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NOTE. It is anticipated that requests may be forthcoming for copies of the computer programs. It may not be advisable to comply with these requests for some time, for several reasons: (l) These programs have grown "like Topsy" and have not been fully documented; (2) The programs are for the simplest basic IBM-650 computer and are in computer language; (3) Experience has indicated that these programs are quite difficult for others to use as it is almost impossible to describe their contents fully; and (4) <sup>a</sup> definite effort has been made throughout this presentation to express all the formulation with sufficientthoroughness so that it can be readily programmed in any one of the several program translation notations.

## **REFERENCES**

- [a] O. von Gruber, Photogrammetry, Collected Lectures and Essays, Chapman & Hall, p. 13, 1932.
- [b] Schmid, Dr. Hellmut H., "An Improvement in Accuracy of the Orientation of a Photogrammetric Camera by Means of Condition Equations," PHOTOGRAMMETRIC ENGINEERING, Vol. XVlJl, no. 5,
- p. 836, 1952. [c] Tewinkel, G. c., "Film Distortion Compensation for Photogrammetric Use," *Coast and Geodetic Survey Technical Bulletin No.* 14, Washington 25, D. c., September 1960.
- [d] Washer, Frances E., "Prism Effect, Camera Tipping and Tangential Distortion," PHOTOGRAMMETRIC ENGINEERING, Vol. XXIII, no. 4, p. 721, 1957.
- [e] Manual of Photogrammetry, American Society of Photogrammetry, second edition, page 321, 1952. [fl Leijonhufvud, Axel, "On Astronomic, Photogrammetric and Trigonometric Refraction," *Kungl,*
- *Boktryckeriet* P. A. Norstadt & Soner, Stockholm, 1950.
- [g] Rosenfield, George H., "The Problem of Exterior Orientation in Photogrammetry," PHOTOGRAMMETRIC ENGINEERING, Vol. XXV, no. 4, (2.7), p. 544, 1959.<br>[h] Tewinkel, G. C., "Mathematical Basis for Analytic Aerotriangulation
- (forthcoming).
- [i] Eshbach, Ovid \1,1., Handbook of "Engineering Fundamentals, John Wiley & Sons, Inc., p. 2-16, Second Edition, 1957.
- [j] Schmid, Dr. Hellmut H., "An Analytical Treatment of the Problem of Triangulation by Stereophotogrammetry," Report No. 961, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, p. 14, October, 1955.<br>[k] Harris, William D., "Aerotriangulation Adjustment of Instrument Data by Conventional Methods,
- 
- *Technical Bulletin* No. 1, Coast and Geodetic Survey, Washington 25, D. C., January 1958.<br>[I] ---, Vertical Adjustment of Instrument Aerotriangulation by Computational Methods, *Technical Bulletin* No. 10, September 1959. [m] Schmid, Erwin, "Transformation of Rectangular Space Coordinates," *Technical Bulletin* No. 15,
- 
- Coast and Geodetic Survey, December 1960. [n] Swanson, Capt. L. W., "Report on Analytic Aerotriangulation," Third United Nations Regional Cartographic Conference for Asia and the Far East, Bangkok, Thailand, October 1961.

## *Tests Of Radar Doppler as a Tri-Lateration Device\**

D R. FRANCE BERGER, Director of Research & Planning of General Precision Laboratory, Inc., describes in Volume II of his Technical Series a simple Doppler Velocity Measuring System. "An antenna directs a microwave beam to the ground ahead of the aircraft. The antenna is fed by a micro-wave radio frequency source. The echo power, through the use of suitable duplexers is directed to a crystal mixer. Some power from the  $(cw)$  source is also fed to the mixer. The peak frequency between the transmitter signal and the Doppler shifted echo is ampli-

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fied. After suitable amplification, the frequency signal is measured."

Aero Service Corporation has used Doppler radar in the conducting of large block-flying missions for several years. Our original purpose in employing Doppler and in becoming the first commercial mapping organization to utilize it was to insure the proper forward overlap, by triggering the camera through a slaved-oscillator, and to insure the proper side-lap by utilizing the associated coordinated computer. Block photography over featureless terrain has now become sim-

\* Presented at the 1961 Semi-annual Meeting of the American Society of Photogrammetry.

plified by punching in the desired coordinates it is desired to reach and, upon the return flight, offsetting the one coordinate in the computer by the desired amount. There has been successfully flown 50,000 line miles of Doppler controlled photography in Libya, 10,000 miles over Morocco and the Spanish Sahara and an excess of 60,000 miles in the jungled-covered country of Surinam.

Aero's planned system is made up of a Doppler sensor, a compass, a course and distance computer, a differential height-sensor and an isobaric-slope data-recorder. The Doppler sensor derives ground speed and drift angle, and the direction of the velocity vector relative to aircraft coordinates. The velocity vector is related to earth coordinates by combining heading information from the compass with the drift angle. This provides track of the aircraft, which is then compared with the desired course. When these are equal, the velocity vector lies along the lines selected and is integrated as distance by the course distance computer. If the track and desired course are not equal, the velocity vector is resolved through this angle and the components along and across the track. The acrosstrack component is integrated to give displacement right or left. The along-track component, when integrated, provides distance. Ground speed is provided as a frequency analog output or cycles-per-second representing distance per unit time. Hence a number of cycles present distance at which time a triggering pulse is produced for a fiducial marking on any type of recording device for strip film or a triggering device for a shutter camera. **In** addition to the Doppler Sensor, the course and distance computer, the compass, the preset counter, the pilot's direction indicator and the true air speed transmitter, Aero proposes to include a Delta-Height Recorder or a device to measure the slant range of the Doppler beams and to compute and record the elevation differences between each aerial exposure.

Specifically, the attempt will be to derive from a Doppler controlled flight:

- 1. The azimuth of the flight-line.
- 2. The nadir-point coordinates of each exposure station.
- 3. The slope characteristic of each flightline.
- 4. The incremental lens-height values for each exposure.

This investigation, if successful, should lead to the determination of new mapping and instrumentation techniques, such as a least square adjustment for each exposure station, individual stereo models rectified by distances, rather than by elevations, a secondary-control system established by the flight-line intersection method, a cantilever control system of high reliability.

An evaluation of Doppler was reported by Hans Meier in a 1959 issue of PHOTOGRAM-METRIC ENGINEERING<sup>\*</sup> wherein a consecutive number of photos was triggered over a prepanelled railroad course. The purpose was to find, after calibration, the degree of consistency with which the Doppler would fire a vertical camera. The photo nadir-coordinates were determined in the A-7 autograph. The exposure intervals over the course were proven to be consistent with each other to within  $+6$  feet. This nadir-base determination has an important potential for use in a mapping system which will minimize or perhaps eliminate the necessity for expensive ground-control in inaccessible areas. Using air-base measurements of this accuracy, a series of reiterative attempts by the stereo operator with the base fixed, and the Delta Z of each model locked, the tilt in the flight direction can be determined with  $\pm$  onehundredth of a grad or  $\pm 32$  seconds of arc.

**In** addition to this effort, studies are being conducted to determine if the Doppler could contribute to the determination of secondarycontrol points by using the Doppler count from the slaved-oscillator. Flown were a quadrilateral, the diagonals and a cross flow through this area. This test was only meant to be an indication test. Portions of the plan are still being conducted. Also flown was a small configuration over eight 1: 24,000 scale quads near Philadelphia Figure 2. The total mileage flown was about 400 miles. The legs of the perimeter of the rectangle are 56,000 feet and 80,000 feet. The longest diagonal is approximately 100,000 feet. Fifteen positions were determined by the following procedure:

- 1. Six positions were selected on the perimeter of the proposed rectangle
- 2. A geodetic-azimuth between all stations was computed by inverse computations between the scaled positions of each point. A total of 15 lines was computed.
- 3. An average magnetic declination of  $9\frac{1}{2}^{\circ}$ was applied to each computed azimuth.
- 4. The Doppler interval was set at 500'.
- 5. The flight altitude was approximately 1,700'.
- 6. The magnetic-azimuth was programmed into the Doppler and the
- \* Vol XXV, no. 4, p. 632 (Sept. 1959).

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FIG. J. Proposed Positional System.

flights were conducted by Doppler navigation.

- 7. The initial approach to each line was manually controlled by the pilot. This was done by observing the selected ground-point and flying directly over it.
- 8. A 35 mm. recording camera was synchronized to the Doppler tone wheel which triggered the camera at 500' intervals.
- 9. The principal-points of the 35 mm. frames at the common intersection of all flight lines were transferred to  $1\frac{1}{2}$ diameter prints of previously existing photography.
- 10. The mean scale of each plot was determined by dividing the scale distance of each line segment into the Doppler measured distance.
- 11. A straight line was fitted to each line segment on each photograph.
- 12. An image-point was then selected on each photograph.
- 13. A perpendicular was dropped from each selected image-point to each line segment.
- 14. The distance from three principalpoints to the selected image-point was measured along each fitted straight

line segment. This was done for each line.

- 15. The perpendicular distance from the perpendicular to the selected imagepoint was measured.
- 16. The scaled distances were converted to feet by multiplying each scale distance by the appropriate scale factor determined previously.
- 17. The Doppler distance between each pair of image-points was computed from the total number of intervals between points and the scaled increments of each end of the line.
- 18. The distance between each point was computed as described using three different points at each end of the line thus providing nine determinations of each distance. The mean of the nine determinations was used as the final Doppler distance. A free adjustment was then made by the variation of coordinates method.
- 19. The adjusted Doppler distances were then compared to the inverse distance between scaled positions.

Preliminary results of the recent tri-Iateration test indicate a probable error  $\pm 39$  feet,



**Elmer Newfield**

| Line    | Observed dist. | Corr.    | Adjusted dist. | Scaled dist. | $S - A$ |
|---------|----------------|----------|----------------|--------------|---------|
|         | feet           | feet     | feet           | feet         | feet    |
| $1 - 2$ | 35351.9        | $\Omega$ | 35351.9        | 35328.7      | $-23.2$ |
| $1 - 3$ | 75723.1        | $+3.6$   | 75726.7        | 75675.5      | $-51.2$ |
| $1 - 4$ | 88687.8        | $-7.2$   | 88680.6        | 88619.6      | $-61.0$ |
| $1 - 5$ | 58672.2        | $+48.6$  | 58720.8        | 58654.6      | $-66.2$ |
| $1 - 6$ | 56297.9        | $-13.1$  | 56284.8        | 56258.0      | $-26.8$ |
| $2 - 3$ | 40505.4        | $-12.1$  | 40493.3        | 40468.9      | $-24.4$ |
| $2 - 4$ | 62173.3        | $-6.2$   | 62167.1        | 62162.2      | $-4.9$  |
| $2 - 5$ | 54048.7        | $+38.1$  | 54086.8        | 54062.2      | $-24.6$ |
| $2 - 6$ | 69674.2        | $-38.7$  | 69635.5        | 69678.1      | $+37.6$ |
| $3 - 4$ | 43987.9        | $+9.8$   | 43997.7        | 43993.6      | $-4.1$  |
| $3 - 5$ | 70154.6        | $-67.6$  | 70087.0        | 70025.9      | $-61.1$ |
| $3 - 6$ | 96465.9        | $+67.0$  | 96532.9        | 96544.7      | $+11.8$ |
| $4 - 5$ | 49365.0        | $+22.3$  | 49387.3        | 49338.2      | $-49.1$ |
| $4 - 6$ | 80700.2        | $-12.1$  | 80688.1        | 80703.7      | $+15.6$ |
| $5 - 6$ | 31335.0        | $-34.5$  | 31300.5        | 31365.5      | $+65.0$ |

FIG. 2. Tests of Radar Doppler as a Tri-Lateration Device over eight 1 :24,000- scale quadrangles near Philadelphia, Pa.

the average line length being 60,834 feet. The proportional part error is less than 1: 1,500 which exceeds the manufacturer's specifications. The probable error stated above refers to the precision of the adjusted Doppler distances, or in other words, reflects the internal agreement of the unadjusted Doppler measurements. It does not indicate the accuracy of the measurements.

The determination of the systematic error inherent in the Doppler system is difficult to

evaluate over a limited test area. The necessary test area should be of a considerably greater magnitude than was used in the New Jersey test. Also-any systematic error determined could be strengthened if several known positions are included in the final net adjustment. The validity of the systematic error can be determined by continuing the forward solution of a least square adjustment and solving for the probable error of the systematic error.