviation of the density due to the granularity may be either greater or less than σ. Thus the observed resolution fluctuates from one measurement to the next. For an accurate determination of the resolution of a system a number of measurements must be averaged.

Selwyn estimated that κ in equation (7) is 0.003 when G is measured in microns and R in lines per mm. If G is measured in millimeters, then κ becomes 3. If the (log e) is absorbed into κ, it becomes

$$\kappa = \frac{3}{\log e} \simeq 7 \quad (14)$$

Preliminary calculations at this laboratory have indicated values of κ lower than 7 for bar charts. Whether this discrepancy is caused by the 7 being based upon a ΔD of maximum density minus the minimum density (rather than the maximum minus the average), the use of different resolution targets or differences in data is not clear. Certainly no accurate determination of κ has been described in the literature. This determination of κ is complicated by its very sensitive dependence upon the measurement of R, its variation with the resolution pattern used, and the precision required in making the measurements of all the quantities in Selwyn's equation. (However, inversely, R can be calculated accurately even if κ is known only approximately.)

Selwyn's equation is illustrated by the straight line in Figure 1. The slope is the signal-to-noise ratio.

2. Discussion of Selwyn's Equation

Selwyn's equation combines the effects of the transfer function and the granularity into one equation. Unfortunately, the literature commonly uses the transfer function only and neglects the granularity. In other cases the emphasis is on graininess, that is, whether the noise can be detected rather than whether the signal can be detected in the presence of noise.

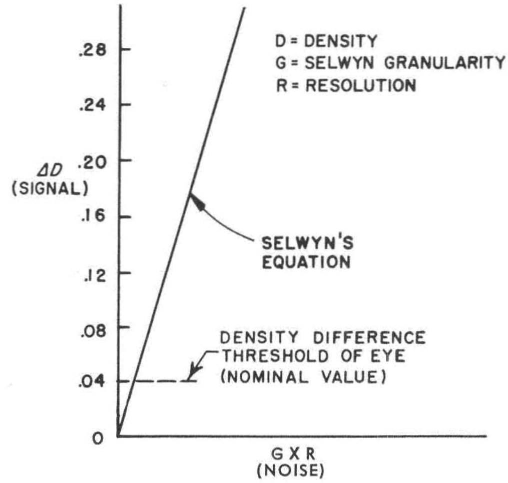

FIG. 1. Minimum signal-to-noise ratio for resolution.

Equation (10) shows that resolution as measured with the eye is a two-dimensional quantity. In contrast to this, a microdensitometer, scanning a resolution chart consisting of long sinusoidal lines, measures the transfer function in its one-dimensional form.

Selwyn's equation shows the effect upon the resolution of varying the object contrast. It shows the importance of obtaining information about the optical properties of the objects before trying to optimize the system. There is not just one resolution but a whole spectrum of resolutions in an aerial photograph, depending upon the particular object contrast.

Carmen and Carruthers[6] have shown that the 1,000:1 maximum-to-minimum brightness ratio for which optical systems are frequently tested does not exist in aerial scenes. A much better method would be to test the resolution at contrasts which are typical of the object contrasts for which the optical system will be used, or a graph of the resolution versus the target contrast should be obtained. A typical type of error is to focus for maximum resolution for 1,000:1 maximum-to-minimum brightness ratio ($C=1$) rather than a value which is characteristic of the intended target. (For a further discussion of this point, see the work of Bousky.[8]) Much of the criticism of resolution as a concept comes from the overworked use of high-contrast resolution.

Selwyn's equation shows directly that, if the gamma is proportional to the granularity, the resolution is constant. This is the reason for the fact, experimentally observed by Perrin and Altman,[9] that the resolution is approximately independent of the development time even though the gamma changes.

A question may be asked, "What is necessary in order to double the resolution of a particular photographic system?" Selwyn's equation shows that one method of doing it is to obtain the same amplitude of the system-transfer-function at twice the frequency and also to use a film with half the granularity. If the same film is to be used then the system-transfer-function must have twice the amplitude at twice the frequency.

For very fine grain film (low G in equations (11) and (13)) the density difference threshold of the eye is the limiting factor in determining the resolution rather than the signal-to-noise ratio of the film. This is shown by the dotted line in Figure 1. In this case the transfer function and γ determine the resolution. If G is neglected then equation (13) reduces to the requirement that a minimum signal is necessary for resolution. The threshold is usually given as 0.02 to 0.04 density-difference. However, for the films usually used in aerial photography, the signal-to-noise ratio, not the density-difference-threshold is the limiting factor. Evidence that the density-difference-threshold of the eye is not the limiting factor is that as the gamma is made greater by increasing the development, the density difference is increased, yet the resolution is constant.

Selwyn's equation is useful even in the case of extremely fine grain film because enhancement methods may be used to improve the system transfer function and then the signal-to-noise ratio sets an upper limit to the resolving power. This upper limit is the resolution in Selwyn's equation.

Selwyn had two objections to his equation. The first is that the wrong optimum magnification was calculated during the course of his derivation. However, the derivation given here of Selwyn's equation was made without calculating the optimum magnification, but the optimum magnification must be determined empirically. Another objection was that some of his data did not agree with his equation. However, Powell's more recent data[4] with improved techniques support Selwyn's equation.

3. Derivation of the Empirical Relationship Between Luminance Ratio and Resolution

Selwyn's equation is useful in deriving many of the properties of photographic systems. It is useful in combining apparently unrelated concepts into a single theory.

For example, an empirical equation obtained by Perrin and Altman[10] for the variation of resolution with the target luminance ratio is

$$R = R_m\left(1 - \frac{1}{\rho}\right) \tag{15}$$

ρ is the luminance ratio of the target, R is the resolution and R_m is the resolution for a luminance ratio of infinity, i.e., the high contrast resolution.

This equation can be shown to be a special case of Selwyn's equation by substituting the special unnormalized transfer function

$$\tau(R) = \frac{\kappa G R_m}{\gamma}\left(1 - \frac{R}{R_m}\right) \tag{16}$$

Also,

$$C = \rho - 1 \tag{17}$$

Examining the question of an appropriate transfer function further, it may be noted that equation (15) is merely an approximation to Perrin and Altman's data. The experimental graphs consisted of curved lines. As a test of the theory, Selwyn's equation was used to calculate the transfer-function from the curves. The results were transfer-functions more realistic in appearance than the straight line relation, equation (16). In particular, the transfer-functions so calculated were higher at low frequencies and lower at higher frequencies than the straight line of equation (16). Therefore, it appears that the variations from equation (15) in Perrin and Altman's data might well be accounted for by the use of Selwyn's equation with the actual system transfer functions.

It is interesting to note that equation (15) and the experimental curves cross near the point ($1/\rho = \frac{1}{2}$, $R = R_m/2$). This is equivalent to the rule-of-the-thumb that the low-contrast resolution is approximately half the high contrast resolution.

4. Maximum Resolving Power Criterion for Optimum Exposure

Selwyn's equation is useful for optimizing photographic systems. The transfer functions of the components of the system are combined to find the transfer function $\tau(R)$ of the system. (Note: only the sine wave response, not the square wave response, of the system can be found by simple multiplication.) The transfer function of the image motion must be included. Then Selwyn's equation may be used to maximize the signal-to-noise ratio.

As an example, the important case of determining the optimum exposure will be

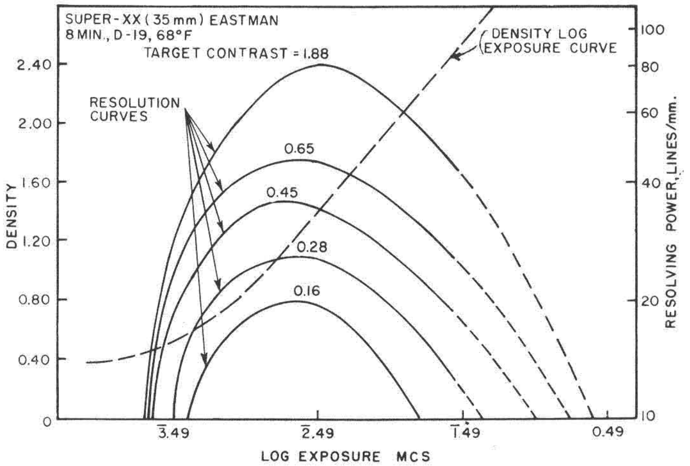

FIG. 2. Resolution as a function of exposure for various target contrasts according to Kardas.[12] (Reprinted by courtesy of *Photographic Science and Engineering.*) MCS is an abbreviation of meter-candle-seconds, which is a unit for luminous exposure.

considered. The particular case of no motion will be considered first.

As the exposure is varied only the slope γ of the operating point (for a particular object) on the characteristic curve and G are varied. According to Lamberts[11] the transfer function is independent of exposure. Applying Selwyn's equation, the optimum exposure is the one which gives a maximum signal-to-noise ratio, a maximum ratio γ/G or maximum resolving power R.

The curves of Kardas[12] (an example is given in Figure 2), show that the resolution rises rapidly as the exposure point approaches the region where γ becomes a maximum, reaches a peak, and then drops down again at only moderate densities. This can be explained on the basis of Selwyn's equation. As the exposure falls off from the peak resolution, the slope γ approaches zero while the granularity approaches a constant determined by the fog level. Also, as the exposure increases from the peak, the slope remains constant up to a fairly high density but the granularity increases. (The granularity increases as the square root of the density or a little slower. This follows from the Beer-Lambert law which states that the density is proportional to the number of grains and their projected opaque area, and the law of probability which states that the standard deviation in counting a random sample is proportional to the square root of the number sampled. The granularity increases more slowly than the square root of the density because the smaller grains are slower so that the average grain size decreases as the density increases.) This explains why the resolution starts to fall off long before the shoulder of the characteristic curve is reached. Selwyn's equation predicts that the exposure for maximum resolution should be near the top of the toe.

This method of determining the exposure yields the maximum resolving power criterion. It appears to be one of the best single-number criteria where maximum detail rendition is necessary, as in aerial photography.

The maximum resolving power criterion for exposure was originally suggested by Howlett[13] in 1946, based upon experimental data. In 1955, Kardas[12] showed experimentally that the optimum exposure using the maximum resolving power criterion is approximately independent of the contrast. This conclusion may also be obtained from Selwyn's equation where the optimization of γ/G is independent of C. It was not until 1961 that Levi[14] tested this criterion against other criteria in flight tests and found it to be the best.

The application of the maximum resolving power exposure criterion is not new in a general sense. For example, the military standard[15] for determining the resolving power of an optical system requires that the exposure which gives the maximum resolution be selected.

The usual (but not the only) procedure in applying the maximum resolving power criterion is to place the average exposure at the peak of the curve. Because the exposure meter indicates the average exposure, this explains the successful use of exposure meters in aerial photography.

The maximum resolving power criterion

leads immediately to a fixed-density-criterion because for a fixed development the point of maximum resolution occurs at a fixed density. For a long time astronomers and people working with microfilm have used fixed-density-criteria.

There is some disagreement in the literature as to exactly what the value of the density for maximum resolution is, but most authorities agree it is about 0.8 above fog. Presumably some of the disagreement occurs because different procedures are used. Some authorities may use the density-above-base while others use the density-above-fog. Some may use dark targets on a light background and others the opposite. Some use the density corresponding to the background while others use the density which corresponds to the average of the background and object. Moreover, the optimum density varies according to the film and development.

It is important to keep the objective in mind when applying this criterion. For example, if the objective is to photograph a particular type of target, the exposure for that type of target, not the average exposure for the terrain, should be placed at the peak of the curve.

An example of the characteristics of the objects affecting the exposure is given by Harris.[16] In case there is less than 25% wooded area and more than 75% sand, he exposes one-half stop less than the exposure meter indicates. Using this procedure, satisfactory detail can be seen in both the wooded and sandy areas and not merely in the sandy areas.

This approach also enables the error in exposure to be separated from the performance of a photographic system when testing it over resolution targets and terrain. The resolution-versus-density curve can be obtained with the camera and collimator on the ground. The resolution and density obtained in flight can be measured. The difference in exposure between the peak of the curve and the average exposure in the flight test is the exposure error. The corresponding drop in resolution is the loss in resolution caused by the exposure error. Many resolution targets located on the ground have a much higher average reflectance than the terrain. The resolution which is measured is not that at which the camera is being used for the terrain, but the resolution at a much greater exposure. One wishes to avoid this condition.

A logical question is, "Why are there several different criteria for the optimum exposure?" Kardas' curves show a peak in the resolution-versus-exposure curve and indicate that there is little exposure latitude. Yet the 0.3 average gradient and other criteria imply that a considerable exposure latitude exists: that no detail is lost with underexposure until a certain point is reached and that a moderate overexposure causes no loss in image quality.

Hariharan[17] found that the 0.3 average gradient and the maximum-resolution-criteria give similar exposures where the log exposure range (logarithm of the ratio of maximum to minimum exposure in the scene) was 1.50. Thus, for ground photography there is little difference in the two methods. However, the exposure range in aerial photography is considerably less. Placing the shadows at the speed point obtained from the 0.3 average gradient method would result in underexposure.

The ASA Standard is based upon the 0.3 average gradient, although less directly with the new Standard.[18] (For the meaning of the ASA Standard see the analysis by Nelson.[19]) However, when an exposure meter is calibrated to the ASA Standard it is implicit that a fixed ratio, based upon statistical data, exists between the average and the minimum luminance. Again, for ground photography, there is little difference between the new ASA Standard and the maximum resolution criterion. In the case of aerial photography the exposure meter (by exposing for average luminance) follows the maximum-resolution-criterion, gives the correct exposure, but departs markedly from the 0.3 average gradient criterion.

Nelson[19] stated that when contact prints were to be made from negatives the exposure latitude of the negative was 32:1 as an average. However, when the negative was to be enlarged by a factor of 10, the exposure latitude was only 4:1. A look at Figure 3 shows

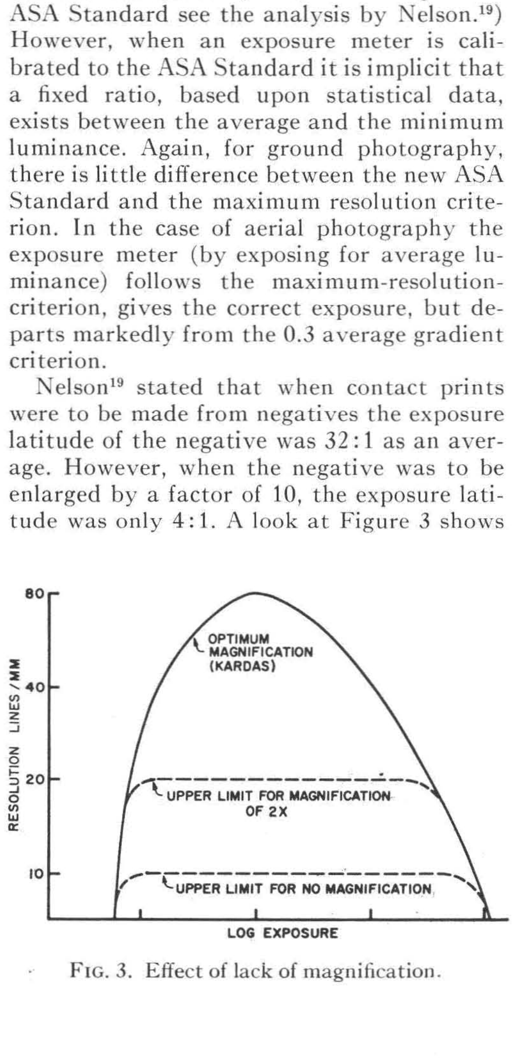

FIG. 3. Effect of lack of magnification.

that if no magnification were used there would be a large plateau over which the eye, limited to about 10 lines/mm., could detect no variation of resolution with exposure. On the other hand, Kardas used the optimum magnification, probably nearly identical to Selwyn's magnification of 0.8 times the number of lines/mm. being observed. The conclusion is that these other exposures criteria were arrived at with less than the optimum magnification. It will be recalled that the 0.3 gradient method was based upon contact prints for pictorial use where the reproduction of tones, rather than the resolution, was important.

Howlett[13] stated that the speed of an aerial film should be defined as inversely proportional to the exposure giving either the maximum-resolving-power or a specified fraction of that resolving power. Based upon the work of Howlett, Kardas and Levi, it is suggested here that this criterion of film speed be used rather than the ASA rating.

The above discussion has neglected the image motion. The maximum-resolving-power-criterion will now be generalized to include the effects of motion. Certainly the exposure must be kept at a minimum to avoid motion.

Howlett observed that a considerable reduction in exposure could be obtained with only a little loss in maximum resolution because the maximum of resolution-versus-exposure curve was somewhat flat rather than peaked. He suggested that the average exposure be placed at the point where the resolution fell to 90% of maximum resolution because of underexposure. Levi[14] tested this and found that a reduction in exposure by a log difference of 0.45 could be obtained by using the 90% point, and that there was no loss of image quality. When the exposure was reduced below the 90% point the image quality fell off rapidly.

Selwyn's equation suggests a more general method of optimizing the exposure with motion present. The transfer function depends upon the amount of motion which in turn depends upon the exposure time. The slope depends upon the exposure which in turn depends upon the time. Also, the Selwyn granularity depends upon the exposure. Therefore it is suggested that the optimum exposure (or exposure time) be found by maximizing the resolution using Selwyn's equation. This procedure may be too complicated for field work, but should be useful in design work.

The optimization with respect to the film may be made similarly.

5. Development

In determining the optimum gamma or development time let us first consider how one might do it classically, and then how it may be done using Selwyn's equation. Although an exact classical procedure has not been worked out, the outline of the procedure appears clear.

The most important effect of development is to fix the latitude or the exposure-range which reproduces detail satisfactorily. The curves of Howlett and Kardas show that the resolution is a maximum for the optimum exposure and falls off as the exposure varies from the optimum on either side. The latitude may be defined as the ratio of the greater to the lesser exposure between the two points where the resolution falls to a certain fraction of the maximum resolution, for example, 50%. This definition makes the exposure range a definite mathematical quantity. The difficulty is that it is not known what fraction would be the most useful.

One possible objection to this method is that the exposure-range depends upon the object-contrast because this contrast affects the position of these points. However, the data of Kardas indicate that the exposure-range, using the 50% resolving power definition, changes quite slowly with object-contrast. Therefore it is not necessary to know the contrast exactly. Carmen and Carruthers[6] have suggested a contrast corresponding to a log-luminance-ratio of 0.1 or 0.2 as a suitable contrast for aerial photography when a definite contrast must be specified.

In addition to the exposure-range of the film, the whole camera system has an exposure-range which may be similarly defined. It is the exposure-range of the camera system in which we are interested.

The exposure-range which is required is determined by the luminance range or brightness scale of the terrain (often called scene contrast), or objects as the case may be. Optimization of the camera system requires that the exposure-range of the camera system match the effective luminance range of the terrain.

The effective luminance range of the terrain is more difficult to specify. It is first necessary to graph the luminance of arbitrarily small areas of the scene versus the frequency of occurrence, as done by Carmen and Carruthers. Probably the best method is to select two points on an area (under the curve) basis, one point representing, for example, 20% of the total integral, the second point 80% of the total integral and then taking the ratio of the luminance at the 80% point to the luminance at the 20% point as the exposure-range.

After determining the exposure-range and effective luminance range it is then necessary to match them. If they have been properly specified then it would be expected that the exposure-range required would be equal to the effective luminance range, or possibly a constant times the effective luminance range. The success of Harris[16] is partly due to his having made such a match for his equipment and type of photography. However, no one has expressed this match mathematically so that it can be made the first time with new equipment and new situations.

When optimizing both development and exposure one procedure is to optimize the development first upon the basis of object contrast and afterward to optimize the exposure. In this connection it should be mentioned that Howlett[13] and Hariharan[17] have found that when the speed of a film is based upon the maximum resolving power, the speed is proportional to the gamma.

One can go one step farther than the above procedure by using Selwyn's equation to optimize the exposure and development simultaneously. The product of the resolution and frequency of occurrence of exposure integrated over the exposure is maximized.

$$I = \int R(E)f(E)dE \qquad (18)$$

$$\frac{d^2I}{dEd\gamma_E} = 0 \qquad (19)$$

The $f(E)$ is the frequency of occurrence of exposure as a function of the exposure. It may be obtained from the frequency of luminance versus luminance curve, for example, the data of Carmen and Carruthers, and then converted to frequency of occurrence of exposure by introducing the f/-number, shutter time and other parameters. The $R(E)$ is the resolution obtained by using Selwyn's equation, the transfer function being regarded as a function of the exposure through the shutter time and image motion and the granularity being a function of the exposure. The contrast may be arbitrarily taken as the contrast corresponding to a log brightness ratio of 0.1 or 0.2 as before. The γ_E is the slope of the straight line portion of the characteristic curve. Maximizing with respect to γ_E is equivalent to maximizing with respect to the development time.

It is assumed that the amount of detail at any one exposure is proportional to the area on the film which receives that exposure.

Conclusions

Maximizing the signal-to-noise ratio by means of Selwyn's equation is a powerful tool for the optimization of photographic systems. In addition to its applicability to specific systems, this procedure leads to the following general conclusions:

1. Optimizing the resolution of a system in which the emulsion is the limiting factor on resolution is equivalent to optimizing the signal-to-noise ratio.
2. A photographic system has not merely one resolution but a spectrum of resolutions depending upon the object contrast.
3. When a system is optimized, the frequency of occurrence of different contrasts and the brightness scale of the object scene must be known.
4. The optimum exposure for detail rendition is given by the maximum resolving power criterion.
5. The final magnification of the image is an integral part of the system.

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*The Wild A-7 Autograph as a Comparator**

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INTRODUCTION

PHOTOGRAMMETRY is commonly defined as, "The science or art of obtaining reliable measurements by means of photography."[1] To obtain the required "measurements" it is necessary to make certain other measurements on the photograph itself. These photo-measurements may be recorded as actual distances on the photograph, or may be converted by some instrument directly to the required results, recorded as a map or drawing. This is the system employed by the many projection type plotters in use today. Analytic photogrammetric methods require actual measurements on the photograph itself, as part of their basic data. It follows then, that the accuracy of the results depends in a large part on the accuracy of the photo-measurements.

In aerial photogrammetry these measurements are usually obtained as two-dimensional rectangular co-ordinates. The origin of the system is commonly the principal-point of the photograph as defined by the fiducial marks. The axes of the system are then the fiducial-axes with positive x being that axis which most nearly coincides with the line and direction of flight.

Many instruments have been adapted or developed for the specific purpose of measuring photoco-ordinates. These range from the simple engineers scale, with an accuracy of about 0.01″ to 0.005″, to the highly-refined monocular and binocular comparators, most of which have accuracies approaching 0.001 mm. or 1 micron.

This paper deals with the adaptation of yet another instrument—the Wild A-7 Autograph—to the function of measuring photo co-ordinates. The investigation was undertaken in order to determine if an organization—Cornell University in particular—which possesses an A-7, can make use of the instrument to perform the measurement task with accuracy sufficient for analytical methods of photogrammetric solution. A discussion of the instrument's construction and possible methods of use is included in this paper as well as the presentation of the test data which was gathered.

[1] American Society of Photogrammetry, MANUAL OF PHOTOGRAMMETRY, *2nd edition* p. 830.

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