Analytic Aerotriangulation in the Coast and Geodetic Survey

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ABSTRACT: *Analytic aerotriangulation as used in this paper means the determination of the positions and elevations of objects from observed linear measurements on single aerial photographs printed on glass and utilizing an electronic computer. The system is based on relatively simple and unconventional apparatus: a point-marking stereoscope, a monocular comparator, and an electronic computer of only moderate size. Accuracy improvement over conventional first-order stereoscopic plotting instruments has been experienced. The presentation includes the complete formulation suitable for computer programming. The paper constitutes a report on the present state of development.*

PREFACE

This paper comprises a condensed version of a forthcoming technical bulletin having the same title.

The authors respectfully acknowledge that this analytic system has been developed through the cooperative efforts of several organizational units and a large number of individuals, whence the authors serve principally as reporters of a new development.

1. INTRODUCTION

ANALYTIC aerotriangulation is a method for accurately determining the ground positions of objects throughout a strip or block of overlapping aerial photographs, using relatively few known ground positions, by means of digital calculations based on coordinate measurements of pertinent image positions on each photograph. This method differs from the more conventional instrumental method that is based on measurements of a stereoscopic model which is perfected or solved through the use of an analog device (first-order stereoscopic plotter). The analytic method offers certain worthwhile advantages accruing from automation, digital accuracy, least-squares adjustment, and freedom from the mechanical discrepancies contributed by the plotting instrument.

In the system developed in the Coast and Geodetic Survey, emphasis has been placed on the use of relatively simple instruments of moderate cost and of ready availability. Consequently, a stereoscopic point transfer device is utilized which is already being manufactured in series lots and which is simple in the sense that it has very few moving parts, and the only critical feature is that a neat, small, round hole be placed in the photographic emulsion at the same location as indicated by the index marks with an accuracy of a few microns.* Also a conventional comparator originally developed for astronomers is used in which the only critical feature is that the X and Y coordinates be indicated with micron accuracy on a single photograph. The third apparatus consists of a common electronic computer of sufficient capacity and of moderate cost, specifically one already on the premises.

The general procedure of the system is illustrated by the accompanying flow dia-

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* 1000 microns is one millimeter; one micron is approximately 1/25,000 inch.

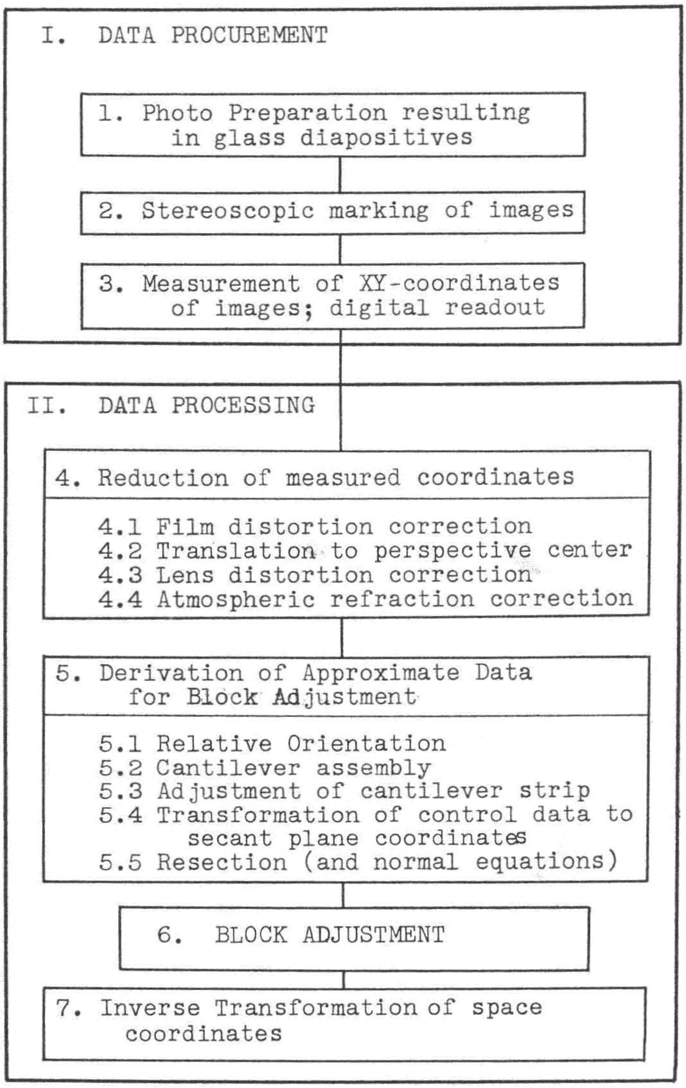

FIG. 1. Flow diagram showing sequence of procedures in analytic aerotriangulation.

gram (Figure 1) which indicates a separation into two general categories: (1) data procurement and (2) data processing. The latter is entirely a computational function. It is pointed out that Steps 4 and 5 consist of preparations for the accomplishment of Step 6, the so-called "block adjustment." This is a simultaneous solution of an entire block of photographs in terms of the orientation parameters of all photographs as unknowns. (The term "block" is used here not only in the usual sense of an area but also for a strip of photographs in the sense that a strip is a simple block. The reason for the special definition arises from the fact that the mathematics is identical for the two, the block adjustment containing merely a larger number of terms than that for a strip.)

The block adjustment is a difficult and time-consuming solution because the number of unknowns is large, being perhaps in the hundreds. Nevertheless, Step 6 need not necessarily be performed on a giant computer: more time may be required on a smaller computer but, if elapsed time is not an important factor, the total cost may well be smaller. Theoretically it is possible to proceed from Step 4 directly into Step 6, skipping Step 5, by iterating Step 6 as many times as necessary. But Step 5 is relatively small in time consumption and yields approximations which then require only a single iteration of Step 6. Thus all of Step 5 serves merely as an economic measure to derive close approximations in preparation for Step 6, so that the latter need be performed but once.

The quality of these approximate values at the end of Step 5.3 is so satisfactory for many current practical applications that Step 6 has not yet been applied in productive work—only in experimental studies. Consequently, *all 25 strips triangulated to date have terminated at Step 5.3 with accuracies better than experienced with first-order stereoscopic plotting instruments.* Also all accuracy reports to date have been made from these strips. Studies are being continued relative to the realm of application of the block adjustment.

It should also be noted that each of the computational items listed in Steps 4 and 5 is a relatively simple one which can be solved with a computer of only moderate size.

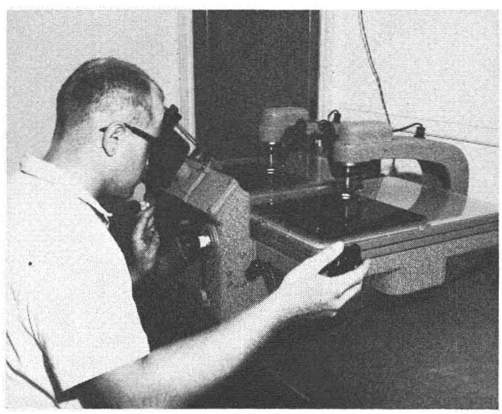

FIG. 2. The Wild PUG 2 Point Transfer Device.

Figure 2 is a view of the Wild PUG 2 stereoscopic point transfer device on which lay a pair of aerial photographic diapositives. The instrument is used to select suitable images common to both photographs and to mark these images with a drilled hole in the plastic emulsion so that the images can be readily identified in the measuring comparator and also later in a map compilation instrument. Actually such images do not exist, but once a mark is made on one photograph, a similar mark can be placed very accurately on the other, in the corresponding pattern of silver grains. The fact that with good photography the computed residual y-parallaxes have had a standard error of only three microns, in the presence of other additional sources of error, indicates that the accuracy of the instrument may be adequate. The photo stages are sufficiently large to accommodate the photos in any orientation so that images can be transferred to adjacent strips as well as in the same strip.

Figure 3 is a view of the precision screw comparator Type 422 D produced by David W. Mann Co. Digital readout is provided by Telecomputer Corp. and a Friden Flexowriter is included to produce a typewritten record and also a punch paper tape. The least count of the digital system is one micron and the standard error is about two microns throughout the measuring area. All results being reported were obtained through the use of the projection screen which has since been augmented by a cathode tube display of the light energy transmitted through the drilled holes. A very simple selection of reticles enables the operator to accommodate any size of hole. A round dot type of reticle is formed just smaller than the projected image of

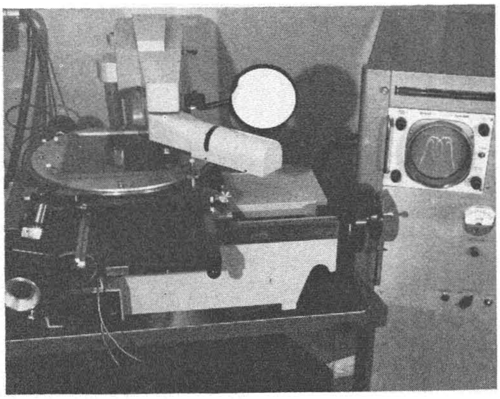

FIG. 3. The Mann Comparator Type 422-D with the visual-electronic centering device.

the drilled hole, and may be drafted directly onto the screen or onto a temporary paper cover without altering the optical system. The comparator is enclosed in a room carefully conditioned to $\pm 1°$ F, and with special provisions for isolating and removing the heat generated by the electronic readout equipment.

Figure 4 shows the console of the IBM-650 data processor which has been used in all of the work so far. It is a basic model having a 2000-word drum memory and the usual peripheral equipment including a tape-to-card converter.

The mathematical basis for the system can be expressed by a pair of equations from projective geometry having the general form.

$$x = \frac{aX + bY + cZ}{dX + eY + 1}. \tag{1}$$

This evidently predates the development of photography. These equations were utilized by O. von Gruber [a] in 1932 and by Schmid in [b] 1952 and in his many subsequent writings. The equations imply that in the absence of photographic distortions, every image-lens-object set of three points lies on a common straight line. If all systematic distortions of the photographs are corrected so that any remaining discrepancies are randomly distributed, then all such three-point collineations in a group of overlapping photographs should be satisfied simultaneously, such that the residual discrepancies in the corrected *image coordinates* obey the principles of least squares.

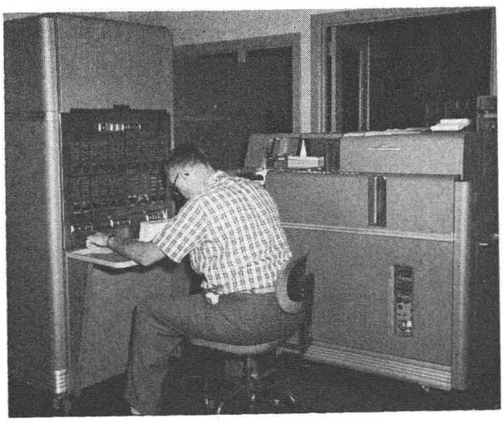

FIG. 4 Console of the IBM-650 data processor.

2. Data Procurement

Data procurement is considered here to include the three phases: aerial photography, image identification and coordinate measurement. These are discussed separately.

2.1 AERIAL PHOTOGRAPHY

All photographs processed to date have been obtained with one of the three cameras: Wild RC-5, RC-8 or RC-9. All of them are film cameras, the first two having focal-lengths of 6 inches and aviogon lenses and the last one, 3.6 inches with a super-aviogon 120° lens. A fiducial mark is automatically photographed sharply in each of the four corners of the picture format (a reseau is not used). Generally the film had the Plus-*X* emulsion on the standard topographic base, except for tests with polyester bases. The exposed film was normally air-expressed to Washington for processing from various project sites throughout the nation. Thus a few days time elapsed between photography and processing. Diapositives were prepared usually about 30 days after development using a LogEtronic printer. Diapositives are printed on the commercial $\frac{1}{4}$-inch super-flat plates. No attempt has been made to use thinner plates.

Lens-distortion data consisted of the manufacturer's reports indicating the radial distortion on each of the two diagonals to the nearest micron together with the calibrated focal-lengths to the nearest 10 microns. These data were verified by testing the Bureau's Ohio Calibration Area.

Aerial photography was usually at 20,000 feet altitude, using the standard camera mount (no automatic stabilization) and without any glass between the camera and ground, nor any special heating arrangements around the camera.

The system is generally independent from the type of photography—any focal-length, format, distortion pattern, tilt, altitude and relief is acceptable. Obviously, the accuracy of the system depends, among other items, on a knowledge of the focal-length and lens-distortion pattern within close tolerances.

2.2 IMAGE IDENTIFICATION

It is considered to be a basic and fundamental concept of photogrammetry that stereoscopic observation is a necessary operation in any aerotriangulation system. Eventually automatic machines may be developed to perform this function, but here the manually operated and relatively simple Wild PUG-2 (Figure 2) is used.* The purpose of the operation is to select and mark images which are unmistakably and accurately identified on two photographs of the same area. The images must also be identified on a third photograph in the aerotriangulation of strips of photographs, and perhaps on as many as nine for blocks of photographs.

Three categories of points are employed: pass-points, control-points, and photo-points. Pass-points are located in the usual rectangular pattern in six locations of each overlap area. Two points, a few millimeters apart, are selected in each of the six locations.

Control-points are those for which survey data are available—either their positions (*X*, *Y*; or latitude and longitude), (horizontal-control) or their elevations, (vertical-control) or both, are known from ground surveys. Ordinarily these points are not premarked except for accuracy studies, although it is recognized that the failure to premark horizontal-control introduces into the system one of the largest sources of error. In lieu of premarking, a system of "substitute" stations is applied—a system in current use in instrumental aerotriangulation. Near each horizontal station the field surveyor identifies at least two natural objects which are discernible on the aerial photographs and determines their positions by short traverses, including the necessary observations for geodetic azimuth. One of these substitute stations

* See advertisement in Photogrammetric Engineering for September 1961, page 498.—Editor.

is treated during data processing as a control station and the other as a check-point. Ordinarily a larger number of control-points is used than is uniquely required to control the computation. The frequency of control stations is governed by the accuracy specifications for the particular project and the availability of stations already established during previous surveys. As an example, the control for a strip of photographs may consist of horizontal stations distributed roughly one in every sixth photograph and a pair of vertical stations situated near the edges of the strip in roughly every fourth photograph.

Photo-points are images whose ground-coordinates are wanted from the computation for any one of several applications in subsequent mapping or charting work. Presumably it is this group of points for which the aerotriangulation is performed, although pass-points frequently serve this purpose.

The instrument contains a "floating" dot in the optical train and a system of prisms enabling images to be viewed in any orientation, i.e., in the normal manner, pseudoscopically, and at right-angles so that y-parallaxes appear as elevational discrepancies. A fine drill can be pressed so that a small hole is formed in the emulsion of the diapositive at the same location as indicated by the floating dot. The drills are honed by the operator to yield desirable sizes. A diameter of 65 microns seems to be about the smallest that can be produced reliably, and 150 microns is needed for eventual mapping with the Kelsh Plotter. As no loss of accuracy in using the larger holes has been indicated in recent results, it may eventually be considered advisable to standardize on this size. Incidentally, the diapositives cannot be stacked after marking without damaging the holes.

2.3 COORDINATE MEASUREMENT

The purpose of coordinate measurement is to assign x and y linear coordinates to all marked images and the four fiducial marks for each photograph. The origin of coordinates is immaterial as later any origin can be readily applied by the computer based on the observed values at the fiducial marks. The accuracy of the measurements is important: the fidelity and stability of the machine, the straightness of the ways, the right-angle validity of the x and y carriages, the acuteness of the viewing system, human fatigue during operation, etc. An important auxiliary is the digitization of the observations so that the operator need not write down the coordinates. Instead, the coordinates are punched into paper tape which is fed directly into the computer processing phase without manual intervention.

A sample form of the digital readout is:

15 16 3 11 02135010 02281120 99.

The first seven digits comprise the point identification number, the next two eight-digit numbers are the x and y coordinate values, and the final two digits are a computer program code number. Photograph numbers are conceived as consisting of only two digits, 00 to 99. The value 15 is the photograph number on which the measure is made; 16 is that of the photo whose center is nearest the point being measured; 3 is the class of point, such as control-point or pass-point; the next 1 is the number of the pass-point location, and the second 1 is the serial number of the pass-point in the location. The initial and terminal zeros in the coordinate values are dummy characters, and the remaining six-digit values are number of microns from the fixed machine origin. The computer code number refers to fiducial marks and is dropped after the first computer program. The identification and code numbers are dialed on the Telecordex prior to pressing the readout switch which actuates both the dialed numbers and coordinate values. These values are also displayed at all times. Identification numbering is normally uniform from plate to plate.

Ordinarily the operator makes five readings on each of the four fiducial marks and three readings on each other point, numbering at least 18 per plate. Averages are determined by the computer and are retained to the nearest tenth of a micron. Seldom does the spread between the three readings exceed two microns, which indicates the repeatability of the device, and also may indicate that plural readings may not be necessary. This comparator model has a motor drive for only one coordinate direction for rough positioning. The reticle consists of a combination of a dot to fit just inside the drilled holes, and also a system of diagonal lines to facilitate setting on the fiducial marks. Incidentally, it is unnecessary to position or orient the plate precisely before observing.

Because of unavoidable backlash in a screw type of comparator, the operator must always approach a mark from the same direction and must not overrun. If overrun does occur, one must back up at least three millimeters before returning. The operator seldom glances at either the display panel or the typewritten record during the reading of an entire plate. If he is cognizant of a mistake, a means is provided for obliterating the erroneous portion of the paper tape record. The typed record is scanned before the tape is released for data processing, and the typed form comprises a record for any subsequent references. Hand notes are added to help identify specific control points, etc.

The cathode ray tube display of the light energy passing through a drilled hole has not yet been rigidly tested, but initial observations indicate that repeatability is excellent and operator fatigue is reduced. The system seems to work equally well with drilled holes, black dots or grid intersections.

The comparator is quite sensitive to temperature variations. Not only is the room temperature closely maintained but an off-duty heater is also provided in the instrument maintaining at all times a heat flow equal to that of the projection lamp. Otherwise two hours of warm-up time is needed until the machine reaches a stable condition. Difficulties of this nature are detected at the end of observing a plate by re-observing the initial point.

3. Data Processing

As indicated in Figure 1, data processing includes all the computer steps beginning with the digital *xy*-coordinates data produced by the comparator and ending with the computed X, Y, Z coordinates of points on the ground in any convenient system, such as a State Plane-Coordinate System. The form of the output is both printed and on IBM cards.

Data processing is considered as being composed of four stages. The *stages* are not identical to computer *programs* as, for example, the first stage is accomplished with a single program and six programs are used in the second stage. Again it is emphasized that no practical work has yet progressed beyond Step 5.3 in the second stage (Step 7 has been employed immediately after Step 5.4) omitting Steps 5.5 and 6.

3.1 Film Distortion Correction

Any type of plastic aerial film changes shape slightly and non-uniformly between the time the photograph is exposed and the time it is printed on glass, when any further changes are considered to be arrested. Evidence of the distortion is revealed by the comparator measurements of fiducial marks as compared to the known fixed measurements in the camera itself. The following formulas [c] are used to correct for the distortion in the best manner available to date.

The corner fiducial marks are considered to be numbered clockwise from one to four. Corner three is arbitrarily selected as an origin with the ordinate passing through two. The given constant camera-coordinates of the corner fiducial-marks and principal-point are designated as Xj, Yj; x_p, y_p where $Y_2 = X_3 = Y_3 = 0, j = 1 \cdots 4$. The values x_p, y_p are the coordinates of the principal point. The observed coordinates of the four corners based on the comparator coordinate system are correspondingly designated by small letters x_j, y_j.

A series of coefficients are defined:

$$
\begin{aligned}
u_j &= x_j - x_3 & v_j &= y_j - y_3 \\
d &= u_2 v_4 - u_4 v_2 & & \\
m &= (X_2 v_4 - X_4 v_2)/d & a &= m u_1 + n v_1 \\
n &= (X_4 u_2 - X_2 u_4)/d & b &= p u_1 + q v_1 \\
p &= v_2 Y_4/d & r &= (X_1 - a)/ab \\
q &= - u_2 Y_4/d & s &= (Y_1 - b)/ab
\end{aligned}
\tag{2}
$$

Once the six coefficients m, n, p, q, r, s have been determined for a given diapositive, then the observed coordinates x_i, y_i, of any image i on the photograph can be transformed into compensated coordinates x_i', y_i' through the application of the formulas:

$$
\begin{aligned}
x_i' &= (m u_i + n v_i)[1 + r(p u_i + q v_i)] + x_p \\
y_i' &= (p u_i + q v_i)[1 + s(m u_i + n v_i)] + y_p.
\end{aligned}
\tag{3}
$$

The effect of the transformation is to apply linear transformations (translation, rotation, dilation) to correct three corners and then to apply a conformal fitting to correct the fourth point without disturbing the other three. Tests so far have indicated smaller remaining residual discrepancies from film distortion than was originally anticipated, but it is nevertheless-recognized that a more effective correction method is needed in order to exploit fully the accuracy potential of the analytic system. It is also recognized that some residual systematic distortions can be absorbed in the relative orientation program and attributed to orientation parameters without indicating any abnormal y-parallax residuals. The effectiveness of the correction is exhibited only by the block adjustment.

3.2 ASYMMETRIC LENS DISTORTION CORRECTION

In all of the aerial cameras used to date, the radial lens distortions are not identical for the different radii, resulting in noticeable residual discrepancies if an average uniform distortion is assumed. Another way of visualizing the condition is that lines of equal distortion are not symmetric or circular with respect to the principal point. However, the pattern in each case closely conforms to an ellipse drifted off-center and, as indicated by Washer [d] is practically identical to a small tilt of the focal-plane. Consequently, a false tilt is introduced here to correct all image-points for the asymmetric effect, after which the total remaining correction for uniform radial distortion is applied.

The false tilt is composed of two parameters: a direction and a magnitude. The "upper" [e] end of the axis of the ellipse is considered to form an angle θ with the x-axis of the photograph in the sense of analytic geometry, and it is defined that $a = \sin\,\theta$, $b = \cos\,\theta$, which become constants for a camera. It is convenient first to rotate the coordinate axis for an image through this angle θ, make the asymmetric correction and then rotate back into the original photographic coordinate system.

The formulas for the initial rotation are:

$$
x_\theta = ax + by \qquad y_\theta = - bx + ay \tag{4}
$$

where x, y are the coordinate values of an image after film distortion compensation.

Based on the analysis in [e] and utilizing the formula

$$
d_x = x^2(\sin\,t)/f
$$

in which d_x is the x-component of the radial tilt displacement of an image having an abscissa x on (the "upper" side of) a photograph of tilt t and focal-length f, it can be shown that the corrected coordinates of the image are

$$
\begin{aligned}
x_t &= x_\theta[1 + x_\theta(\sin t)/f] \\
y_t &= y_\theta[1 + x_\theta(\sin t)/f]
\end{aligned}
\tag{5}
$$

The term $(\sin t)/f$ becomes a constant for a camera. The angle t is determined from an analysis of the radial distortions on four or more diagonals. The small values for t of 10, 17, and 18 seconds, respectively, for the three cameras used to date, allows certain approximations in deriving the simplified formulas (5).

Then the final rotation is the inverse form of the initial one:

$$
\begin{aligned}
x' &= ax_t - by_t \\
y' &= bx_t + ay_t.
\end{aligned}
\tag{6}
$$

3.3 RADIAL LENS DISTORTION CORRECTION

Uniform radial lens distortion is corrected along with atmospheric refraction through the use of the formulas

$$
\begin{aligned}
r^2 &= x^2 + y^2 \\
x' &= x[1 + (rd)/r^2 + k_1 + k_2r^2] \\
y' &= y[1 + (rd)/r^2 + k_1 + k_2r^2]
\end{aligned}
\tag{7}
$$

in which the k_1 and k_2 terms relate to atmospheric refraction, which is discussed later. Here the interest is confined to the term $(rd)/r^2$. The coordinates x, y are those of any image after they have been corrected for the asymmetric condition, x' and y' are the resulting corrected values, and d is the uniform radial distortion factor at radius r after the removal of the asymmetric portion and is essentially the average distortion factor along all four radial directions. The term $(rd)/r^2$ is obviously d/r, but rewritten in a form easier to program for computation.

The value rd is obtained in the computer through table lookup and interpolation based on r^2 as the independent argument. Values of the product rd are supplied as constants for each camera for 150 values of r, that is, one for each millimeter ranging from the principal-point to any corner. The values are determined by desk calculator from the lens distortion data.

The numerical error of these methods for distortion compensation for asymmetric and uniform radial lens distortion is in the order of 0.1 micron, which is somewhat better than the validity of the distortion data.

3.4 ATMOSPHERIC REFRACTION CORRECTION

As indicated previously, the k_1 and k_2 terms in Equation (7) relate to the atmospheric refraction for near-vertical photographs only: if oblique photographs are used, this method of correction will need to be revised.

Based on the tables by Leyonhufvud [f], the values of the k's have been determined by desk calculator to correct the coordinates through their radial distances in accordance with the simplified power series

$$
\begin{aligned}
x' &= x(1 + k_1 + k_2r^2) \\
y' &= y(1 + k_1 + k_2r^2)
\end{aligned}
\tag{8}
$$

where r is as defined by (7). The values of the k's have been determined by desk calculator so that a very close agreement is obtained with the tables [f]. The photogrammetrist who submits the comparator data for computer processing is supplied with iarge scale graphs of the k-values in terms of the camera altitude and terrain elevation as independent arguments. He reads off the two values and enters them as constants on the record form which accompanies the coordinate data for a strip of photographs.

Thus Equation (7) is used to compensate image-coordinates both for radial lens distortion by table-lookup, and for refraction by using the two constants.

It is realized that the refraction correction is based on the assumption that nadir-point of the photograph coincides with the principal-point, that is, that the tilt is zero. Thus a small error is introduced perhaps because the *rms* tilt is a little less than 1°. However, this error is probably not as great as the assumption of a standard atmosphere and not applying any factors for surface and aerial variations in temperature, humidity and barometric pressure.

3.5 RELATIVE ORIENTATION

Relative orientation is defined both here and in conventional instrumental photogrammetry as the determination of the three angular and two linear parameters that specify the attitude and position of one photograph (camera-station) with respect to another (overlapping) one that shows a sufficiently large common area. Relative orientation is perhaps the most important item in this analytic system: it embodies all the basic mathematics that is peculiar to the system, is utilized again later in resection and the block adjustment, and requires the second-largest computer effort next to the block adjustment (nevertheless it is accomplished through a single IBM-650 program). It is in relative orientation that the principles of projective geometry are applied wherein the mathematics of the system may differ from that cf other engineering and computational disciplines.

A classic geometric rotation of the axes in three dimensions (Figure 5) is needed in relative orientation to express the attitude of one photograph to another. Instead of using the three angles between the respective axes as in analytic geometry, a system of three sequential rotations are used, the primary one ω about a horizontal x-axis, the secondary one ϕ about the once rotated y-axis, and the tertiary one κ about the camera axis, as explained by Rosenfield [g]. The rotation equations are

$$\begin{aligned} x &= x^* \cos\phi\cos\kappa + y^*(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa) \\ &\quad + z^*(\sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa) \\ y &= x^*(-\cos\phi\sin\kappa) + y^*(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) \\ &\quad + z^*(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa) \\ z &= x^*\sin\phi + y^*(-\sin\omega\cos\phi) + z^*(\cos\omega\cos\phi). \end{aligned} \tag{9}$$

where x, y, z are the coordinates of any image on a photograph, and x^*, y^*, z^* are the corresponding coordinates in an erect, untilted (rectified) system in which the x^*, y^*, z^* axes may also be conceived as being parallel, respectively, to those of a ground survey coordinate-system, X, Y, Z. The coordinate z corresponds to focal length. Equations (9) may be written in the form

$$\begin{aligned} x &= a_{11}x^* + a_{12}y^* + a_{13}z^* \\ y &= a_{21}x^* + a_{22}y^* + a_{23}z^* \\ z &= a_{31}x^* + a_{32}y^* + a_{33}z^*. \end{aligned} \tag{10}$$

In matrix notation this becomes

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix}. \tag{11}$$

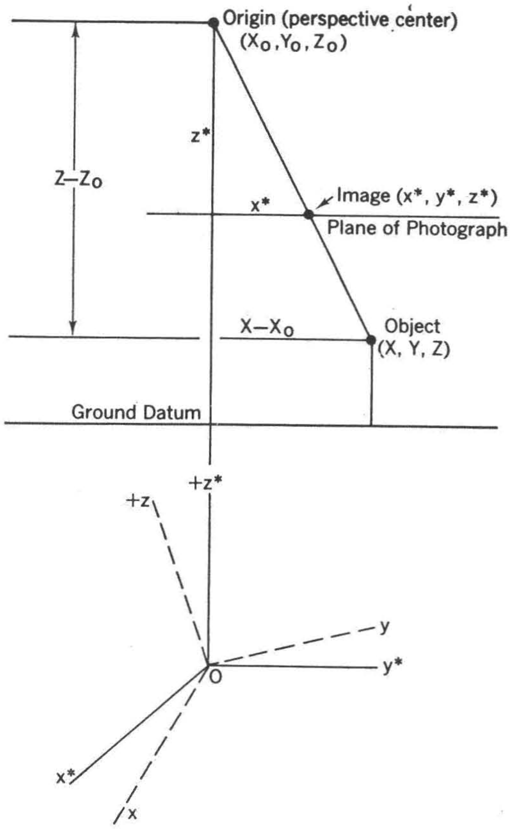

FIG. 5. Geometry of colineation and rotation.

The *inverse* notation is also useful:

$$\begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{12}$$

It is also convenient to write (11) and (12), respectively, as

$$\begin{aligned} \mathbf{x} &= \mathbf{A}\mathbf{x}^* \\ \mathbf{x}^* &= \mathbf{A}^{-1}\mathbf{x} = \mathbf{A}^T\mathbf{x}. \end{aligned} \tag{13}$$

in which explicitly

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{pmatrix}. \tag{14}$$

It is convenient to note that the values of the nine elements of $\mathbf{A}$ are handily formed in a computer by matrix multiplication as indicated by Rosenfield [g] which demonstrates that $\boldsymbol{A}$ is composed of the three sequential plane rotations:

$$\boldsymbol{A} = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \tag{15}$$

It is also useful later to form the product of the last two matrices first inasmuch as the order of formation is otherwise irrelevant:

$$\boldsymbol{A} = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\omega\sin\phi & -\cos\omega\sin\phi \\ 0 & \cos\omega & \sin\omega \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{pmatrix}. \tag{16}$$

As derived in [h], the basic projective transformation equations are (Figure 5):

$$\begin{aligned} \frac{x}{z} &= \frac{(X - X_0)a_{11} + (Y - Y_0)a_{12} + (Z - Z_0)a_{13}}{(X - X_0)a_{31} + (Y - Y_0)a_{32} + (Z - Z_0)a_{33}} \\ \frac{y}{z} &= \frac{(X - X_0)a_{21} + (Y - Y_0)a_{22} + (Z - Z_0)a_{23}}{(X - X_0)a_{31} + (Y - Y_0)a_{32} + (Z - Z_0)a_{33}} \end{aligned} \tag{17}$$

where X, Y, Z are the coordinates of an object on the ground, X_0, Y_0, Z_0 are the coordinates of the camera-station in the same system and x, y, z are the image-coordinates, in which $z = -f$, the camera focal-length. (Compare Equation 1.) By clearing fractions and transposing,

$$\begin{aligned} & x[(X - X_0)\sin\phi + (Y - Y_0)(-\sin\omega\cos\phi) + (Z - Z_0)\cos\omega\cos\phi] \\ &\quad - z[(X - X_0)\cos\phi\cos\kappa \\ &\quad + (Y - Y_0)(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa) \\ &\quad + (Z - Z_0)(\sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa)] = 0 \\ & y[(X - X_0)\sin\phi + (Y - Y_0)(-\sin\omega\cos\phi) + (Z - Z_0)\cos\omega\cos\phi] \\ &\quad - z[(X - X_0)(-\cos\phi\sin\kappa) \\ &\quad + (Y - Y_0)(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) \\ &\quad + (Z - Z_0)(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa)] = 0. \end{aligned} \tag{18}$$

If $\boldsymbol{A}_i$ is defined as representing the three elements in row i of the matrix in Equation (11)

$$\boldsymbol{A}_i = (a_{i1} \quad a_{i2} \quad a_{i3})$$

and also

$$\boldsymbol{B} = \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix},$$

then Equation (17) can be expressed in determinant notation:

$$\begin{vmatrix} x & z \\ \boldsymbol{A}_1\boldsymbol{B} & \boldsymbol{A}_3\boldsymbol{B} \end{vmatrix} = 0, \qquad \begin{vmatrix} y & z \\ \boldsymbol{A}_2\boldsymbol{B} & \boldsymbol{A}_3\boldsymbol{B} \end{vmatrix} = 0. \tag{19}$$

It should be noted that Equations (17), (18), and (19) are merely different forms of the same equation which expresses the condition that the image, object and perspective center are colinear. It is this condition one seeks to enforce, and if perchance the condition does not exist, the wish is to allow incremental corrections to the observed coordinates x, y such that sum of the squares of the corrections is minimum.

Equation (16) is transcendental and, in the most general case, all twelve terms are considered as unknowns. Consequently a form of Newton's Method [i] is used to solve them. This is an iterative method based on initial approximations which are quite easily obtained for all the unknowns. Experience indicates that about 95% of the problems require three iterations. Applying partial differentiation and rearranging the terms, using $v_x = dx$ and $v_y = dy$, the following *observation* equations can be formed

$$\begin{aligned} v_x &= p_{11} + p_{12}dw + p_{13}d\phi + p_{14}dk - p_{15}dX_0 - p_{16}dY_0 - p_{17}dZ_0 \\ &\quad + p_{15}dX + p_{16}dY + p_{17}dZ \\ v_y &= p_{21} + p_{22}dw + p_{23}d\phi + p_{24}dk - p_{25}dX_0 - p_{26}dY_0 - p_{27}dZ_0 \\ &\quad + p_{25}dX + p_{26}dY + p_{27}dZ \end{aligned} \tag{20}$$

in which the p-coefficients are defined by Equation (21) (Figure 6).

The partial derivatives of $\boldsymbol{A}$ are formed from Equation (14):

$$\begin{aligned} \frac{\partial \boldsymbol{A}}{\partial \omega} &= \begin{Bmatrix} 0 & -\sin\omega\sin\kappa + \cos\omega\sin\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa \\ 0 & -\sin\omega\cos\kappa - \cos\omega\sin\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa \\ 0 & -\cos\omega\cos\phi & -\sin\omega\cos\phi \end{Bmatrix} \\ &= \begin{Bmatrix} 0 & -a_{13} & a_{12} \\ 0 & -a_{23} & a_{22} \\ 0 & -a_{33} & a_{32} \end{Bmatrix} \end{aligned} \tag{22}$$

$$p_{11} = \begin{vmatrix} x & z \\ \boldsymbol{A}_1\boldsymbol{B} & \boldsymbol{A}_3\boldsymbol{B} \end{vmatrix} \qquad p_{21} = \begin{vmatrix} y & z \\ \boldsymbol{A}_2\boldsymbol{B} & \boldsymbol{A}_3\boldsymbol{B} \end{vmatrix}$$

$$p_{12} = \begin{vmatrix} x & z \\ \frac{\partial \boldsymbol{A}_1}{\partial \omega}\boldsymbol{B} & \frac{\partial \boldsymbol{A}_3}{\partial \omega}\boldsymbol{B} \end{vmatrix} \qquad p_{22} = \begin{vmatrix} y & z \\ \frac{\partial \boldsymbol{A}_2}{\partial \omega}\boldsymbol{B} & \frac{\partial \boldsymbol{A}_3}{\partial \omega}\boldsymbol{B} \end{vmatrix}$$

$$p_{13} = \begin{vmatrix} x & z \\ \frac{\partial \boldsymbol{A}_1}{\partial \phi}\boldsymbol{B} & \frac{\partial \boldsymbol{A}_3}{\partial \phi}\boldsymbol{B} \end{vmatrix} \qquad p_{23} = \begin{vmatrix} y & z \\ \frac{\partial \boldsymbol{A}_2}{\partial \phi}\boldsymbol{B} & \frac{\partial \boldsymbol{A}_3}{\partial \phi}\boldsymbol{B} \end{vmatrix}$$

$$p_{14} = \begin{vmatrix} x & z \\ \frac{\partial \boldsymbol{A}_1}{\partial \kappa}\boldsymbol{B} & \frac{\partial \boldsymbol{A}_3}{\partial \kappa}\boldsymbol{B} \end{vmatrix} \qquad p_{24} = \begin{vmatrix} y & z \\ \frac{\partial \boldsymbol{A}_2}{\partial \kappa}\boldsymbol{B} & \frac{\partial \boldsymbol{A}_3}{\partial \kappa}\boldsymbol{B} \end{vmatrix}$$

$$p_{15} = \begin{vmatrix} x & z \\ a_{11} & a_{31} \end{vmatrix} \qquad p_{25} = \begin{vmatrix} y & z \\ a_{21} & a_{31} \end{vmatrix}$$

$$p_{16} = \begin{vmatrix} x & z \\ a_{12} & a_{32} \end{vmatrix} \qquad p_{26} = \begin{vmatrix} y & z \\ a_{22} & a_{32} \end{vmatrix}$$

$$p_{17} = \begin{vmatrix} x & z \\ a_{13} & a_{33} \end{vmatrix} \qquad p_{27} = \begin{vmatrix} y & z \\ a_{23} & a_{33} \end{vmatrix}$$

FIG. 6. Equation 21: The basic coefficients of the observation equations used in relative orientation, resection, and block adjustment.

$$\frac{\partial A}{d\phi} = \begin{bmatrix} -\sin\phi\cos\kappa & \sin\omega\cos\phi\cos\kappa & -\cos\omega\cos\phi\cos\kappa \\ \sin\phi\sin\kappa & -\sin\omega\cos\phi\sin\kappa & \cos\omega\cos\phi\sin\kappa \\ \cos\phi & \sin\omega\sin\phi & -\cos\omega\sin\phi \end{bmatrix} \tag{23}$$

(Note that the first two rows can be formed from the third row of Equation (16) by multiplying by $(-\cos\kappa)$ and $(+\sin\kappa)$, respectively, and that the third row of (23) is the first row of (16)).

$$\frac{\partial A}{\partial\kappa} = \begin{bmatrix} -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ -\cos\phi\cos\kappa & -\cos\omega\sin\kappa - \sin\omega\sin\phi\cos\kappa & -\sin\omega\sin\kappa + \cos\omega\sin\phi\cos\kappa \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ -a_{11} & -a_{12} & -a_{13} \\ 0 & 0 & 0 \end{bmatrix}. \tag{24}$$

Equation (20) is in a sense a "universal" type of formula for analytic photogrammetry. It is used here to solve three somewhat different problems: relative orientation, resection, and in the block adjustment of either a strip and a genuine block of photographs. If the approximate values of X, Y, Z are sufficiently near correct (which is the function of Step 5 in Figure 1), the dX, dY, dZ terms may be neglected for a time, leaving six unknowns. In relative orientation, however, these three terms cannot be neglected, and also the term in dX_0 has no significance. It is shown presently how the three terms can be eliminated, leaving five independent unknowns.

It is shown by Schmid [j] that the elevation Z of an object whose images appear on two photographs can be expressed in general as

$$Z = \frac{(X_0'' - X_0')z^{*\prime}z^{*\prime\prime} + Z_0'x^{*\prime}z^{*\prime\prime} - Z_0''x^{*\prime\prime}z^{*\prime}}{x^{*\prime}z^{*\prime\prime} - x^{*\prime\prime}z^{*\prime}} \tag{25}$$

where the primes refer to the first and second photographs and the asterisks are defined by Equation (12). Once Z is evaluated,

$$\begin{aligned} X &= X_0' + x^{*\prime}(Z - Z_0')/z^{*\prime} \\ Y &= Y_0' + y^{*\prime}(Z - Z_0')/z^{*\prime}. \end{aligned} \tag{26}$$

In the relative orientation problem these equations simplify to

$$Z = \frac{Z_0x^* - z^*}{x^* - (x/z)z^*}, \qquad X = xZ/z, \qquad Y = yZ/z \tag{27}$$

because the first photograph is considered as untilted, the first camera-station can be selected as the origin, and the abscissa of the second station can be selected as unity. As a consequence of Equations (27), the terms in dX, dY, dZ in Equation (20) can be eliminated by substitution and expressed in terms of the other unknowns $d\omega$, $d\phi$, $d\kappa$ and dZ_0 resulting in the special observation equations for relative orientation

$$\begin{aligned} v_x &= p_{11} + (p_{12} + S_1T_2)d\omega + (p_{13} + S_1T_3)d\phi + (p_{14} + S_1T_4)d\kappa \\ &\quad - p_{16}dY_0 - (p_{17} - S_1T_7)dZ_0 \\ v_y &= p_{21} + (p_{22} + S_2T_2)d\omega + (p_{23} + S_2T_3)d\phi + (p_{24} + S_2T_4)d\kappa \\ &\quad - p_{26}dY_0 - (p_{27} - S_2T_7)dZ_0 \end{aligned} \tag{28}$$

in which

$$T_2 = \begin{vmatrix} X - 1 & Z - Z_0 \\ (\partial \boldsymbol{A}_1{}^T/\partial\omega)\boldsymbol{C} & (\partial \boldsymbol{A}_3{}^T/\partial\omega)\boldsymbol{C} \end{vmatrix}$$

$$T_3 = \begin{vmatrix} X - 1 & Z - Z_0 \\ (\partial \boldsymbol{A}_1{}^T/\partial\phi)\boldsymbol{C} & (\partial \boldsymbol{A}_3{}^T/\partial\phi)\boldsymbol{C} \end{vmatrix}$$

$$T_4 = \begin{vmatrix} X - 1 & Z - Z_0 \\ (\partial \boldsymbol{A}_1{}^T/\partial\kappa)\boldsymbol{C} & (\partial \boldsymbol{A}_3{}^T/\partial\kappa)\boldsymbol{C} \end{vmatrix}$$

$$T_7 = \boldsymbol{A}_1{}^T\boldsymbol{C}$$

$$\boldsymbol{C} = \begin{Bmatrix} x_2 \\ y_2 \\ z_2 \end{Bmatrix}$$

$$S_1 = (1/u_2)(p_{15}u_3 + p_{16}u_4 + p_{17})$$

$$S_2 = (1/u_2)(p_{25}u_3 + p_{16}u_4 + p_{27})$$

$$u_1 = Zx^* - z^* \qquad u_3 = x/z$$

$$u_2 = x^* - u_3z^* \qquad u_4 = y/z.$$

The subscript 2 in $\boldsymbol{C}$ indicates the image coordinates on the second photograph, $\boldsymbol{A}^T$ being defined by Equations (13) and (12). Specifically

$$\boldsymbol{A}_1{}^T\boldsymbol{C} = x^* = a_{11}x_2 + a_{21}y_2 + a_{31}z_2$$

$$\boldsymbol{A}_3{}^T\boldsymbol{C} = z^* = a_{13}x_2 + a_{23}y_2 + a_{33}z_2$$

and the partial terms refer to elements of *columns* in Equations (21), (22), (23).

The pair of observation Equations (28) occurs for each image on each photograph. In the resection and aerotriangulation problems both equations apply, whereas in relative orientation only the y-equation is significant. Nevertheless both equations are utilized for programming uniformity. The problem then becomes one of solving a large number of simultaneous linear equations, applying least squares, for unique values of the five unknowns $d\omega$, $d\phi$, $k\kappa$, dY_0, dZ_0. These are changes which need to be made in the five approximate values of the unknowns themselves, after which the computation is repeated using the corrected values. The iteration terminates when the corrections become insignificant, specifically, when each of three $d\omega$, $d\phi$, $d\kappa$ is less than 10^{-5} radians.

Initial approximations consist of zeros for all five parameters but other values can be entered if they are considered to be more appropriate.

The output of the computation consists of (1) the values of the five parameters, (2) the values of the nine elements in the final $\boldsymbol{A}$-matrix, (3) provisional or "model" coordinates for each object, (4) and the residuals (p_{11}, p_{21}) for each image.

The purposes of the relative orientation procedure consists of: (1) grossly mistaken data are indicated by the residuals enabling discarding and re-computing; and (2) an interrelated system of cartesian coordinates of ground objects which can be transformed into a single continuous coordinate-system for an entire strip or block of photographs.

In relative orientation, each pair of photographs is treated entirely independent from the others. Consequently no special problem occurs if one or more intermediate pairs are recomputed.

Relative orientation is arranged as a single IBM 650 program requiring about five minutes per model.

3.6 CANTILEVER ASSEMBLY

After the completion of the relative orientation computation for each model of a strip of photographs, the models are connected into a single continuous chain by means of successive coordinate transformations of rotation, dilation and translation for all the objects in the models, based on the coordinates of common objects in adjacent models, along with the orientation data, and without any ground-control information, but based on the first model as a reference system. It is termed "cantilever" because of the successive attachment of each model to the preceding one. As one might expect, the assembly is affected by an accumulation of systematic errors, but these errors are corrected insofar as is possible as described in Section 3.7.

If $\boldsymbol{R}_i$ denotes a rotation matrix for the model coordinates of the ith model in the same sense that $\boldsymbol{A}$ is used in Equations (13), (12), (11), (10) and

$$\boldsymbol{X}' = \boldsymbol{R}\boldsymbol{X} \tag{29}$$

such that $\boldsymbol{X}$ consists of the model coordinates before rotation and $\boldsymbol{X}'$ those after rotation, it can be shown that

$$\boldsymbol{R}_i = \boldsymbol{R}_{i-1}\boldsymbol{A}_{i-1}{}^T. \tag{30}$$

In words, the rotation matrix for any model can be determined by forming the matrix product of that of the previous model and the transpose matrix from relative orientation. In order to get the system started, (1) the model coordinates in the first model are considered to be already in the desired system and require no further rotation; (2) those in the second model are rotated by the application of Equation (29) in which $\boldsymbol{R} = \boldsymbol{A}^T$ of the first model; (3) thereafter Equation (30) applies. It may be noteworthy that $\boldsymbol{R}$ is a function only of angular parameters, ω, ϕ, κ.

A scale factor m is determined from

$$m^2 = \frac{(X_2' - X_1')^2 + (Y_2' - Y_1')^2 + (Z_2' - Z_1')^2}{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2} \tag{31}$$

in which X, Y, Z are the model coordinates of a selected common object and X', Y', Z' are its final coordinates in the previous model. The two objects are selected by a photogrammetrist from opposite sides of the model on the basis of favorable residual parallax values from relative orientation. All coordinates of objects in a model being attached are then changed by multiplying by the common factor m. The coordinates X_0, Y_0, Z_0 of both camera stations are also multiplied by this factor. (In the present program the initial camera station in each model is assigned arbitrary, constant, non-negative coordinates, whereas some simplification might have been achieved by assigning zeros.)

A translation of coordinates is needed so as to form a single continuous system. The elements a, b, c translation are based on the coordinates of the camera-station which is common to the two models:

$$a = X_0' - X_0, \qquad b = Y_0' - Y_0, \qquad c = Z_0' - Z_0 \tag{32}$$

where the primed coordinates are those of the previous model and the unprimed coordinates are those of the subject model after rotating and scaling. Then the translated coordinates of any object-point are

$$X' = X + a, \qquad Y' = Y + b, \qquad Z' = Z + c \tag{33}$$

which is applied to all the object-coordinates in the subject model *and also those of the second-camera station* for use in the next model.

After the three operations of rotation, scaling, and translation are complete, those object-points common to both models will obviously have two slightly different sets of coordinates. The mean value is computed and adopted as the final value, and the deviation from the mean for each of the three coordinates is printed out. The function of the deviations is to detect by visual scanning any unusually large errors which might be a sufficient reason to disregard a given point in subsequent mapping applications.

3.7 ADJUSTMENT OF THE CANTILEVER DATA

For the adjustment of the cantilever strip, two computer programs normally used in the adjustment of data from plotting instruments are applied without change. These were described by Harris in [k] and [l] and the formulation is repeated here. The programs are relatively short and might easily be combined into a single step. The formulas provide a generally conformal transformation of the coordinates of points using quadratic and cubic terms to allow for the accumulation of systematic errors. The input of the horizontal program [k] consists of a list of cantilever coordinates [Equation (33)] for four to ten control points and also a list of the State Plane-Coordinates of the same points, in addition to a list of the cantilever-coordinates of all the other points in the strip of photography. The vertical program [l] similarly consists of a list of coordinates of points in the two systems, including the ground-elevations of the objects. The positions of bench marks are obtained from the horizontal adjustment; and the vertical program furnishes elevations of horizontal-control stations which are usually not known. The output consists of the State Plane-Coordinates and elevations of all the points used in the strip. The coordinates are converted to geographic positions by a separate computer program.

As the programs were devised for use with plotting instrument data, some features and precautions might be unnecessary in analytic work, but the cost of reprogramming and the work load on the programming staff precludes unneeded reprogramming for the present since this program serves adequately.

Horizontal Adjustment. The origin of the coordinate system is preferred to be at the center of a strip because of symmetry and computer scaling, and the abscissa is assigned to the longitudinal axis of the strip. To transform the cantilever strip coordinates x, y, into such a system, a standard type of combined rotation and translation is introduced:

$$x' = ax - by + c, \qquad y' = bx + ay + d \tag{34}$$

in which x', y' are the transformed values. The constants a, b, c, d are determined from

$$a = \frac{(x_1 - x_2)(x_1' - x_2') + (y_1 - y_2)(y_1' - y_2')}{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$b = \frac{(x_1 - x_2)(y_1' - y_2') - (y_1 - y_2)(x_1' - x_2')}{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$c = x_1' - ax_1 + by_1, \qquad d = y_1' - bx_1 - ay_1$$

in which x_1, y_1, x_2, y_2 are the coordinates in the cantilever of points near the centers of the first and last photographs of the strip, respectively, and x_1', y_1', x_2', y_2' are the transformed coordinates of these two centers having the values

$$x_2' = +\tfrac{1}{2}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]^{1/2}$$

$$x_1' = -x_2, \qquad y_1' = y_2' = 0.$$

Once a, b, c, d are determined, all the points in the strip are transformed using Equation (34).

The next part of the solution uses the same type of transformation to convert the new strip-coordinates into ground-coordinates, and the inverse transformation to convert the State-Plane-Coordinates of horizontal ground-control stations into the strip coordinate-system for purposes of comparison. Actually the inverse is utilized first, and then the direct transformation is used later as a final step in the procedure. This routine was considered necessary because of the use of diagonally flown photographs. The transformation is based on the State-Plane-Coordinates X_1, Y_1, X_2, Y_2 of two control-points near the ends of the strip, together with their strip-coordinates x_1, y_1, x_2, y_2. The direct transformation is

$$X = ax - by + c, \qquad Y = bx + ay + d \tag{35}$$

where the inverse is

$$x = a'X + b'Y - c', \qquad y = -b'X + a'Y - d' \tag{36}$$

and the values of the coefficients are

$$\begin{aligned} a &= \frac{(x_1 - x_2)(X_1 - X_2) + (y_1 - y_2)(Y_1 - Y_2)}{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ b &= \frac{(x_1 - x_2)(Y_1 - Y_2) - (y_1 - y_2)(X_1 - X_2)}{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ c &= X_1 - ax_1 + by_1, \qquad d = Y_1 - bx_1 - ay_1 \\ a' &= a/(a^2 + b^2), \qquad b' = b/(a^2 + b^2) \\ c' &= (ac + bd)/(a^2 + b^2), \quad d' = (ad - bc)/(a^2 + b^2) \end{aligned} \tag{37}$$

Consequently, the constants are determined after which Equation (36) is used to transform the coordinates of all the horizontal-control stations into the cantilever-coordinate system.

The cantilever-coordinates x, y ordinarily differ from the transformed control-coordinates x', y' for all the control-points except the two that are used to determine the constants in Equations (37). A conformal polynomial transformation is used [k] to express the relationship between these differing coordinate values:

$$\begin{aligned} x' &= Ax^3 + Bx^2 + (C + 1)x - 2Dxy - Ey + F \\ y' &= 3Ax^2y + 2Bxy + (C + 1)y + Dx^2 + Ex + G \end{aligned} \tag{38}$$

These equations are used first with control-point coordinates to determine the values of the seven unknown coefficients $A \cdots G$. Ordinarily more than four control-points are used, making more than eight equations which are solved applying least squares in the same routine as Equations (28) earlier. Then the equations are applied to determine corrected cantilever coordinates for all the other points in the strip. Next, Equation (35) is applied to transform the cantilever coordinates into the State-Plane-Coordinates. This is the place where the program for horizontal-coordinates has terminated for all production jobs to date.

Vertical Adjustment. The vertical-adjustment follows the horizontal-adjustment although it is considered that they should be done simultaneously. The horizontal data from the previous program is utilized. Specifically, the values after the application of Equations (38)—and not Equation (35)—are used because they are symmetrically arranged with regard to the center of the strip.

As a preliminary measure, the ground elevations are transformed into the cantilever system through scaling and translation using

$$z' = Z/g + k \tag{39}$$

whose inverse form is

$$Z = g(z' - k) \tag{40}$$

where

$$g^2 = \frac{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}{(x_1 - x_2)^2 + (y_1 - y_2)^2}, \tag{41}$$

the coordinates being those used to determine the constants in Equation (37), and where k is a rough constant determined by solving Equation (39) with any normal vertical-control station. Once g and k are evaluated, all the control elevations in the strip are transformed using the same equation.

As with the horizontal-coordinates, the vertical cantilever-coordinates z at control-points do not agree with transformed values z' except for the single point used in deriving k. A similar polynomial is used to indicate the relationship of the different values:

$$z' = z + Hx^3 + Ix^2 + Jx + Kx^2y + Lxy + My + N \tag{42}$$

which introduces seven new unknown coefficients $H \cdots N$. Using at least seven vertical-control points, Equation (42) is formed for each point in which the coefficients are unknowns. Solving these as simultaneous linear equations using least squares—as for Equations (28) and (38)—unique values are obtained for the coefficients. Then with the coefficients known, Equation (42) is applied to all the points in the strip to determine z'. The next step is to find the corrected ground elevation Z (or h) for each of the points with Equation (40).

This until now completes the adjustment, yielding coordinate values which are ready for use in map compilation or other applications as survey data. The print-out includes the computed-coordinates at control-stations which should be identical to known correct values, and allows one to scan the results to detect faulty data. Large differences are due to erroneous data. As the computer program requires but ten minutes for a long strip, little cost is involved by recomputing with any erroneous or questionable data deleted.

Where strips have been very long and involve mountainous terrain, the secant plane system of the next section has been applied. These cantilever adjustments are repeated with the different system of control data. The initial adjustments furnish sufficiently accurate elevations of horizontal-control stations and the positions (which ordinarily do not result from field surveys) of vertical-control stations for the secant plane transformation. The readjustments to the secant plane data fully recognize the effect of earth curvature. Obviously, the secant plane-coordinates need to be transformed back into the State-Plane-Coordinate system for mapping uses as indicated by Step 7 (Figure 1).

3.8 TRANSFORMATION OF CONTROL DATA TO A SECANT PLANE SYSTEM

As pointed out heretofore, the purpose of the secant plane-transformation is to account for the curvature of the earth. The term "secant plane" is used instead of "geocentric" to maintain a correct nomenclature inasmuch as the classic geocentric system results in coordinate values which are too large for convenient handling in a computer, and also bear little resemblence to map-coordinates except in special cases. The secant plane system is a local system pertinent to the map project and in which the coordinate directions are comparable to map directions.

The formulation for the block adjustment introduces a three-dimensional Cartesian coordinate system in which X and Y are comparable to horizontal grid coordinates and Z is comparable to elevation. Geographic positions and elevations of control points (data obtained from Section 3.7) may be transformed into such a space system in which the Z-axis is the extension of the normal to the ellipsoid of a point near the center of the mapping project. This point (the origin) may be any selected value of latitude and longitude and not necessarily a control or pass-point. The XY-plane should be "secant" to the ellipsoid in order to avoid negative Z-values. The Y axis is in the plane of the meridian of the origin and thus may be considered as the north-south axis.

Formulas for this transformation are derived by rotating and translating modified geocentric coordinates. Classical geocentric coordinates [m] are

$$\begin{aligned} X &= (N + h) \cos \phi \sin \lambda \\ Y &= (N + h) \cos \phi \cos \lambda \\ Z &= [N(1 - e^2) + h] \sin \phi \end{aligned} \tag{43}$$

where ϕ and λ are the latitude and longitude of any point, h is the elevation (the geoid separation may be included with h if thought essential), and N is the length of the normal through ϕ, λ. (Consider λ negative in the western hemisphere.)

The center of rotation of this system is the intersection of the normal and the earth's axis of rotation (Z-axis) (Figure 7). Therefore a small increment, $N_0 e^2 \sin \phi_0$,

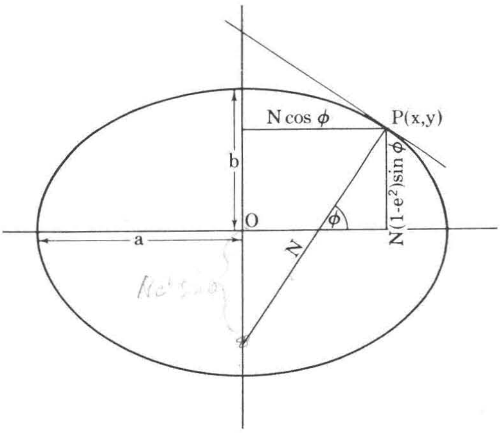

FIG. 7. Geometric elements used in the secant plane transformation.

should be added to the Z-coordinates. N_0 is the length of the normal for the origin and ϕ_0 is the latitude of the origin. Thus,

$$Z = [N(1 - e^2) + h] \sin \phi + N_0 e^2 \sin \phi_0. \tag{43a}$$

The computation of the rotation process depends on the angular relation in space of the two sets of axes. The rotated X-axis will remain in the original XY-plane and will make an angle of λ_0 with the original X-axis. The rotated Z-axis will make an angle of $(90° - \phi)$ with the original Z-axis. The direction of the Y-axis should be reversed so that positive Y would indicate north. Also, positive X indicates east.

The nine cosines of the angles in space between the two sets of axes can be expressed as

$$\begin{pmatrix} X_G \\ Y_G \\ Z_G \end{pmatrix} = \begin{pmatrix} \cos \lambda_0 & -\sin \phi_0 \sin \lambda_0 & +\cos \phi_0 \sin \lambda_0 \\ -\sin \lambda_0 & -\sin \phi_0 \cos \lambda_0 & +\cos \phi_0 \cos \lambda_0 \\ 0 & \cos \phi_0 & \sin \phi_0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{44}$$

in which the subscript G is used to indicate the modified geocentric coordinates of Equations (43) and (43a), and from which the inverse expansion may be stated as

$$X = X_G \cos\lambda_0 - Y_G \sin\lambda_0$$
$$Y = - X_G \sin\phi_0 \sin\lambda_0 - Y_G \sin\phi_0 \cos\lambda_0 + Z_G \cos\phi \tag{45}$$
$$Z = X_G \cos\phi_0 \sin\lambda_0 + Y_G \cos\phi_0 \cos\lambda_0 + Z_G \sin\phi.$$

These Z-coordinates are too large for practical computation, being of the order of N or 6,380,000 meters $\pm$. Therefore, the system may be translated along the Z-axis to a "secant" position. The amount of translation is a quantity slightly less than N so that all values of Z will remain positive. The X and Y values in Equation (45) do not change during the translation to this "secant" position.

These X, Y, Z-coordinates are in the form required for either the readjustment described in Section 3.7 or for the next Section 3.9 on Resection.

After the block adjustment has been completed (Section 3.10), the final adjusted X, Y, Z's need to be transformed back to latitudes, longitudes and elevations. (Conversion to State, Plane-Coordinates is accomplished by a separate computer program.)

The direct expansion of Equation (44), after applying the constant of translation to the Z-coordinates, is

$$X_G = X \cos\lambda_0 - Y \sin\phi_0 \sin\lambda_0 + Z \cos\phi_0 \sin\lambda_0$$
$$Y_G = - X \sin\lambda_0 - Y \sin\phi_0 \cos\lambda_0 + Z \cos\phi_0 \cos\lambda_0 \tag{46}$$
$$Z_G = + Y \cos\phi_0 + Z \sin\phi_0.$$

From Equation (43)

$$\tan\lambda = X_G/Y_G \tag{47}$$

from which the longitude is calculated. Also from (43) and (43a),

$$\tan\phi = Z_G/(X_G^2 + Y_G^2)^{1/2} \text{ approximately.} \tag{48}$$

By an iterative process involving two iterations, the second term of (43a) can be corrected to be consistent with the latitude of the point being transformed rather than based on the origin; then the Z of (43a) is

$$Z = (N + h) \sin\phi \tag{49}$$

and Equation (48) is adequate to compute an accurate latitutde. The elevation h may be computed from (43),

$$h = (Y_G/\cos\phi \sin\lambda) - N \quad \text{or}$$
$$h = (Y_G/\cos\phi \sin\lambda) - N, \tag{50}$$

using the equation with the larger function involving λ.

The computer program for the direct transformation may be modified to make the inverse become the similarity of operations and equations. The input and output of the two types of computation are merely reversed.

3.9 RESECTION

A solution of the resection problem is needed for each photograph as initial approximations for the block adjustment. However, this portion of the program is designed not only to yield the six parameters, but also to furnish the coefficients of the normal equations for the block adjustment, inasmuch as the routine is a relatively short one, and as this is a favorable stage for terminating one program and beginning the next one.

Resection in photogrammetry is defined as the determination of the six fundamental parameters, ω, ϕ, κ, X_0, Y_0, Z_0 of a single photograph from the given positions and elevations of at least three non-colinear points imaged on the photograph. Although most of these parameters were considered in the Section 3.5 on relative orientation, these elements were referred to different assumed and unrelated reference systems for each separate photograph. Now after Section 3.8, on each photograph are ordinarily available 18 points whose ground-coordinates are known with a fair degree of accuracy expressed in a single secant plane system for the entire block of photographs. Inasmuch as all large mistakes have been detected, and as the accuracy cannot be improved appreciably at this stage, a unique solution using only three points is applicable.

$$\begin{aligned} v_x &= p_{11} + p_{12}d\omega + p_{13}d\phi + p_{14}d\kappa - p_{15}dX_0 - p_{16}dY_0 - p_{17}dZ_0 \\ v_y &= p_{21} + p_{22}d\omega + p_{23}d\phi + p_{24}d\kappa - p_{25}dX_0 - p_{26}dY_0 - p_{27}dZ_0. \end{aligned} \tag{51}$$

These equations with the coefficients already defined by Equation (21) apply to the resection problem. The S and T terms of Equation (28) are not used because at this stage it can safely be assumed that dX, dY, dZ are insignificantly small. Thus by using an abbreviated form of the routine for relative orientation, six simultaneous linear observation equations can be formed for a photograph and solved in the same iterative manner for the unknowns as used before.

The initial approximations of the angular terms ω, ϕ, κ can again be zeros, but the linear terms X_0, Y_0, Z_0 are different. As an image near each principal-point is invariably selected and carried with the others through Section 3.8, the X, Y-coordinates of this point are satisfactory first approximations for X_0, Y_0. In the first photograph of a block, Z_0 is taken initially as the reported flight altitude. For all the other photographs, the finally iterated value of the previous photograph is used as the initial approximation.

Thus no new problem is involved in determining the coefficients of the six observation equations and solving them for the unknowns using the same routine as already used for Equation (28), (38) and (42).

But an important principle at this point facilitates the computation: *the coefficients of the observation equations on the last iteration of resection are exactly the ones that are also required in block adjustment!* It is necessary to include not only the three points used in resection but also all those wanted in the computation.

Then also it is not necessary to store the coefficients themselves of each observation equation, but as each equation is formed, its contribution to the normal equation system is readily computed and accumulated insofar as possible into a system of "normal" terms and "partial" normals. (However, the coefficients are punched out, one equation per card, for later use in determining the values of the residuals.) As a simple example, suppose the observation-equations had only three unknowns in the form

$$ax + by + cz = d. \tag{52}$$

Regardless of how many observation-equations there were, the normal equations would consist of exactly three equations of the same form and just 12 numbers. The first term of the first normal-equation would consist of what had accumulated previously plus the product aa; the second term consists of what had accumulated previously plus the product ab. In a similar way, all the terms of the normal-equation system are accumulated from the contributions of each observation-equation at a time, after which the equation can be "forgotten" and then the next one formed. Thus after each resection solution for a photograph, the output is not only the six parameters, but also the contributions to the normal-equation system for the block adjustment.

One of the principal parts of the resection solution is then a systematic method for numbering the normal-equation coefficients. This becomes somewhat involved in a pure, large block of photographs, but has been well organized in Triangulation Branch for the conventional geodetic problem.

3.10 BLOCK ADJUSTMENT

The normal-equations computed in the resection procedure are very similar to those which are developed in the adjustment of area triangulation. The matrix consists of a series of disconnected sets of three unknowns each (for the ground-points) and blocks of six-column data for the camera unknowns (Figure 8).

The normal equations are solved by the classical Gauss elimination with the Cholesky modification in order to reduce the computer storage requirement and to improve the numerical significance. These sets of equations are quite large but may be solved by direct methods without unreasonable effort.

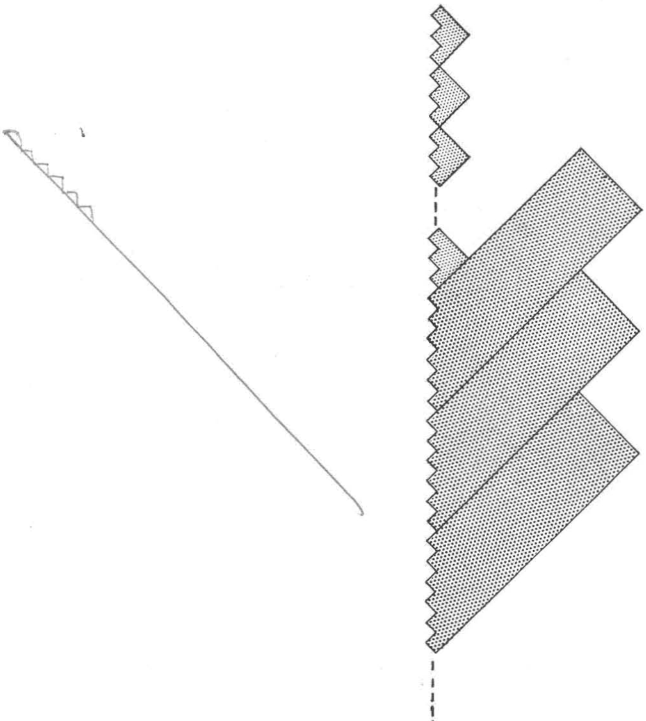

FIG. 8. Diagram of non-zero coefficients in the system of normal equations for the block adjustment.

An optimum number of pass-points should be used to avoid developing an unduly large network. The number of unknowns in the set is equal to six times the number of photographs plus three times the number of ground-points. For example, a project with 20 photographs and an average of 5 additional ground points per additional plate would produce a network of $(20\times 6)+(5\times 20\times 3)$ or 420 unknowns. Consequently it is planned to use only one of the two pass-points observed in each of the locations. If a side overlap of 60% is used in a block of photographs, then the number of ground-points need be only a few more than the number of photographs instead of five times the number.

The control-points may be entered as observation equations and added directly to the normal-equations prior to solution. If a point is a triangulation station without a known elevation, two equations $X=0$ and $Y=0$ are used. These may be weighted to suit the problem. For example, if a weight of 4 is used, 4^2 or 16 may be added to each respective diagonal term in the normal-equations prior to solution. Similarly, $Z=0$ may be used as the equation for elevation.

When control data are used in this manner, the X, Y, Z secant plane-coordinates consistent with the control (Section 3.8) should be used in Section 3.9 for developing the respective observation equations, rather than use the X, Y, Z coordinates for the same points which resulted from the cantilever adjustment and subsequent transformation.

4. Report of Results

As noted earlier, all productive results and tests completed prior to 1962 terminated with the adjustment of the cantilever strip (Section 3.7) whereas the block adjustment is only in the stage of preliminary testing. However, 25 strips of various lengths and different locations in the United States have been satisfactorily aerotriangulated in this manner for productive work, along-side of conventional methods using first-order plotting instruments.

The initial trial of this analytic aerotriangulation system [n] consisted of two strips of 17 photographs each of a test area 35 miles in length flown at an altitude of 20,000 feet with the RC-8 camera using cronar film base. Horizontal-control points and horizontal test-points were premarked but the vertical points were not premarked: The

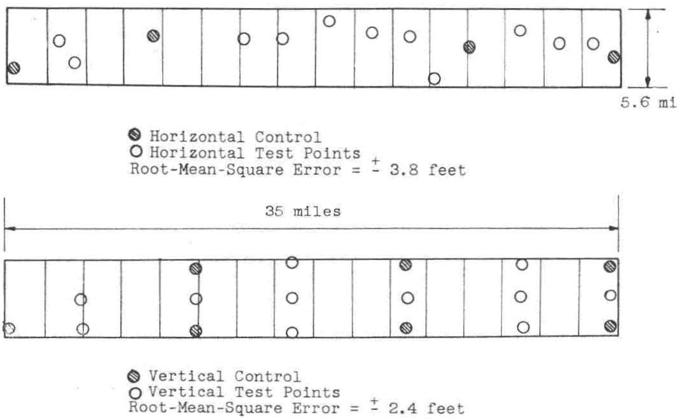

Fig. 9. Distribution of control points and test points in the test strip.

rms horizontal error was 3.2 feet (24μ; μ = microns) on one strip and 4.4 feet (33μ) on the other; the corresponding vertical errors were 1.6 and 3.1 feet (note that the scale of photography was 1:40,000). The horizontal errors are based on the use of 4 control-points and 11 additional test-points in each instance, and the vertical was based on 7 control and 11 additional test elevations. As indicated in Figure 9, the test points were situated in those places in the strip where the errors were expected to be maximum, namely, midway between control points. The *rms* value of the computed y-parallax for 600 images was 5μ for those points used in relative orientation (24 points per model in this test) and 7μ for other images. It may be noteworthy that the largest span between horizontal-control was 8 models (18 miles) and between vertical-control the largest span was 5 models (11 miles). It has since become evident that operator accuracies have improved appreciably with experience both in pass-point identification and coordinate measurement. Inasmuch as this test was the initial project, it is conceivable that the above accuracies might be improved if the work were to be repeated today as evidenced by the results obtained on more recent productive work.

On two productive jobs the abundance of existing control data allowed limited accuracy assessments. In both instances no control was premarked (substitute stations were used as described in Section 2.2), and the usual topographic film base was used in the RC-8 camera. In one instance the project involved four irregularly overlapping strips and 27 photographs taken as an altitude of 15,000 feet. A count on 288 consecutive pass-point images indicated an *rms* residual y-parallax of 3.0μ. In the strip adjustments the *rms* discrepancies in horizontal positions was 3.9 feet (39μ) based on 27 scattered control-points of which 11 were used in the adjustments. Eleven tie points between strips showed an *rms* deviation from the mean values of 2.4 feet (24μ). In some of the photography on this project taken at 12,500 feet, 6 more tie-points indicated a similar discrepancy of 1.4 feet (17μ).

In another instance in which the photography involving 30 pictures was taken at 5,000 feet, the corresponding horizontal error was 2.1 feet (63μ) where 43 control-points were tested of which only 9 were used in the adjustment. The *rms* deviation at 11 tie-points was 0.8 feet (24μ).

A test was conducted using the RC-8 and RC-9 cameras with topographic film over the Ohio Calibration Area in a grossly over-controlled short strip of 5 photographs (four models). Horizontal-control, vertical-control and test-points were all premarked. The RC-8 altitude was 10,000 feet (1:20,000 scale) and the RC-9 altitude was 6875 feet (1:23,800). In the RC-9 strip, the *rms* horizontal error was 1.2 feet (15μ) and the vertical error was 1.2 feet. For the RC-8 strip, the horizontal error was 1.1 feet (16μ) and the vertical was 1.0 feet. Eight horizontal and eight vertical-control points were used in the adjustments. Residual parallax for the RC-8 was about 5μ, and 8μ for the RC-9.

Several observations resulting from the tests and work to date may be of interest. In the adjustment of cantilever strips (Section 3.7), all three of the primary corrections [11, 12] were significantly smaller than those experienced with conventional plotters, *with the azimuth correction approaching non-existence.* Discrepancies for all components are "better behaved" and less erratic than formerly experienced. Vertical discrepancies are generally smaller than horizontal ones contrary to the geometry of the model, but this may be partly explained by the better relative distribution of control in the strip: minimum vertical-control consists of 7 distinct elevations; horizontal, only 4 distinct positions. Discrepancies at tie-points between strips are generally smaller than at control-stations possibly indicating that substitute control-points, which are selected in the field with the aid of a pocket stereoscope, cannot be as accurately identified as are tie-points, which are selected in the office with the aid of the PUG-2.

A large spread exists between the residual parallax value of about 5μ and ground discrepancies varying from 15 to 63 μ. The smaller ground values accrue from the premarked, over-determined tests, and the larger ones from unmarked control and during productive work. It is estimated that 15 to 25μ derive largely from the residual film-distortion that escapes correction, and that further errors are due to uncertain control-point identification in the absence of premarking. This estimate is supported by the fact that 22 tie-points in 67 photographs had a discrepancy of 24μ and 6 other tie-points in 10 photographs had 17μ. A further source of the spread might be due to the nonconformity of the quasi-empirical equations for the adjustment of strips given in Section 3.7. This latter spread source may, or may not, be reduced later during block adjustment through an increase in the residual parallax values.

Based on the above results, some of which were unexpected, one is naturally cautious about predicting what benefit will be derived when the block adjustment is applied. Although it seems evident that ground errors may be decreased somewhat and parallaxes may be increased, the degree of improvement is scarcely conjecturable, pointing to the necessity of repeated tests and experience in order to derive a practical answer. However, it can be stated that, at the present stage of development, the errors are only about one-third or one-fourth as large as those being experienced with the conventional plotting instruments. Future development may consist of economic aspects such as attempting to discover where the law of diminishing returns applies with regard to (1) block adjustment, (2) control of film distortion, (3) density of ground control, (4) premarking, and (5) automation.

NOTE. It is anticipated that requests may be forthcoming for copies of the computer programs. It may not be advisable to comply with these requests for some time, for several reasons: (1) These programs have grown "like Topsy" and have not been fully documented; (2) The programs are for the simplest basic IBM-650 computer and are in computer language; (3) Experience has indicated that these programs are quite difficult for others to use as it is almost impossible to describe their contents fully; and (4) a definite effort has been made throughout this presentation to express all the formulation with sufficient-thoroughness so that it can be readily programmed in any one of the several program translation notations.

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*Tests of Radar Doppler as a Tri-Lateration Device**

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DR. FRANCE BERGER, Director of Research & Planning of General Precision Laboratory, Inc., describes in Volume II of his Technical Series a simple Doppler Velocity Measuring System. "An antenna directs a microwave beam to the ground ahead of the aircraft. The antenna is fed by a micro-wave radio frequency source. The echo power, through the use of suitable duplexers is directed to a crystal mixer. Some power from the (*cw*) source is also fed to the mixer. The peak frequency between the transmitter signal and the Doppler shifted echo is amplified. After suitable amplification, the frequency signal is measured."

Aero Service Corporation has used Doppler radar in the conducting of large block-flying missions for several years. Our original purpose in employing Doppler and in becoming the first commercial mapping organization to utilize it was to insure the proper forward overlap, by triggering the camera through a slaved-oscillator, and to insure the proper side-lap by utilizing the associated coordinated computer. Block photography over featureless terrain has now become sim-

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