Kelsh Plotter Notes

G. C. TEWINKEL,
*Coast & Geodetic Survey**

ABSTRACT: *A few tests relative to the operation of the Kelsh Plotter are described in an effort to assist users in improving instrument performance. These ideas may also be applicable to some other types of instruments. The tests are not new ones and are generally easy to apply, requiring only the usual knowledge and capabilities of the interested operator. The application of these tests normally has the effect of improving accuracy—increasing the effective C-factor** of the system. In fact, one-foot and two-foot contouring, requiring large C-factors, should not be attempted without first performing some or all of these tests.*

G. C. TEWINKEL

INTRODUCTION

WITH few if any exceptions each plotting instrument delivered from the factory can be improved in performance by applying a short series of tests and studies. This is quite understandable if one considers the many sources of errors, and the difficulties in overcoming them, during and after manufacture. The makers are ordinarily free from blame because they have essentially exhausted their mechanical and optical possibilities in attempting to deliver a faultless instrument. Although they have approached the limits of mechanical perfection, unresolved discrepancies still persist. For example, most so-called distortion-free lenses used in aerial cameras contain residual distortion of sufficient amount to warp a stereoscopic model. The two projector lenses of a plotter are not without distortion, not exactly identical to the distortion of the lens in the aerial camera, or not even exactly the same. Likewise, correction devices are not precisely akin to the aerial camera lens, not exactly alike, nor take into account the difference in the projector lenses. Table tops are never perfectly flat; the shape of the unflatness may augment other discrepancies. Earth curvature and atmospheric refraction are not negligible, and their effects differ with the flight altitude.

A few years ago when the Kelsh plotter was credited with a *C*-factor of 900 or 1,000 the discrepancies might possibly have been neglected without serious damage to the results. But today some owners are attempting to operate at *C*-factors of 1,200, 1,500 and even 1,700; these exceed basic capabilities of the instruments unless special efforts, such as those mentioned herein are applied.

Model flatness may be only one of several problems of the user:

1. Projection-distance
2. Zone of sharp focus for each projector
3. Pantograph errors
4. Model flatness
   4.1 As determined with test grids
   As determined with ground control.

PROJECTION-DISTANCE

Klesh plotters are ordinarily delivered with a gage for obtaining the projection-distance setting of the projectors. Ordinarily these gages are correct and applicable, but one can and should make certain of this by applying

* Mr. Tewinkel is employed in the Office of Research and Development, U. S. Coast & Geodetic Survey, Washington 25, D. C.

** *C*-factor may be defined as the number obtained by dividing the flight altitude by the contour interval where the contouring meets U. S. map accuracy standards and is considered to be the limiting ability of the photogrammetric procedure.

the "approximate determination" routine described in "Kelsh Plotter Procedure," Topographic Instructions, U. S. Geological Survey, Chapter 3C8, 1960. The description is not repeated in this paper inasmuch as the publication is already in use by many operators, and is readily obtainable. Actually the "approximate" determination is in fact more precise than the name implies, while the gage method is subject to several sources of gross human blunders. A grid plate is required for this test—the manufacturer can ordinarily furnish the grid. Presumably, in place of a grid plate one might use a pair of scratches on a diapositive where the distance between them is measured to the nearest 0.001 inch and the scratches are symmetrically located relative to the center of the diapositive.

Zone of Sharp Focus

One assumes that the sharp-focus projection distances for a pair of projectors are equal. However there have been reported several instances of a significant amount of inequality. The red and green filters sometimes affect the sharp-focus setting and the manufacturer attempts to ensure this agreement. But an operator may occasionally interchange filters without realizing its possible effect since normally such effect is not detectable. In a few cases this indiscriminant interchange has seriously affected image sharpness, causing the zone of sharp focus to be different for the two projectors.

The test is very simple. Without wearing the filter spectacles, with only one projector illuminated, and with the space rods disconnected from the tracing table, a sheet of white paper can be raised and lowered until the sharpest image of a diapositive is obtained. The distance from the table top to the paper is then measured to the nearest inch with a ruler. One should obtain the same distance with the other projector; possibly a difference of one inch in the two determinations is not significant. A difference of two inches or more is a cause for concern. By interchanging the projector filters one may find the difficulty lessened, unaffected or worsened. If the difficulty is large and persists, the manufacturer is usually willing to make any necessary exchanges.

The author has found most projectors to be satisfactory and unaffected by the change in filters, but in one instance where a difference of four inches occurred for the zones of sharp focus, the difficulty was completely corrected by the filter interchange.

Pantograph Errors

Happily, recent models of pantographs have exhibited insignificant discrepancies. However some of the older ones, which were more or less improvised, had considerable error. The largest error is ordinarily encountered in the extreme corner of the model opposite from the anchor point where the pantograph is in its most unfavorable working position—where the arms form a long, narrow parallelogram having minimum geometric "strength of figure." In this orientation a small angular misalignment of any of the six vertical bearings can cause a greatly magnified error in the position of the tracing point.

"Pantograph Adjustment"* presents a method for studying, analyzing and adjusting a pantograph based on a series of measurements and computations. Not only does the result yield corrections to be added to the three vernier settings, but it also indicates the limitation of accuracy which may be expected after such corrections are applied. The most complicated part of the system is the solution of nine observation equations in three unknowns by least squares using a desk calculator. Detailed instructions for the measurements and solution are included in the bulletin No. 7; also the author has prepared a supplement giving a step-by-step solution of the equations.

Model Flatness

The product of the model flatness test consists of plot of isograms (Figure 1 herein and Figure 12 in Kelsh Plotter Procedure cited above) depicting the number of units to increase or decrease the elevation counter in different areas of the model for contouring, with the four corner-points regarded as being correct. The use of the graph consists of drawing it on the table top of the plotter in bold lines one-fourth inch wide so that the operator will be aware when the tracing stand crosses a line, whereupon he can change his elevation counter accordingly. If the contour map is being compiled on transparent plastic or by means of a pantograph, the application of the graph presents no particular problem. But if opaque material is used, it may be necessary to trace the graph on each map sheet using some convenient color, such as non-photographic blue.

The units of correction are immaterial except that they should correspond to the

* Tewinkel, G. C., Technical Bulletin No. 7, Coast and Geodetic Survey, 1959.

graduation on the elevation counter. For example, the units may be tenths of millimeters, thousandths of an inch, or feet at the model scale. A graph usually contains very few lines, such as those for corrections of zero, +0.05, +0.10 and +0.15 mm., with an occasional rare graph extending as far as ±0.30 mm.

(a) THE GROUND CONTROL METHOD

The ground-control method for compiling the correction graph for the vertical deformation of the model is perhaps the easier to explain and understand; accordingly it is first described. This method also has the theoretical advantage of simultaneously accounting for all the sources of error, with the least danger of committing gross mistakes in the analysis. However, this method may be more costly, inasmuch as it depends on the field establishment of the relative elevations of 23 to 50 points in a single model area with an accuracy that cannot be criticized. But one can select a model which offers the easiest ground operations.

Presumably the model area should be one of those from each work project to account for (1) the particular camera and magazine, (2) earth curvature and (3) atmospheric refraction. It is naturally desirable, but not necessary, that the points be signalized prior to aerial photography to assure no mistakes due to misidentification. The results, of course, apply to all photography taken under the same conditions. For work at different altitudes either (1) a test area can be established for each altitude or (2) curvature and refraction corrections can be applied to the data for one given altitude; but this may be risky as theoretical and operational mistakes are possible, and one begins to lose some of the basic advantages of this direct method.

Once diapositives have been prepared for a model in which appear the images of an appropriately large number of points whose elevations and identifications are faultless, one can use the same plates to test each plotter. It should be pointed out that the test applies to a given pair of projectors on a specific table top, and that the exchange or interchange of projectors invalidates the test. It is also assumed that the overlap is 60% and that any deviations are minor and insignificant.

Relative orientation is performed in the usual manner.

Absolute orientation amounts to (1) setting the projector separation for the ordinary overlap if horizontal-control points are not available to scale to and (2) levelling the

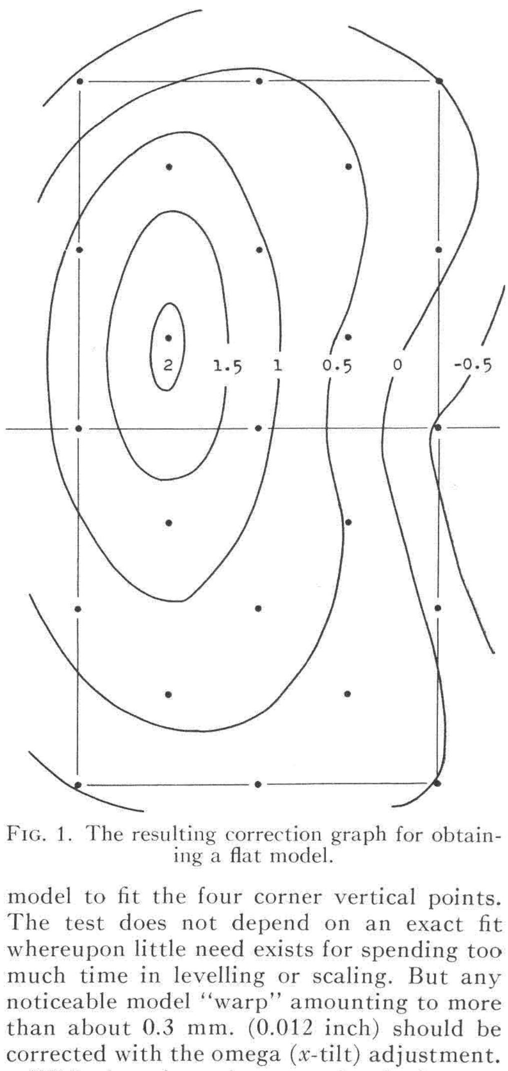


FIG. 1. The resulting correction graph for obtaining a flat model.

model to fit the four corner vertical points. The test does not depend on an exact fit whereupon little need exists for spending too much time in levelling or scaling. But any noticeable model "warp" amounting to more than about 0.3 mm. (0.012 inch) should be corrected with the omega (x-tilt) adjustment.

With the orientations completed, the next step is observing the elevations of the many test points. It is suggested that these three individuals make these observations:

1. The stereo operator who sets the dot on the image
2. An observer who reads aloud the numbers on the counter
3. A recorder.

Five readings (Figure 2) are desirable on each point. As a temporary graphic record for later use, the positions of the test points should also be plotted on a sheet of paper. See Figure 1. It is immaterial whether the five readings be taken on one point before moving to the next point, or taken in some other order. It is suggested that the recording be made on a form similar to the actual geometric distribution of the test points. The next step is to determine the average value of each list of five observations and to enter the averages on another similar form.

To yield numbers simpler to deal with, subtract a constant from all the averages such that the remainder is zero at the upper right corner. If decimal points are ignored momentarily, all discrepancies will then be one and two digit numbers ranging up to about 100 (Figure 3).

At this stage it will probably become apparent that the model is (1) not exactly level and (2) not entirely free from omega warp. But these effects can now be removed through linear interpolation such that the numbers represent only instrumental model deformations, and on the contrary do not reflect any arbitrary decision of the operator. This application of linear interpolation is the only operation that may seem to be weird, complicated or confusing, although it consists of only a reasonable adjustment of the numbers at the four corners, along with proportional adjustments at all the other intermediate points. The system may be conceived as simply tipping or rocking the model about some line in the model.

58.8 58.9 58.9 58.9 58.8 294.2 58.84		59.6 59.55 59.65 59.7 59.6 298.10 59.62		59.2 59.2 59.3 59.3 59.2 296.2 59.24
	60.25 60.15 60.1 60.3 60.2 301.00 60.20		59.8 59.8 59.8 59.8 59.7 298.9 59.78	
59.9 60.05 60.1 60.0 60.05 300.10 60.02		60.4 60.3 60.3 60.4 60.5 301.90 60.38		59.5 59.45 59.5 59.4 59.5 297.35 59.47
	61.4 61.3 61.3 61.3 61.3 306.6 61.32		59.6 59.8 59.75 59.8 59.7 298.65	
60.2 60.2 60.3 60.1 60.2 301.0 60.20		60.45 60.45 60.5 60.4 60.5 302.30 60.46		58.9 58.7 58.9 58.85 58.8 294.15 58.83
	60.4 60.6 60.6 60.55 60.5 302.65 60.53		59.7 59.8 59.9 59.9 59.8 299.1 59.82	
59.8 59.8 59.8 59.9 59.9 299.2 59.84		60.1 60.2 60.0 60.2 60.1 300.6 60.12		59.3 59.3 59.2 59.25 59.3 296.35 59.27
	59.9 59.8 60.1 60.0 59.9 299.7 59.94		59.7 59.7 59.85 59.7 59.8 298.75 59.75	
59.4 59.15 59.35 59.2 59.3 296.40 59.28		59.7 59.6 59.8 59.6 59.7 298.4 59.68		59.4 59.3 59.6 59.5 59.4 297.2 59.44

FIG. 2. The list of elevation readings at 23 places in the stereoscopic model and their averages.

-40		+38		0
	+96		+54	
+78		+114		+23
	+208		+49	
+96		+122		-41
	+129		+58	
+60		+88		+3
	+70		+51	
+4		+44		+20

FIG. 3. The remaining values after (1) subtracting the upper-right value from the average readings and (2) deleting the decimal point.

In performing this arithmetic adjustment one adds small positive or negative amounts to the rows and columns of numbers shown on the form (Figure 4). To obtain the values to be applied at the corners of the model, Equations 4, 5 and 6 of a previous paper* are applied for levelling and flattening the model:

$$x = \tfrac{1}{2}(a - b - c) = \tfrac{1}{2}(-40 \quad -4 \; - 20) = -32$$

$$y = \tfrac{1}{2}(c - b - a) = \tfrac{1}{2}(\;\; 20 + 40 - \;\; 4) = +28$$

$$\omega = \tfrac{1}{4}(b - a - c) = \tfrac{1}{4}(\;\; 4 + 40 - 20) = + \;\; 6$$

$$\omega 2 = + 12.$$

* G. C. Tewinkel, Levelling the Stereo Model, PHOTOGRAMMETRIC ENGINEERING, v. 26, n. 5, p. 810, December 1961.

	(+28)	(+21)	(+14)	(+7)	(0)
After averaging and subtracting X-correction Y-correction 2w-correction	-40 0 +28 +12 0		+38 0 +14 + 6 58		0 0 0 0 0
(-4)		+96 - 4 +21 + 8 121		+54 - 4 + 7 + 2 59	
(-8)	+78 - 8 +28 + 9 107		+114 - 8 + 14 + 3 123		+23 - 8 0 + 3 18
(-12)		+208 - 12 + 21 + 4 221		+49 -12 + 7 + 2 46	
(-16)	+96 -16 +28 + 6 114		+122 - 16 + 14 0 120		-41 -16 0 + 6 -51
(-20)		+129 - 20 + 21 + 2 132		+58 -20 + 7 + 4 49	
(-24)	+60 -24 +28 + 3 67		+88 -24 +14 + 3 81		+ 3 -24 0 + 9 -12
(-28)		+70 -28 +21 + 2 65		+51 -28 + 7 + 8 38	
(-32)	+ 4 -32 +28 0 0		+44 -32 +14 + 6 32		+20 -32 0 +12 0

FIG. 4. Application of corrections for x-tilt, y-tilt and omega-warp to the values shown in Figure 3.

The x-correction of -32 is then prorated among the rows of points as shown, and the y-correction is prorated among the columns.

For the omega correction, double the correction which was indicated by the formula is applied at the two opposite corners for ease in the arithmetic application, and zeros at the other two corners. A line of zero correction then passes diagonally through the zero corners, as shown in Figure 5. A maximum correction of $+12$ is noted, which establishes a scale for interpolation perpendicular to the zero axis. Then the corresponding correction for any other point can be determined with a pair of dividers, depending on how far the point is from the line of zero correction.

Finally at each location in Figure 4 is added the list of quantities consisting of (1) the initial observed error, (2) the correction for tilting the model about the x-axis, (3) about the y-axis, and (4) for flattening model warp. The sum at each point is the residual model error, free from any arbitrary decisions of the operator, and from which the correction graph (Figure 1) can be drawn. This concludes the study.

(b) THE GRID PLATE METHOD

A correction graph equivalent to that shown in Figure 1 can also be compiled by using a pair of diapositive grids in the plotter in place of actual aerial photographs. Inasmuch as these diapositives do not embody the distortion characteristics of the aerial camera, different values will be observed in the plotter and the list of corrections shown in Figure 4 will require a fifth term in each sum to take the distortion into account. The determina-

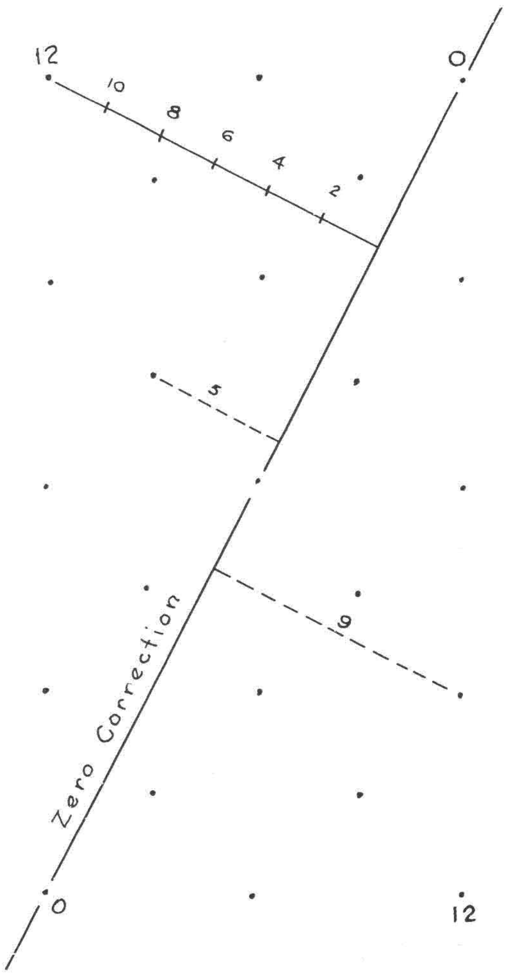


FIG. 5. Graphical scheme for determining the distribution of the omega-warp correction.

tion of this added term from lens distortion data comprises the principal difference in the two methods. The computation is simplified by the fact that one needs to consider only one-fourth of the model area because of the strict symmetry in the four quadrants.

The initial operations are identical to the previous method, including relative orientation, observing the elevations at 23 to 50 grid intersections, averaging, reducing, levelling and flattening. The base length on the grid between photo centers is considered to be 90 mm. and the model as 90 by 180 mm.

Lens data may consist of a record similar to that shown on page 36 of the 1952 edition of the MANUAL OF PHOTOGRAMMETRY. It is probably necessary to convert the angular notation to radial distances on the photograph using the formula

$$r = f \tan \alpha$$

where f is the calibrated focal length. Then it is convenient to compile a graph of the distortion values at the various radial distances as shown in Figure 6. Now the radial distortion at any test point on the glass grid can be read directly from the graph. It is to be noted that two distortions affect each test point—one arising from each of the two centers of the grid plates.

Only the distortion component that is parallel to the x-axis (air base) is associated with height error. That error is directly proprotional to the algebraic sum of the two x-components at each point. If the sum is plus, the effect is to cause the model image to appear *higher*. The x-component d_x of the distortion can be obtained using the formula

$$d_x = d \cos \phi$$

where ϕ is the angle the radial line makes with the x-axis and d is the radial lens distortion value from the graph. Also

$$\cos \phi = x/(x^2 + y^2)^{1/2}$$

where x and y are the coordinates of a test point relative to its associated photo center.

So now one can find the distortion component at each point from each of the two photo centers and secure their algebraic sum. If this sum is p, then the height error h is given by the formula

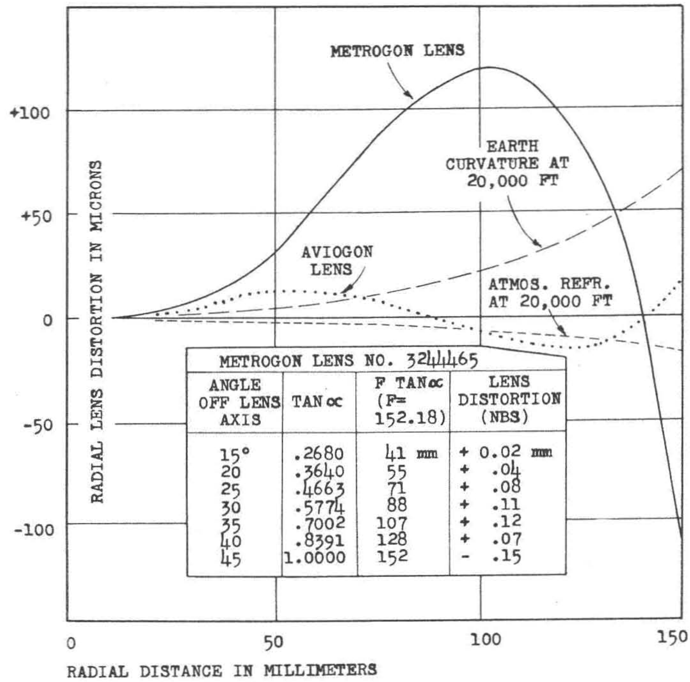
$$h = 5fp/b$$

where b is the air base (90 mm.) and the factor 5 is present because the Kelsh model is five times the size of the diapositive. Thus one can compute the height errors due to lens distortion at any number of points in the model area.

It is probably obvious that the errors at the four corners are equal and different from zero. To obtain zeros at the corners it is necessary to subtract the corner value from each of the values at all of the test points in the area. The result is then in such a form that the respective quantities can be applied to the elements of the table in Figure 4 as a fifth term in each list of corrections. These values of height errors also correspond to that shown graphically on page 19 of *Kelsh Plotter Procedures.* The drawing of the correction graph follows in the same manner as in the previous ground control method.

CURVATURE AND REFRACTION

A note or two about earth curvature and atmosphere refraction may be appropriate. As a numerical example, consider a flight altitude of 10,000 meters (32,808 feet) with the usual 90° aerial camera. If the corners of



METROGON LENS NO. 3244465

ANGLE OFF LENS AXIS	TAN α	F TAN α (F= 152.18)	LENS DISTORTION (NBS)
15°	.2680	41 mm	+ 0.02 mm
20	.3640	55	+ .04
25	.4663	71	+ .08
30	.5774	88	+ .11
35	.7002	107	+ .12
40	.8391	128	+ .07
45	1.0000	152	- .15

FIG. 6. A graphical display of lens distortion, earth curvature and atmospheric refraction.

the model area are regarded as a datum, then the other four parts of the model midway between then have apparent elevations in feet due to the curvature of the earth, as follows:

0	+ 2	0
+8	+10	+8
0	+ 2	0.

For any other flight altitude, the values vary as the square of the ratio in altitude. For example, at 1,000 meters altitude the values are only 1/100 as great, or only 0.1 of a foot high in the center, and can possibly be ignored.

Refraction is sometimes said to be one-seventh as great as earth curvature and in the opposite direction. So perhaps if the above values are multiplied by $^6/_7$, both curvature and refraction are accounted for. Obviously, more accurate refraction data are available.

It is therefore possible to add a sixth term to the items listed in Figure 4 to correct for the curvature and refraction. The formula for the earth curvature effect h is

$$h = \frac{D^2}{2R}$$

where D is the distance from a designated center to a point and R is the earth radius (6.367×10^6 meters or 20.89×10^6 feet).

CONCLUSION

An attempt has been made in the foregoing pages to demonstrate a logical and effective procedure by which any photogrammetrist can calibrate his stereoscopic plotting instrument and compensate it for elevation errors. As in other instances of this sort, the description appears more formidable than the operation itself—it is easier to do than to describe it. The procedure may seem to be lengthy partly because efforts were made to make sure that anyone can understand the description and apply the ideas in the correct manner. Some might have hoped that mathematics could have been avoided altogether, but this wish does not seem to be in logical accord with the plotter itself which serves to transform three-dimensional projectivities (photographs) into a useful orthogonal system (a map) for an enterprising customer (engineer; builder).