PANEL
COMPUTATIONAL
PHOTOGRAMMETRY*

MODERATOR
Sumner B. Irish

PARTICIPANTS

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Introduction

SUMNER B. IRISH
Moderator

THE purpose of this panel is to explore the role of the electronic computer in photogrammetry. The wide spread availability and use of this tool is rapidly changing the complexion of photogrammetry. It is hoped that as a result of this panel, a technical committee of the American Society of Photogrammetry will be constituted to continue the exploration of the applicability of computational techniques to the photogrammetric problem, and to point out the areas in which further research is in order.

This group is heavily indebted to Mr. Leonard I. Sherry of the Fairchild Camera and Instrument Corporation, Syosset, L. I., who sparked the original presentation of this material at the 1961 Semi-Annual Meeting of the Society in New York City, and who has been largely responsible for the complexion and direction of this panel. The panel membership has been chosen to represent both industry and government, defense and non-defense activites, in an effort to give it as broad a base as possible and to show the wide-ranging significance of this subject.

SUMNER B. IRISH

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The Computational Relationship of Geodesy to Photogrammetry

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ABSTRACT: *Photogrammetry and geodesy are very closely related. Extensive experience in adjusting geodetic surveys can be applied to analytical aero-triangulation problems. Past geodetic experience in computing procedures and in adjustment distortions can be related directly to photogrammetric situations.*

GEODESY and Photogrammetry are so closely related that when a person has spent thirty years as a geodesist he is appreciative of the mathematical problems relating to Photogrammetry. As the first speaker on this panel I was asked to present a few basic thoughts showing the relationship of methods used in the adjustment of geodetic surveys to the methods used in analytical aerotriangulation. It is not necessary to present details of mathematical techniques used in adjusting geodetic networks but, rather, I wish to emphasize a few points of common interest to geodesists and photogrammetrists.

CHARLES A. WHITTEN

There are two types of equations which may be used in adjusting triangulation, trilateration or traverse. The final results are identical, irrespective of the method used. If the computing equipment is limited to desk calculators or small electronic devices, condition equations should be used. The term "condition equation" should not be confused with an "observation equation" which has been given infinite weight. A condition equation in triangulation is one which refers to the geometrical properties of the figure, and each equation will contain as many residuals or corrections as are required for the particular condition. If medium-sized or larger electronic calculators are available, observation equations based on the concept of variation of coordinates should be used.

In either of these methods I want to emphasize the necessity for giving careful attention to the size of the numbers or what might be called "computer scale." It is the basic problem involving significance, number of digits, placement of decimal point, etc. When working with surveying problems involving matrices, relative weights, errors and residual corrections, the use of fixed-point arithmetic is preferred to any floating-point technique. The unknowns in the equations and units in the absolute terms of the equation should be defined in such a way that the coefficients of the unknowns and the absolute terms as used are never larger than 10 and preferably smaller than 3. Neither should they be too small. Trouble will be encountered if very many terms are less than unity. The number of decimal places to be used may vary but if coefficients are correct to three or four decimal places normal equations with six decimal places will be used.

Another point of interest involves the solution of large sets of matrices. By large, I mean several hundred to even thousands of unknowns. We have found that the Cholesky modification of the classical Gaussian elimination offers several advantages in geodetic and photogrammetric work. André Louis Cholesky, a colonel in the French Artillery, during World War I, proposed a method sometimes referred to as a square root method. When using desk calculators his method seems a bit awkward but with digital computers there are several advantages. Even though it is essential that the leading or diagonal term in a matrix generally be the dominant term, it is also essential that this term be neither too large nor too small. When we take a square root the term is reduced in size. If the diagonal term is small and by some chance is less than unity, taking the square root increases its size. In both cases round-off errors and accumulation of these errors are kept to a minimum.

Another significant fact is that only one term is required in a reduced equation whereas in the classical Gauss-Doolittle (method) two terms are needed for further computation. This cutting of the storage requirements of a computer to one half actually doubles the size of the computer in its application to matrix solutions.

The final point I wish to bring to you involves distortion in adjustment of arcs or nets. If a north-south arc of triangulation has a closure in latitude so that the arc needs to be lengthened, it is not stretched like a rubber band but, rather, it is blown up like an elongated balloon. If the position closure is 1/50,000 and the length closure essentially zero, the adjustment processes will increase the lengths in the middle of the arc by some amount appreciably greater than 1/50,000 and in some instances approaching 1/25,000. If the arc is long and a base line inserted in the middle, and again the length checks essentially zero, the adjustment distorts with a double balloon action, and the lengths between the fixed ends and the base line will be seriously distorted.

In recent years we have adopted the general practice of giving less weight to measured bases and placing some correction on the base line so that the triangulation itself would not be subjected to this distortion.

If the closure on a north-south arc is in longitude the adjustment does not turn the arc as though the ends were on hinges. Rather, it twists it in an *S* shape. Thus the azimuths in the middle of the arc are distorted and twisted by an amount almost twice that required if a hinge action had taken place.

These distortions are the result of the use of least squares. This is a useful tool for making observational data mathematically consistent and yet a rather dangerous instrument when used indiscriminately. It has been our experience that area networks of triangulation have greater strength than arcs.

All of these problems relating to distortion and strength of adjustment are common to those in adjusting photogrammetry whether the adjustment be by strip or blocks. We have much in common; the geodesists have a long history of mathematical experience that is available to the photogrammetrists in their present development of analytical aerotriangulation. I am glad to have the opportunity of cooperating with my colleagues in this new endeavor.

*The Case for First-Order Photogrammetry**

THOMAS M. SCHAFER,
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Lansing, Mich.

(*Abstract is on next page*)

THE modern day enthusiastic acceptance of the processes of photogrammetry point up the need for continuing development to keep pace with ever-increasing sophisticated engineering.

Although an individual engineer may not be too familiar with our techniques, he is usually considering their application in new and imaginative ways. This further stimulates our research and progress.

A careful look by the engineer would reveal the present-day development of a new mapping system, immediately applicable to everyday engineering problems, and offering limitless benefits for our combined future efforts. The system is formulated through the exclusive use of first-order, heavy-type plotting instruments, teamed with distortion-free

* At the 1962 Annual Meeting, this was presented as a separate paper. It was included as a part of the Panel at the request of Moderator Irish.

photography and modern ground survey operations; this system is suitably termed "First-Order Photogrammetry."

In keeping with today's emphasis on progress, the term "First-Order Photogrammetry" connotes the most modern, the most elaborate and the most advanced method of compiling maps. To the map maker and the map user, however, it has a far more serious and significant meaning. The first-order system, when properly applied, can mean less expensive and more accurate maps, greater latitude in project planning and several excitingly new applications of our science of photogrammetry.

The most vital component of the system is the first-order, heavy-type plotting instrument of the type costing $30,000 or more per unit. There is nothing very new about the design and manufacture of this equipment, of which several excellent models have been available for some years. The new aspect concerns the application of the equipment.

It has been common practice for both commercial and governmental agencies to utilize the first-order, heavy-type instruments for stereo triangulation, and occasionally for some types of photogrammetric compilation.

However, the exclusive use of this equipment for mapping of all types necessarily means the implementation of considerably different techniques and supporting equipment, with the resulting new approach to project planning, prosecution and progress. The technical ramifications of first-order photogrammetric systems require the solution of several significant problems in order to effect a successful application of the system to every-day mapping operations. These same technical factors enable the realization of a multitude of advantages for the agency which can solve the problems.

The implementation of the system means a considerable investment, and this must be justified. This justification is found in the rapidly expanding requirement for unusually high map accuracies, coupled with the real need for the application of our science in fields not necessarily related to aerial mapping. These requirements exist today, and they will become more pressing in the future.

Very often, we are being asked to prepare site maps as large as 1/1,200 in horizontal scale, with equally detailed contour information. There is a need for our services in the fields of traffic studies, crime detection, medicine and architecture. We are being given a glimpse of the tremendous mapping work which will be needed to support exploration and research in the Space Age. These many sign posts of the technically challenging future confronting our science of photogrammetry should indicate to us the need for continuing development in both equipment and techniques. The first-order photogrammetric system represents such development to a significant degree.

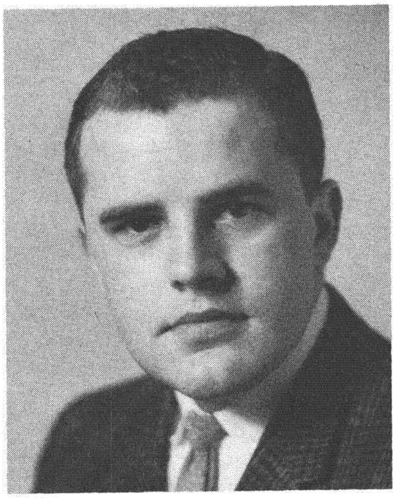
It is my intention to discuss the advantages of the first-order photogrammetric system, and how it can affect this business of mapping. I do not intend to imply that other techniques, or other plotting instruments of any type, classification, or price range, are in any way unsatisfactory.

It is natural for mapping equipment and systems to be judged on the basis of orders of accuracy. The question of such orders of accuracy of respective stereo compilation instruments has been the subject of considerable discussion and varied conclusions. There is little or no doubt of the mechanical and technical capabilities of any one individual instrument; it is when all of the various types are classified with respect to general orders of accuracy that conclusions become as numerous as the experts who make them.

Although accuracy is a very important consideration, there are several other facets to this matter of judging plotting instruments. The type of work for which the equipment is intended becomes most important, as well as the lesser considerations such as instrument versatility and operator comfort.

We can assert with a reasonable degree of certainty that there are several advantages to the system of first-order photogrammetry

ABSTRACT: *This paper reviews the technical and economical advantages of first-order, heavy-type plotting instruments. It relates the development of first-order photogrammetric systems for commercial survey organizations, including reference to advanced photographic and control survey techniques. The paper points out the need for continued development of systems and techniques in view of constantly increasing requirements.*



Thomas M. Schafer

wherein the advanced plotting equipment is teamed with high-quality distortion-free photography and modern ground-survey equipment and techniques. Most of the factors which make such a system desirable are centered around the stereo compilation instrument and include the following:

1. Increased photo-scale to map-scale enlargement or reduction capabilities.
2. Significantly greater *C*-factor ranges.
3. Better detail identification made possible by the increased enlargement of the viewing area through the refined optical system.

These factors are reflected in technical or economical advantages throughout the survey operation.

As an example, much more latitude is possible in the planning and prosecution of flight operations. The restrictions on photo-scale and flight altitude, necessitated by the enlargement capabilities and *C*-factor ratings of other types of instruments are not necessarily applicable to first-order, heavy-type equipment. To be sure, these instruments do have restrictions in these respects, but they are considerably less than those of other types. The problem of detail identification, and of accurately plotting that detail, is considerably lessened due to the improved optical system. Here again, there are limits which must be recognized. Actually, flight altitudes are usually more restricted by consideration for this requirement than by mechanical capabilities of the instruments.

A far greater selection of aerial cameras is usually possible, thanks to the increased focal-length of the plotting instrument. Certainly, a 6″ focal-length camera is the usual standard and is used in the clear majority of photogrammetric projects. Occasionally, it is desirable to utilize photography from different focal-length cameras, especially when using existing photography which may have been flown without consideration to any requirement for photogrammetric mapping. On some first-order, heavy-type equipment, focal-length ranges of from 98 mm. to 230 mm. permit mapping without the added inconvenience of replacing any part of the projection equipment to accommodate other than 6″ focal-length cameras.

There is another area in flight operations that is sometimes a serious consideration and thus deserves some attention. Suppose we are confronted with a need for compiling an extremely large-scale map of a proposed urban expressway. Let us assume the desired map-scale would be 1/240. Immediately, we are faced with a problem of FAA minimum altitude restrictions in urban areas, usually 1,200 feet. Our problem is solved if we can use a first-order, heavy-type instrument with an enlargement capability of 10 or 12 diameters and *C*-factor capabilities in the neighborhood of 2,500. We can fly at or above the FAA established minimum altitudes and still compile our map at the desired large scale, thanks to the capabilities of our equipment.

Probably the most striking case for first-order photogrammetry is found in the type of work which is most common to the commercial survey organizations, that of preparing maps at scales ranging from 1/4,800 to 1/480. For instance, a mapping program at 1/1,200 scale would mean that some agencies would have to utilize photography at 1/6,000 while a first-order photogrammetric system could accommodate photographic scales of 1/9,600 or even 1/12,000. At first glance, this might not appear to be very startling. From this fact, however, emanates the most significant advantages in terms of money that can be attributed to first-order photogrammetry.

Speaking in terms of a square mile of stereoscopic coverage, when a photo-scale is reduced by $\frac{1}{2}$ the reduction in the number of aerial negatives is quadrupled. A 1/12,000 photo-scale means four times less negatives than the 1/6,000 photoscale. This can mean lower initial photographic cost and lower film processing cost. It will probably not mean, however, a corresponding lower cost in stereo compilation, although it should be somewhat less.

Because the ground-control survey plan is based on the photographic pattern except when special circumstances or requirements are involved, there will be a sizable reduction in the amount of both horizontal and vertical-control needed to support photogrammetric compilation. The wider coverage of a higher altitude negative permits more latitude in control planning, making it somewhat easier to place control lines along roads and other easily accessible routes.

The first-order photogrammetric system readily accommodates the use of techniques aimed at supplementing basic horizontal and vertical controls, especially through the use of stereo triangulation.

Briefly stated, this is a recognized system of coordinate reading and control point stationing employing basic high-quality ground-control, the use of first-order stereo plotting equipment and electronic data reduction processes. The supplemental control thus established is in every respect as accurate and dependable as field-control, and offers the further advantage of guaranteeing immediate isolation of any systematic or procedural errors. This technique has a great many applications in commercial and governmental mapping operations. It is consistently used as a means of supplementing basic horizontal and vertical-control needed to support the accuracy of both large and small-scale photogrammetric maps and photographic mosaics. On a project of sufficient scope to warrant the use of stero triangulation, it is possible to realize significant savings of both time and money through the use of this technique.

Once the ground-survey net has been established and the photogrammetric map completed, the first-order system can minimize the requirement for extensive field annotation or field checks on the accuracy and completeness of the map. It cannot be denied that the advantages of a highly refined optical system enabling better detail identification, teamed with significantly increased enlargement of a model area viewed by the instrument operator will, in many cases, completely preclude the necessity of field editing the map.

I have pointed out only some of the technical advantages of a first-order photogrammetric system. Of course, these technical advantages are translated into several other equally important benefits such as the savings of both time and money in the actual mapping operation. Application of the system to related fields such as industrial and medical photogrammetry will result in the realization of equally significant advantages, limited only by our own ingenuity in applying the system to the various problems.

It is understood that completely first-order photogrammetry will mean high cost both in procuring equipment and in training personnel. But it will also mean a considerable change in flight operations, ground-control layout, and indeed, the formulation of a mapping program, because the stereo compilation instrument is the heart of the photogrammetric system and all other components, including supporting equipment and techniques are, to varying degrees, dependent upon the characteristics and capabilities of the plotting instrument.

Many people will correctly point out that, while such a step forward may be desirable, such advances must be tempered with a practical business approach. This is very true, but it should not restrict our consideration of a better way to conscientiously serve the people who look to us for the information they will use as the basis for planning and engineering. Necessity is the Mother of Invention, but it is also the inspiration for progress. Indeed, the necessity for better accuracy in mapping has not decreased; it has gone far in the other direction. This expanding requirement is the impetus for applying the technically improved equipment and methods now available.

Status of Computational Photogrammetry at the Air Force Missile Test Center

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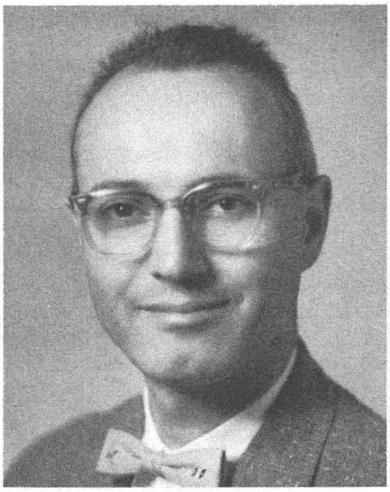
(*Abstract is on the next page*)

I HAVE been asked to present a progress report on the activities to date of the Air Force Missile Test Center in the field of Computational (Analytical) Photogrammetry. Also requested is that I mention the problems not yet solved and the difficulties



GEORGE H. ROSENFIELD

likely to occur in the near future. For accomplishing this mission less than 10 minutes has been allotted.

The present goals at the RCA Missile Test Project in the Data Processing of Photogrammetric Records is to expedite the development and production use of automatic editing techniques and of automatic single-pass computer reduction routines. In keeping with these goals, the following Data Reduction system is in effect for each of the major metric optical data systems.

The Cine Theodolite system data reduction technique presently utilizes semi-automatic, visual reading of the film records, with automatic readout from the Telecomputing theodolite film reader. A single-pass computer reduction system is in use for the production of theodolite position data.

The Fixed Camera system data reduction technique presently utilizes semi-automatic, visual reading of the film records, with automatic readout from the Gaertner microscope comparator and from the Coleman powered comparator. The system is rapidly approaching the capability of a single-pass computer reduction with a readout option after the camera orientation routine.

The Ballistic Camera system data reduction technique presently utilizes semi-automatic, visual reading of the plate records with automatic readout from the Mann comparator to 1 micron, and from the Wild stereocomparator to 2 microns. By semi-automatic readout (keyboard entry of the additional digits), tape recording to 0.1 micron is available from all precision comparators. A single-pass computer reduction capability is available using series run techniques.

Forthcoming developments are presently under procurement, with expected delivery within the next year. These include: a high-speed Cine Theodolite film reader with automatic readout of the azimuth and elevation dial recordings. A high-speed CZR film reader with automatic reading of the fiducial images and semi-automatic reading of the data image points. The equipment also includes automatic pick-off of the timing records. Modification of the Camera equipment to allow the automatic timing is in progress.

An automatic high-accuracy comparator has been under development for reduction of the Ballistic Camera plates. The original concept included a low-accuracy visual-reading plate previewer; as well as a tape drive, automatic scanning, high-accuracy interferometer comparator. The original concept has been modified for immediately converting the plate previewer to a 1 micron comparator with a Ferranti fringe counting system, and for eventually converting the interferometer comparator to a laboratory instrument for calibrating grids and other high-accuracy study and calibration operations.

ABSTRACT: *Three data reduction techniques are mentioned. Forthcoming developments are also described. The need for 2 micron total error is developed, along with the probable major equipment considerations included, and the difficulties which are likely to occur.*

One major problem falls within the "Not Yet Solved" category: Accuracy, Accuracy, Accuracy!!!—especially for the Ballistic Camera System. A major advantage of Analytical Photogrammetry is that it allows correction of all error sources to the state-of-the-art of human knowledge. The problem now is that operations are not quite up to the state-of-the-art, and in some cases the state-of-the-art does not go far enough. Requirements already exist for accuracies of the order of 2 parts per million. This corresponds to a total error on the plate of 2 microns with a focal-length of 1,000 millimeters.

Major items for consideration to achieve this 2 micron total error on the plate include:

1. Camera evaluation and calibration.
2. True geometric position of the latent image.
3. Stabilization and calibration of the emulsion and base.
4. Vibrations introduced by shutter operation.
5. Stability of the pedestal and mount.

Difficulties which are likely to occur in the present and proposed systems are:

1. Problems resulting from long focal-length lenses and wide aperture.
2. Problems resulting from non-diffraction limited optics.
3. Poorly formed images from point sources of light. These include problems of secondary spectrum effects and residual color aberration, as well as problems in determining the standard deviation of the spread function and applying this concept in the design and evaluation of the optical system.
4. Problems caused by atomospheric shimmer.
5. Errors introduced by atmospheric refraction.

*Report on U.S.G.S. System of Analytical Aerotriangulation**

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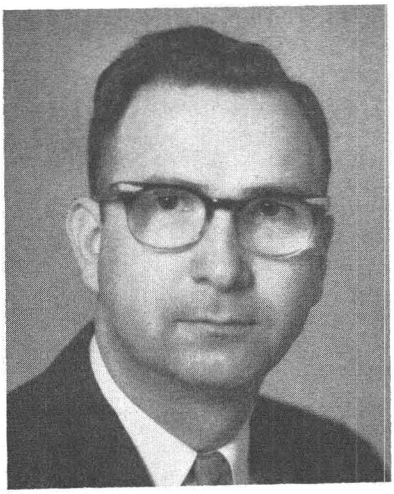
ABSTRACT: *The Direct Geodetic Restraint method is discussed as to type of problem handled, and as to the results which have been obtained theoretically and practically. Modifications recently developed by M.I.T. will be incorporated into the U.S.G.S. system.*

IN JANUARY 1957 the Topographic Division of the U. S. Geological Survey initiated research with the objective of developing an analytical system of aerotriangulation based on a method derived by Dr. Paul Herget. Mr. Hugh F. Dodge of our organization was selected to be a member of a research team at Cornell University. This was organized to develop a solution to the general analytical aerotriangulation problem under a U. S. Army Engineer Research and Development Laboratories contract. Subsequently Messrs. Dodge, Robert C. Eller, and David S. Handwerker continued the development at the Geological Survey. Important modifications were made in the Cornell approach and the revised system was named "The Direct Geodetic Restraint Method."



MORRIS L. MCKENZIE

Problems consisting of single models, single strips, or blocks of models can be solved with the system. The method in general consists of analytically accomplishing simultaneous relative and absolute orientations of photographs using the directed rays emanating from the perspective centers of the photographs and passing through image points. The primary objective is to arrive at the most probable orientations and positions in space of the several camera stations through an iterative process and from these to compute the network points. Estimates for the angular orien-

* Approved for publication by the Director, U. S. Geological Survey.

tation and position elements of the camera stations are used in the initial iteration. Corrections to these elements are determined and applied after each iteration until a solution is reached. Three iterations are usually required.

After establishing the orientations and positions of the various camera stations in the geocentric coordinate system, the geographic positions and elevations of all image-points involved in the solution are determined.

In 1958 a computer program in Datatron 205 code was prepared to determine the capability of the method. The prototype computer program can solve problems containing nine photographs in any configuration of adjoining models using only the internal memory of the computer in the simultaneous solution. A series of 26 problems, using fictitious photographic data developed by Professor Earl Church and the minimum amount of control, were solved for the spatial positions of the camera stations to test the method and computer program. The Church data had been based on a camera focal-length of 150 millimeters and a flight height of 40,000 feet. The root mean square error in the determinations of the camera station positions for the 26 problems was 0.87 feet.

Two problems containing actual data were solved and the ground positions for all image-points involved in the solutions were determined. In the first of these two tests, comprising 7 photographs exposed at a flight height of 39,000 feet, the root mean square error in determining the horizontal positions of image-points was found to be 16.0 feet with a maximum error of 44.2 feet. The root mean square error in determining the elevations of the image-points was 13.5 feet with a maximum error of 29.1 feet.

The second test involved nine photographs exposed at a flight height of 9,000 feet controlled by three horizontal points and eight vertical points. In this test the root mean square error in determining the horizontal positions of image-points was 7.9 feet, with a maximum error of 11.8 feet. The root mean square error in determining the elevations of the image-points was 4.5 feet, with a maximum error of 11.9 feet.

The horizontal accuracies obtained in these two tests are well within the limits required for control in standard topographic mapping, but improved vertical accuracies are desirable. Results from tests with perfect data strongly indicate that the results were adversely influenced more by errors in the input data than by the computational method. Before the final solutions were attempted, each problem was solved using the minimum number of constraints. By examining the magnitudes of the condition equation constants and the discrepancies of control point positions computed under these limited constrained conditions, it was possible to detect and eliminate large input errors and gross blunders. These errors and blunders are believed to be the result of misidentification of ground-control, incorrect transfer of image-points, and inaccuracies in the ground-control coordinates. It was not feasible to confirm these suspicions since much of the input data had been furnished through the courtesy of other agencies.

The Geological Survey replaced the Datatron 205 computer with a Burroughs 220 computer in November 1959. The Topographic Division is now in the process of preparing a computer program in the code of this computer. Since the internal memory capacity of the Burroughs 220 computer is 10,000 cells, it is believed that it will be possible to solve problems consisting of from 1 to 22 photographs without having to resort to tape storage in the simultaneous solution.

In March 1960, under a U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency contract, the Massachusetts Institute of Technology began research in evaluating and improving the general approach and computer program for this system. Two interim reports have been issued covering two phases of this investigation.

In the prototype program, the angular relationships between rays emanating from the perspective center of a photograph are held fixed throughout the solution. The position of values of the ground control are assumed to be absolute. Weights are empirically determined and applied to the condition equations according to the type of equation involved.

MIT proposes that the angular relationship between rays and the coordinate values of control points be adjusted by amounts inversely proportional to the reliability of the observations involved. Using this approach, the observed coordinate values of image-points and the spatial coordinates of control points are weighted by predetermined amounts, which are subject to verification through statistical tests.

It is hoped that the accuracy of our method can be improved by incorporating these refinements into the system during the current reprogramming operations.

Plans call for the completion of the recoding operations by July 1, 1962. This, of course, must be followed by tests using fictitious input data to prove the correctness of the revised geometrical approach and the fidelity of the computer program. To evaluate the ability of the revised system to arrive at most probable solutions with input errors present, additional tests will be run in which known random errors have been introduced into the fictitious input data. Finally, it is hoped that a few problems can be run in which actual photographs covering a reliable and specially designed test area are used.

Progress on Computational Photogrammetry at IBM

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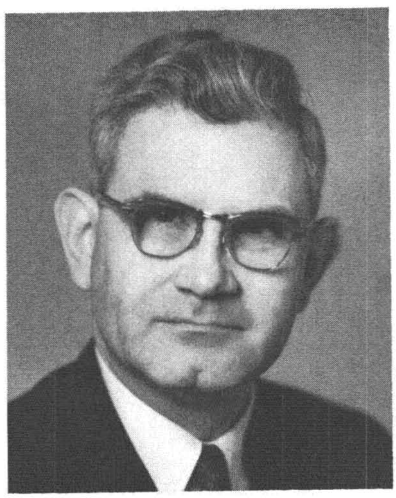
ABSTRACT: *The flexibility of the IBM Digital Mapping System as regards data input is mentioned. The computational operations are listed. The possible final formats are also listed.*

RECENT advances in computational photogrammetry have been occurring at IBM's Command Control Center at Kingston, New York. These advances have been a result of over three years of effort sponsored by GIMRADA. The work has been in connection with a Digital Mapping System being developed by IBM. This system is capable of processing digital photo data of varying quality. It ranges from TV satellite and x-ray camera quality to reconnaissance and precision mapping camera quality. These photos contain 10 million to 10 billion bits of digitalizable information depending on camera photo-quality flown. Associated with these data is digitalized control data obtained from ground or vehicle control instrumentation.



JOHN V. SHARP

In precise mapping operations, these data include geodetic, ground control measurements, and camera calibration data. In less precise systems, vehicle attitude and associated sensor data are used.

These digitalized photo and control data are recorded on magnetic tape and processed in an IBM 704 data processing system. The following computational operations are being performed:

(1) Resection and Orientation of Photos.
(2) Rectification of Photos.
(3) Scaling of Photos to Print-Out Scale.
(4) Orthographic Correction of Photos.
(5) Contouring the Orthophoto.
(6) Producing Contour Maps.
(7) Transformation of Maps to any Desired Map Projection.
(8) Preparing Profiles of Terrain Models.

These typical digital processing operations are performed automatically by the computer, and are stored on magnetic tape. From these tapes a scanning printer produces these data in several photo forms as desired by the user. Present outputs from this mapping system onto film chits or glass plate which have proven feasible are:

(1) Rectified and scaled photos.
(2) Ortho corrected photos.
(3) Orotho photo-maps with contours.
(4) Contour maps.
(5) Annotated map or photo overlays.

The human being is still required in standard accuracy mapping for stereoscopic measurement of control points. Also, final editing of maps to conform to topographic practice requires the skill of experts.

Mathematical Photogrammetry at the Army Map Service

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ABSTRACT: *The continuing role of analogical triangulation at Army Map Service including strip and block adjustments is discussed. The present experimental nature and probable forthcoming operational possibilities of analytical triangulation are detailed.*

THERE are many of those at the Army Map Service who are keenly interested in the prospects of a working group within the Society whose members have as their common interest, the formulation and solution of mathematical problems in photogrammetry. As I have previously mentioned to Professor Irish, I have no significant accomplishments with which to overwhelm you; however, I have taken advantage of this opportunity to acquaint you with what we are doing in terms of the application of numerical solutions to photogrammetric problems and what our plans are for the future.

CARL J. BORN

ANALOGICAL TRIANGULATION

Virtually all of our experience revolves around the adjustment of photogrammetric data from analogical triangulation. Certainly the adjustment equations are well known, since, with minor variations, they are used by many commercial and government mapping organizations. The mathematical representation of the problem is a good approximation of the physical conditions; however, the accuracy of the results is still dependent upon the reliability of the input data. For example, the accuracy of the individual model orientation is generally not determined; and if it were, there would be no way of using this information in the adjustment since the stereoscopic models are formed instrumentally and the adjustment is to the coordinates in image space and not through the model orientation. Perhaps a method can be devised for distributing the corrections according to the standard error of unit weight for the y-parallax of each model. The primary source of error, however, results from the misidentification of geodetic control on which the solution is based.

At the Army Map Service (AMS), these adjustments for analogical triangulation, both horizontal (strip and block) and vertical (strip) have been programmed for the UNIVAC. Similar programs will soon be written for the Honeywell H-800. The efficient utilization of these strip-adjustment programs on the large computers depends upon the processing of many strips at one time, as well as the combining of minor operations into one general program. Along with this, we are in need of methods for pre-testing our input data to avoid re-scheduling for computer time. Acceptability of solutions based on goodness of fit to the control data is included in the computer program, and although it is helpful in eliminating bad data, human judgement is frequently necessary. Ready and continuous access to a small computer would offset some of the disadvantages which are presently encountered.

ANALYTICAL TRIANGULATION

Now with respect to analytical triangulation, I should like to make a few brief comments. At present, the Army Map Service is not using analytical triangulation in production. Progress to date has not shown the urgent need for an immediate conversion from analogical to analytical triangulation. On the other hand, some investigations, as early as 1955, were made of the Herget method as programmed by Cornell University. Our later decision was, however, that the method of Dr. Hellmut Schmid was the one which would most probably satisfy our requirements for rigor and generality. For the last several years, therefore, we have kept in close contact with the Ballistic Research Laboratories.

In July 1959, we were fortunate enough to have Dr. Schmid ask us to supply the computer services for checking out the UNIVAC programs being written by Franklin Institute, under contract to the Ballistic Research Laboratories. In return for the use of our UNIVAC, Dr. Schmid agreed to supply us with complete information on each of the five cases being programmed. These five cases are: 1) Single camera resection, 2) Single stereo model, 3) Five camera case (of particular interest in ballistics work), 4) General strip, 5) General block.

Following this initial agreement, AMS broadened its participation in July, 1960, by establishing a formal assignment to evaluate Dr. Schmid's method, and to determine its application to our production operations. To evaluate the method, AMS plans to check out each of the five cases with measurements made on aerial photography, and to compare the results with those obtained by present analog methods.

AMS has a stereocomparator on order, but delivery is not expected until January 1963. To overcome this obstacle, Dr. Schmid has allowed us to use his Wild STK-1 stereocomparator at Ballistic Research Laboratories (BRL). Measurements are currently being made which will permit the evaluation of the entire program. The photography on which the measurements are being made include wide-angle, super-wide-angle and convergent photography. The largest block of photography that is available consists of three adjacent twenty-model strips of KC-1B photography. All of the aforementioned photography has been flown over the Phoenix test area.

With the acquisition of the Honeywell H-800 Computer, AMS further broadened its participation by agreeing to check out not only the UNIVAC programs, but also programs written in FORTRAN language for use on the Honeywell. To date, only the single camera orientation has been checked. The five-camera case, programmed in FORTRAN will be available by the end of June and the block program, by the end of March 1963.

It is, therefore, anticipated that within the next year we will be equipped to perform analytical triangulation on an operational basis. In the meantime, we will note carefully the results obtained by other organizations.

Obviously, it is impossible to predict the ultimate future of analytical triangulation at the Army Map Service. Perhaps it will be used for special projects or for bridging control flights due to its potential for increased accuracy. It is conceivable that it would completely replace analogical triangulation, if the efficiency of the method warrants it and there is a good market for "used" first-order stereoplotting instruments.

With regards to my comments of the past few minutes, you have probably observed that I have posed problems as well as described present procedures and future plans; however, I feel that this is not inappropriate in a preliminary session, such as this, where our primary objective is to stimulate interest and provoke thought on this subject of mathematical photogrammetry.

Computational Photogrammetry

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(*Abstract is on the next page*)

ONE of the ultimate aims of this panel, as outlined by Prof. Irish, is to establish a technical committee on computational photogrammetry within the ASP. If there is justification for founding such a committee, then—in my opinion—it cannot be based on topics which, in the past, have dominated the various symposia, panel discussions and technical papers concerned with the subject of analytical photogrammetry. By now it should be clear that with respect to computational photogrammetry, one only has to apply some simple geometrical principles. Similarly, there is no need for principles beyond the proven basic principles of the Gausian least squares method.

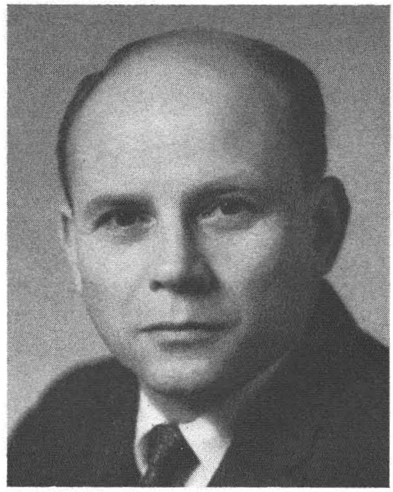
Obviously, it is necessary to understand thoroughly both subjects and to apply the corresponding rules correctly. However, such a demand is basic for any mensuration problem, including those of metric photography—and therefore is not a justification for establishing a special scientific or technical committee.

However, two things are new in computational photogrammetry:

(a) the electronic computer as a tool in photogrammetry and
(b) the elimination of certain restrictions which were imposed on the photogrammetric evaluation methods by specific design characteristics of the analogue type universal plotters.

To elaborate on (a): When one introduces computers into a specific field of scientific or engineering activity, as in this case into photogrammetry, one needs to produce humans who can use them. By using, I mean technically understanding them, not so much from the standpoint of the complex interconnections of the logical elements of a specific computer design, but with respect to the principles of electronic computing.

Thus far, none of the proposed methods of numerical treatment of photogrammetric triangulation contains any basically new concepts, no new geometrical principle has been introduced, nor has there been suggested any new statistical treatment. It is true that a certain number of approximative solutions have been suggested, and in my opinion computational photogrammetry can easily do without them. The present day attitude in organizing tests for deciding on the comparative usefulness of various possible approaches to computational photogrammetry is lacking in imaginative ideas as to what an optimum method should resemble.



HELLMUT SCHMID

One clearly understands today why we can automatize the problems of mathematical analysis. These are problems, and certainly the photogrammetric problem is no exception, which can be arranged in fairly simple but highly repetitive steps. The problems encountered are predominantly multiplicative and therefore the electronic computer, because of its potential for performing multiplications efficiently turns out to be such a marvelous device for this purpose. Consequently, the criteria for an optimized structure of a specific program are (1) to be as short as possible and (2) to the extent feasible, using the speed of the machine to produce repetitive action.

However, the present generation of computers are usable by human beings only through a very detailed set of instructions, each step being of almost micrological nature. On the other hand, a human being has a tremendous memory, and some organization to it. Therefore, the human can react quite wisely if confronted by an entirely new set of circumstances. By making use of previous experience, he is able to make an educated guess as to the significance of the new situation. In other words, the automation of a problem of specific complexity, e.g. photogrammetric triangulation, can be optimized

ABSTRACT: *The fundamental nature of the electronic computer as applied to the photogrammetric problem is outlined. The limitations encountered when extending present analogue computer concepts are mentioned. Finally, the tasks facing computational photogrammetry are listed.*

only if we learn to decide how much complexity can be handled by a specific amount of computer memory. This then is the point where, at the present, human inspection and judgment must be inserted. An optimum automation is obtained by an effective use of this man-machine network, that is, we must not over-automatize but provide human judgment at critical levels.

In my opinion, this judgment is even more important when we consider as mentioned under (b), the additional flexibility which computational photogrammetry offers if compared with the classical restitution techniques. It is true that the computing power inherent in today's general-purpose digital computers is so great that users can apply them in an inefficient and unimaginative fashion and may still obtain superior results. Translated into the language of computational photogrammetry, this means that even the digital simulation of the analogue simulator, which was created to avoid computations, paradoxically may provide some advantage in regard to accuracy and economy. However, such an approach certainly cannot be the ultimate aim of computational photogrammetry.

It is essential that we realize that our traditional photogrammetric restitution methods, without exception, are methods of interpolation. A numerical treatment of the photogrammetric triangulation process does not necessarily change this situation. However, the numerical approach offers the potential to convert the photogrammetric method to an absolute measuring method, that is, a method where all parameters comprising the model can be determined in a geometrically and physically significant manner. Such an approach changes radically the basic theoretical concept of photogrammetry, by providing the means for combining classic photogrammetric raw data with independent measurements of any of the parameters which describe the spatial triangulation process or of any functions of those parameters.

In this connection, it is of importance that theoretical studies already have proven the need for such hybrid systems in order to improve the otherwise unfavorable laws of error propagation. In essence, the computational approach allows us to see our problem in the broader aspect of spatial triangulation, thus eliminating the shortcomings of a measuring method which is more or less restricted to the use of only one specific group of data, e.g. the classical method of photogrammetric triangulation.

It is hardly necessary to point out that a corresponding scientific and engineering effort must be directed not only toward the solution of the associated software problems but also must deal with the emerging hardware problems which will predominantly arise in the area of data acquisition.

Considerations of the justification for a special committee on computational photogrammetry should include a look into the future!

As already stated, even the most elaborate concept of spatial triangulation can be handled by the rules of Euclidean geometry in combination with the techniques for a general least squares treatment. Neither concept is likely to change. Future spatial triangulation problems may very well be characterized by increased complexity. But it is hard to conceive of a need for basically new geometrical or statistical principles. However, the tool of numerical analysis—the computer—will change. The next generation of computers which have already been conceived, may come closer to the goal of an information processing machine, featuring computer languages which are oriented toward both the machine and the human. Thus, by coupling the machine to ourselves, we might be able to achieve another degree of sophistication, enabling us to plan our efforts at a higher level of strategy.

In Photogrammetry a single photograph has easily 10^8 bits of information. In addition there is a family of customers which differ widely as to the purpose and significance of the information contents of a specific photograph. Consequently, the required transfer rate of handling these data is formidable. The resulting problems are not only multiplicative but to a certain extent, of a combinatorial type. Without long range planning we will not be able to take advantage of the possibilities inherent in the next generation of computers, which will feature nanosecond operations and million-word memories.

Concluding, I feel that computational photogrammetry must occupy itself with the tasks of:

(a) formulating its general problem in such a way that the present day's concept of photogrammetric triangulation is being merged into the broader concept of spatial triangulation.
(b) optimizing its approach to the problem of numerical analysis by proper consideration of the basic characteristics of electronic computing, and
(c) creating imaginative ideas for handling the large amount of information,

typical for photogrammetric records, by anticipating the trend of electronic computers toward human oriented information processing machines.

In order to approach these problems successfully I suggest applying a well-known engineering by-word which, slightly modified says: "Conceive it big—but—keep it simple."

Mathematical Analysis of Blur on a Photographic Film Caused by Motions of a Vehicle

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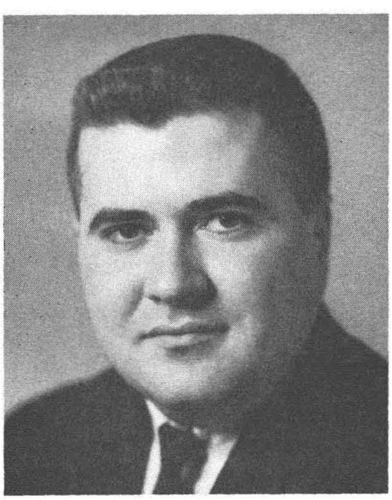
ABSTRACT: *As the orientation of a space vehicle is usually given in terms of pitch, roll, and yaw angles, it is desirable to express the blur produced on a camera aboard such a vehicle in the same angles. By introducing a rotational matrix of the form* $X' = EX$, *the blur equation is found to be* $D = E\dot{X}T + \dot{E}XT$. *The term in* $\dot{E}$ *describes the blur produced by pitch, roll, and yaw rate while the term in* $\dot{X}$ *describes the blur produced by forward motion error and side slip error.*

1. STATEMENT OF THE PROBLEM

A PHOTO reconnaissance mission performed by an aircraft or space vehicle can be totally useless if image-motion on the film causes sufficient blur to render identification of targets impossible. There are, of course, several effects which cause image-motion and consequent blur. Among these can be listed the forward motion of the vehicle, the rotation of the earth and the rotations of the vehicle. The purpose of this paper is to determine the equations governing the image-motion produced by rotational and translational motions of the vehicle.



PETER NOLAN

The most appropriate way to describe the blur on a film is to utilize matrices which describe the motion. We assume that the camera is situated so close to the center of the mass of the vehicle that the pitch, roll, and yaw of the camera can be assumed to be identical to the pitch, roll, and yaw of the vehicle. The coordinate system is shown in Figure 1.

It is next assumed that a method of measuring the angles of rotation is available in the vehicle, and that Eulerian angles can be measured. (1) A rotation about the y axis is pitch, a rotation about the new x axis is roll, and a rotation about the new z axis is yaw. This can be expressed mathematically as:

$$X' = A_{\text{pitch}} X \tag{1}$$

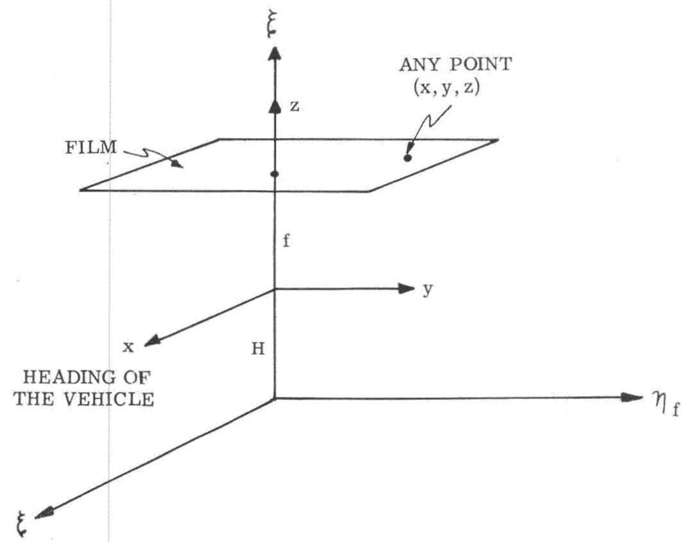


FIG. 1. Coordinate system.

or

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{1a}$$

Equation (1) and (1a) can be interpreted physically as a rotation of the coordinate axis through an angle ϕ transforming any point (x, y, z) into a new point (x', y', z'). If this new coordinate axis is rolled through an angle ω about the new x' axis, the point (x', y', z') is transformed into the point (x'', y'', z'') by Equation: (2)

$$X'' = B_{\text{roll}} X' \tag{2}$$

or

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{2a}$$

In the same manner a yaw through the angle κ transforms the point (x'', y'', z'') into the new point (x''', y''', z'''). In matrix notation:

$$X''' = C_{\text{yaw}} X'' \tag{3}$$

or

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} \tag{3a}$$

Equations 1, 2, and 3 can be combined into

$$X''' = C_{\text{yaw}} B_{\text{roll}} A_{\text{pitch}} X \tag{4}$$

Equation (4) states that after a combination of pitch, roll, and yaw, the original point

X is located at the new point X'''. If we consider the original point X to be an image-point on the film of an object-point on the ground, then after the rotational motions occur, the same object-point on the ground appears as the same image-point, but located at a different point on the film, namely, the point X'''.

Performing the matrix multiplication, equation (4) reduces to:

$$\begin{pmatrix} x''' \\ y''' \\ z''' \end{pmatrix} = \begin{pmatrix} \cos\phi\cos\kappa - \sin\phi\sin\omega\sin\kappa & \sin\kappa\cos\omega & \cos\kappa\sin\phi + \sin\omega\sin\kappa\cos\phi \\ -\sin\kappa\cos\phi - \cos\kappa\sin\omega\sin\phi & \cos\kappa\cos\omega & -\sin\phi\sin\kappa + \cos\phi\sin\omega\cos\kappa \\ -\cos\omega\sin\phi & -\sin\omega & \cos\omega\cos\phi \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (5)$$

calling the above transformation E, we can write the equation as:

$$X''' = EX \quad (6)$$

Since we are interested in the velocity of the image across the film we must differentiate Equation (6) with respect to time, while assuming that the film is not moving and, therefore, $\dot{X} = 0$

Therefore,

$$\dot{X}''' = \dot{E}X, \quad (7)$$

the velocity of the image across the format. The blur or smear of the photograph is caused by this motion when the shutter is opened. The displacement D, of the point which is the length of the blur is given by:

$$D = \int_{t_o}^{t_c} \dot{X}''' dt \quad (8)$$

where

t_o = time of opening of shutter to exposure threshold
t_c = time of closing of shutter to exposure threshold

In most applications, but not all, the approximation

$$D = \dot{X}''' T \quad (9)$$

can be used, where T is the exposure time, $t_c - t_o$. Combining Equations (7) and (8) yields:

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} T \quad (10)$$

Where

$$a_{11} = -(\sin\phi\cos\kappa + \cos\phi\sin\omega\sin\kappa)\dot{\phi} - (\sin\phi\cos\omega\sin\kappa)\dot{\omega}$$
$$- (\cos\phi\sin\kappa + \sin\phi\sin\omega\cos\kappa)\dot{\kappa}$$

$$a_{21} = (\sin\phi\sin\kappa - \cos\phi\sin\omega\cos\kappa)\dot{\phi} - (\sin\phi\cos\omega\cos\kappa)\dot{\omega}$$
$$+ (\sin\phi\sin\omega\sin\kappa - \cos\phi\cos\kappa)\dot{\kappa}$$

$$a_{31} = -(\cos\phi\cos\omega)\dot{\phi} + (\sin\phi\sin\omega)\dot{\omega}$$

$$a_{12} = -(\sin\omega\sin\kappa)\dot{\omega} + (\cos\omega\cos\kappa)\dot{\kappa}$$

$$a_{22} = -(\sin\omega\cos\kappa)\dot{\omega} - (\cos\omega\sin\kappa)\dot{\kappa}$$

$$a_{32} = -(\cos\omega)\dot{\omega}$$

$$a_{13} = (\cos\phi\cos\kappa - \sin\phi\sin\omega\sin\kappa)\dot{\phi} + (\cos\phi\cos\omega\sin\kappa)\dot{\omega} + (\cos\phi\sin\omega\cos\kappa - \sin\phi\sin\kappa)\dot{\kappa}$$

$$a_{23} = -(\cos\phi\sin\kappa + \sin\phi\sin\omega\cos\kappa)\dot{\phi} + (\cos\phi\cos\omega\cos\kappa)\dot{\omega} - (\sin\phi\cos\kappa + \cos\phi\sin\omega\sin\kappa)\dot{\kappa}$$

$$a_{33} = -(\sin\phi\cos\omega)\dot{\phi} - (\cos\phi\sin\omega)\dot{\omega}$$

Equation (10) can now be used to evaluate the smear of any photograph for different combination of pitch, roll, and yaw, roll rate, pitch rate, yaw rate, and exposure time. The calculations to be performed, however, are long and tedious.

2. THE SMALL-ANGLE APPROXIMATION

When a computer is not available, or deemed not necessary for the particular job, the small-angle approximation may sometimes be used. It assumes the angles are small enough so that:

$$\left.\begin{array}{l}\cos b = 1 \\ \sin b = b\end{array}\right\} \quad b = \text{small angle} \tag{11}$$

and products of small angles are themselves negligible. Ex. $\phi\omega = \kappa\phi = \kappa\omega = 0$. Equation (12) for the small-angle approximation reduces to

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = T \begin{pmatrix} -\phi\dot{\phi} - \kappa\dot{\kappa} & \dot{\kappa} & \dot{\phi} + \kappa\dot{\omega} + \omega\dot{\kappa} \\ -\omega\dot{\phi} - \phi\dot{\omega} - \dot{\kappa} & -\omega\dot{\omega} & -\kappa\dot{\kappa} - \kappa\dot{\phi} + \dot{\omega} - \phi\dot{\kappa} \\ \dot{\phi} & -\dot{\omega} & -\omega\dot{\omega} - \phi\dot{\phi} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{12}$$

Equation (12) can be used to determine smear when the roll, pitch, yaw, roll rate, pitch rate, and yaw rate of the vehicle and exposure time of the camera are known. It is to be noted; however, that the assumption of small angles restricts the use of Equation (12) to photographs taken in the vicinity of the nadir-point. As the angles ϕ, and κ, and ω become larger, the error induced also becomes larger.

3. ANALYSIS OF SMEAR FOR ROLL, PITCH, AND YAW SEPARATELY

For the particular case where Eulerian angles or the small-angle approximation cannot be used, a recourse to a separate analysis can still be made. With this approach all rotations occur about the original coordinates axis, but only one rotation is possible at a time.

a. Pitch about the y axis.

The blur equation for pitch is given by:

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}_{\text{pitch}} = T \begin{pmatrix} -\sin\phi\dot{\phi} & 0 & +\cos\phi\dot{\phi} \\ 0 & 0 & 0 \\ -\cos\phi\dot{\phi} & 0 & -\sin\phi\dot{\phi} \end{pmatrix} \begin{pmatrix} x \\ y \\ f \end{pmatrix} \tag{13}$$

or

$$\begin{cases} D_x = T\dot{\phi}[f\cos\phi - x\sin\phi] \\ D_y = 0 \\ D_z = T\dot{\phi}[-f\sin\phi - x\cos\phi] = 0 \end{cases} \tag{14}$$

The D_z term is zero since the optical system requires the image to be focused on the film plane. Equation (14) now states that the blur in the x direction depends on the pitch angle.

b. Roll about the x axis.

The blur equation for roll is given by:

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}_{\text{roll}} = T \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sin\omega\dot{\omega} & \cos\omega\dot{\omega} \\ 0 & -\cos\omega\dot{\omega} & -\sin\omega\dot{\omega} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{15}$$

or

$$\begin{cases} D_x = 0 \\ D_y = T\dot{\omega}[f\cos\omega - y\sin\omega] \\ D_z = 0 \end{cases} \tag{16}$$

c. Yaw about the z axis.

The blur equation for yaw is given by:

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}_{\text{yaw}} = T \begin{pmatrix} -\sin\kappa\dot{\kappa} & \cos\kappa\dot{\kappa} & 0 \\ -\cos\kappa\dot{\kappa} & -\sin\kappa\dot{\kappa} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{17}$$

Equations (14), (16), and (17) are vector equations. The total distance of smear is given by:

$$\vec{\boldsymbol{D}} = \vec{i}(D_{x\text{pitch}} + D_{x\text{roll}} + D_{x\text{yaw}}) + \vec{j}(D_{y\text{pitch}} + D_{y\text{roll}} + D_{y\text{yaw}}) \tag{18}$$

with magnitude

$$|\vec{\boldsymbol{D}}| = [(D_{x\text{pitch}} + D_{x\text{roll}} + D_{x\text{yaw}})^2 + (D_{y\text{pitch}} + D_{y\text{roll}} + D_{y\text{yaw}})^2]^{1/2} \tag{19}$$

The advantage to this method of approach is that the small-angle approximation is not made, and therefore, the equations can be used even when considering oblique photography.

4. THE RECIPROCAL PROBLEM OF ATTITUDE STABILIZATION REQUIREMENTS

Up to now the main consideration has been that of determining blur when the angular rates of motion are known. There are times, however, when the problem is somewhat different. Suppose it is necessary to keep the blur to a certain value, or at least never greater than some certain value. The problem is then to determine the maximum value permissible for any angular rate. Equations (10) and (12) cannot be used, so recourse must be made to the separate analysis using equations (14), (16), and (17).

The attitude stabilization requirement for pitch is found by solving equation (14) for $\dot{\phi}$, i.e.;

$$\dot{\phi} = \frac{D_{x\text{pitch}}}{T[-x\sin\phi + f\cos\phi]} \tag{20}$$

Equation (20) says that for some minimum known value of $D_{x\text{pitch}}$, with parameters T, ϕ, x, and f, values of $\dot{\phi}$ can be determined such that the blur is never greater than the known $D_{x\text{pitch}}$. The vehicle must now be stable in pitch rate to these values of $\dot{\phi}$.

In the same manner the attitude stabilization requirement for roll is found by solving equation (16) for $\dot{\omega}$, i.e.;

$$\dot{\omega} = \frac{D_{y\text{roll}}}{T[-y \sin \omega + f \cos \omega]} \tag{21}$$

Similarly for the case of yaw, equation (17) yields the two values of $\dot{\kappa}$; namely

$$\dot{\kappa} = \frac{D_{x\text{yaw}}}{T[-x \sin \kappa + y \cos \kappa]}$$

$$\dot{\kappa} = \frac{D_{y\text{yaw}}}{T[-x \cos \kappa - y \sin \kappa]} \tag{22}$$

the stabilization requirements for yaw rate is, therefore, determined. The vehicle must, therefore, be held stable to the values of $\dot{\phi}$, $\dot{\omega}$, and $\dot{\kappa}$ just determined. If it is obvious that the vehicle cannot be held to these values then the possibility of using a stable mount for the camera must be investigated.

5. EFFECT OF ROLL, PITCH, YAW, AND THEIR RATES ON IMC

In the transformation equations treated so far the relative motion of the vehicle has been neglected. The original transformation equations

$$\begin{cases} X'_{\text{pitch}} = A_{\text{pitch}}X \\ X'_{\text{roll}} = B_{\text{roll}}X \\ X'_{\text{yaw}} = C_{\text{yaw}}X \end{cases} \tag{23}$$

when differentiated with respect to time yield the equations

$$\begin{cases} \dot{X}'_{\text{pitch}} = A_{\text{pitch}}\dot{X} + \dot{A}_{\text{pitch}}X \\ \dot{X}'_{\text{roll}} = B_{\text{roll}}\dot{X} + \dot{B}_{\text{roll}}X \\ \dot{X}'_{\text{yaw}} = C_{\text{yaw}}\dot{X} + \dot{C}_{\text{yaw}}X \end{cases} \tag{24}$$

Previously the terms in $\dot{X}$ were neglected since it was assumed that the film itself did not move. Figure 2 shows the geometry of the coordinate system in relation to the ground for a vertical camera.

Since the vehicle is moving with respect to the ground any object point on the ground, denoted by a vector $\boldsymbol{R}$, is moving with relation to the vehicle. The image point of $\boldsymbol{R}$, denoted by $\boldsymbol{r}$, also moves, therefore, this motion causes a blur on the film. From the geometry of Figure 2, we have the relation

$$\overrightarrow{\boldsymbol{r}} = S\overrightarrow{\boldsymbol{R}} \tag{25}$$

where S is a proportionality constant. For a vertical photograph

$$S = \frac{f}{H} \tag{26}$$

and for a pitched photograph

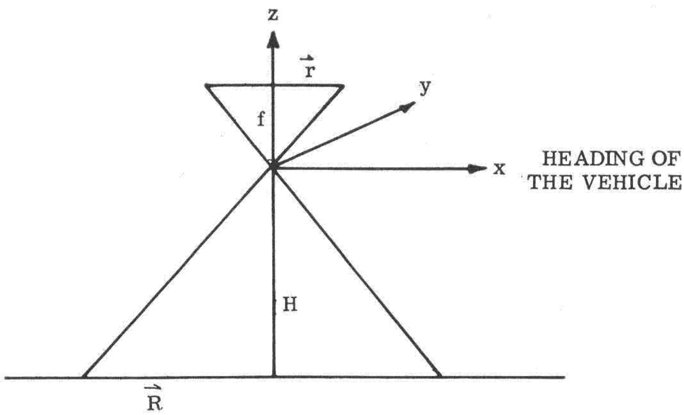


FIG. 2. Geometry of coordinate system.

$$S = \frac{1}{H}\left[f \cos \phi - x \sin \phi\right], \tag{27}$$

where ϕ is the pitch angle from the horizontal.

Since there is a relative motion between the ground and the film we have

$$\vec{V}_f = S\vec{V}_g \tag{28}$$

where $\vec{V}_f$ is the velocity of the image across the film, and $\vec{V}_g$ is the ground velocity.

To compensate for this image-motion IMC is usually utilized. Let us assume for the moment that we have a three-dimensional IMC. That is, we will move the film at the velocity $\vec{V}_f$, so that there will be no image motion across the film. Since we will not be able to match these velocities exactly there will be an error given by:

$$\vec{V}_e = \vec{V}_f - \overrightarrow{\mathbf{IMC}} \tag{29}$$

Calling the efficiency of the IMC, E_{ff}, equation (29) becomes:

$$\vec{V}_e = (1 - E_{ff})\vec{V}_f \tag{30}$$

If we consider the accuracy to be k where $k = 1 - E_{ff}$

$$\vec{V}_e = k\vec{V}_f \tag{31}$$

Therefore

$$\vec{V}_e = kS\vec{V}_g \tag{32}$$

If we now compare Equation (24) and (32) we see that the matrix term $\dot{X}$, is actually $\vec{V}_e$.

From Equation (32) we have for V_{e_x}, V_{e_y}, V_{e_z} the scalar equations

$$\begin{cases} V_{e_x} = k_1 S V_{g_x} & \text{Forward Motion Error} \\ V_{e_y} = k_2 S V_{g_y} & \text{Side Slip Error} \\ V_{e_z} = 0 \end{cases} \tag{33}$$

The first equation of (33) is nothing more than the Forward Motion Error. The second equation of (33) is the Side Slip Error. If there is no velocity, V_{g_y}, then the Side Slip Error is zero.

Since there usually is no IMC in the y direction k_2 is equal to 1.

Therefore, equation (33) reduces to:

$$\begin{cases} V_{e_x} = k_1 S V_{g_x} \\ V_{e_y} = S V_{g_y} \\ V_{e_z} = 0, \end{cases} \tag{34}$$

The image-motion on the film due to the translational motion of the vehicle.

If we now multiply equation (24) by the exposure time, T, we get the smear equations:

$$\begin{cases} D_{\text{pitch}} = \dot{X}'_{\text{pitch}} T = A_{\text{pitch}} \dot{X} T + \dot{A}_{\text{pitch}} X T \\ D_{\text{roll}} = \dot{X}'_{\text{roll}} T = B_{\text{roll}} \dot{X} T + \dot{B}_{\text{roll}} X T \\ D_{\text{yaw}} = \dot{X}'_{\text{yaw}} T = C_{\text{yaw}} \dot{X} T + \dot{C}_{\text{yaw}} X T \end{cases} \tag{35}$$

Equation (35) now represents blur produced either by angular rates and/or their angles. Since we have already discussed the blur caused by pitch rate, roll rate, and yaw rate, it behooves us to determine the blur produced by pitch, roll, and yaw. As a special case, therefore, let us assume that the pitch rate, roll rate, and yaw rate is zero, so that Equation (35) reduces to:

$$\begin{cases} D_{\text{pitch}} = A_{\text{pitch}} \dot{X} T \\ D_{\text{roll}} = B_{\text{roll}} \dot{X} T \\ D_{\text{yaw}} = C_{\text{yaw}} \dot{X} T \end{cases} \tag{36}$$

For the case of pitch

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}_{\text{pitch}} = \begin{pmatrix} \cos\phi & 0 & +\sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} k_1 S V_{g_x} \\ f V_{g_y}/H \\ 0 \end{pmatrix} T \tag{37}$$

or

$$\begin{cases} D_x = k_1 S V_{g_x} T \cos\phi & \text{Forward Motion Error} \\ D_y = \dfrac{f V_{gy} T}{H} & \text{Side Slip Error} \\ D_z = 0 \end{cases} \tag{38}$$

In Equation (37) and (38) the x components contain S while the y components contain f/H. The reason for this is that as the vehicle pitches S is no longer f/H for the x component, but is instead the pitched proportionality factor given in equation

$$S = \frac{-1}{H} [f \cos\phi - x \sin\phi] \tag{39}$$

The forward motion error is, therefore, a function of the pitch-angle. As the pitch-angle increases the forward motion error decreases. However, the error introduced into the scale factor prohibits the use of a programmed pitch to reduce the forward motion error. The side slip error is independent of the pitch-angle.

For the case of roll

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} k \dfrac{f}{H} V_{gx} \\ S V_{gy} \\ 0 \end{pmatrix} T \tag{40}$$

or

$$\begin{cases} D_x = \dfrac{k_1 f V_{g_x} T}{H} & \text{Forward Motion Error} \\ D_y = S V_{g_y} T \cos\omega & \text{Side Slip Error} \\ D_z = 0 \end{cases} \tag{41}$$

The Forward Motion Error is independent of the roll-angle. The Side Slip Error is, however, a function of the roll-angle ω. As the roll-angle increases the Side Slip Error decreases. However, as in the case of pitch, the error introduced into the scale factor prohibits the use of a programmed roll to minimize the side slip error. In Equations (40) and (41) the y components contain S while the x components contain f/H. As in the case of pitch, the proportionality factor is no longer simply f/H for the y component, but is instead

$$S = \frac{-1}{H}\left[f\cos\omega - y\sin\omega\right]$$

For the case of yaw

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}_{\text{yaw}} = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dfrac{k_1 f V_{g_x}}{H} \\ \dfrac{f V_{g_y}}{H} \\ 0 \end{pmatrix} T \tag{42}$$

or

$$\begin{cases} D_x = T\dfrac{f}{H}\left[k_1 V_{g_x}\cos\kappa + V_{g_y}\sin\kappa\right] \\ D_y = T\dfrac{f}{H}\left[-k_1 V_{g_x}\sin\kappa + V_{g_y}\cos\kappa\right] \\ D_z = 0 \end{cases} \tag{43}$$

Equation (43) gives the amount of blur produced for a yaw-angle of κ.

For satellites it would be necessary to program a yaw into the vehicle to compensate for the rotation of the earth. The velocity of rotation of the earth as seen from a polar orbit is given by:

$$V_{g_y} = V_e \cos\theta \tag{44}$$

where

V_e = Velocity of earth at the equator
θ = Latitude angle
V_{g_y} = Velocity of a point on the earth due to the rotation of the earth (for a polar orbit V_{e_y} is perpendicular to the direction of flight).

The velocity of the earth is substituted for V_{g_y} in Equation (43) since this velocity is in the y direction.

To minimize the Side Slip Error a programmed yaw is utilized. For a given latitude the geometry is given in Figure 3.

If the satellite yaws through the angle κ given by:

$$\kappa = \tan^{-1} \frac{V_{gy}}{V_{gx}} \tag{45}$$

then the film x axis is lined up with the direction of flight and the Side Slip Error is eliminated.

In aircraft applications the same type of technique can be employed. A camera mount can be used to yaw the camera or the film platen.

Cancellation of Side Slip Blur

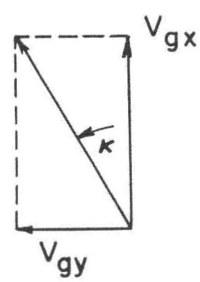


FIG. 3. Geometry of a programmed yaw.

The combined blur is that due to the translational and rotational motions. For the case of pitch the addition of Equations (14) and (38) gives:

$$\begin{cases} D_x = T[f \cos \phi - x \sin \phi] \left(\dot{\phi} + \dfrac{k_1 V_{gx}}{H} \cos \phi \right) \\ D_y = \dfrac{f}{H} V_{gy} T \end{cases} \tag{46}$$

For the case of roll the blur is found by the addition of Equations (16) and (41) gives:

$$\begin{cases} D_x = \dfrac{k_1 f V_{gx} T}{H} \\ D_y = T[f \cos \omega - y \sin \omega] \left(\dot{\omega} + \dfrac{V_{gy}}{H} \cos \omega \right) \end{cases} \tag{47}$$

The total blur due to yaw is simply equation (17) plus (43).

$$\begin{cases} D_x = T\left[\left(\dfrac{f}{H} k_1 V_{gx} + y\dot{\kappa} \right) \cos \kappa + \left(\dfrac{f}{H} V_{gy} - x\dot{\kappa} \right) \sin \kappa \right] \\ D_y = T\left[\left(\dfrac{f}{H} V_{gy} - x\dot{\kappa} \right) \cos \kappa + \left(\dfrac{-f}{H} k_1 V_{gx} - y\dot{\kappa} \right) \sin \kappa \right] \end{cases} \tag{48}$$

Table 1 compares the effective blur due to pitch, and roll, with the vertical case.

By using Table 1 and Equation (48) the total distance of blur can be determined. With this value of D the dynamic system resolution can be determined. An example of its use is shown by the aid of Figure 4, the graph of Resolution vs Image Blur.[2]

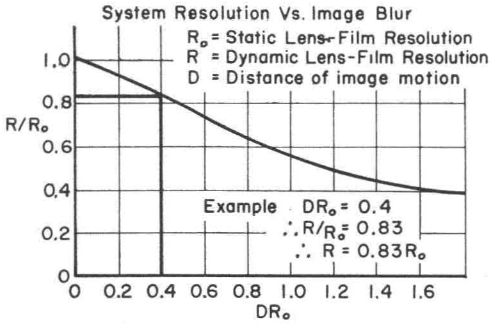


FIG. 4. Graph of system resolution vs. image blur.

TABLE 1

	Oblique	*Vertical*
D_x pitch	$T[f \text{ Cos } \phi - x \text{ Sin } \phi]\left[\dot{\phi} + \frac{k_1 V_{gx}}{H} \text{ Cos } \phi\right]$	$T\left[f\dot{\phi} + k_1 \frac{f}{H} V_{g_x}\right]$
D_y pitch	$\frac{f}{H} V_{g_y} T$	$\frac{f}{H} V_{g_y} T$
D_y roll	$T[f \text{ Cos } \omega - y \text{ Sin } \omega]\left(\dot{\omega} + \frac{V_{gy}}{H} \text{ Cos } \omega\right)$	$T\left[f\dot{\omega} + \frac{f V_{gy}}{H}\right]$
D_x roll	$k_1 f \frac{V_{gx}}{H} T$	$k_1 f \frac{V_{gx}}{H} T$

When D is known the product DR_0 is calculated, where R_0 is the static Lens-Film Resolution. From the graph the ratio R/R_0 is found and since R_0 is known, R the dynamic Lens-Film Resolution is determined.

There are, of course, other causes of image blur such as vibration and shock, but for these types of image motion vibration envelopes of the specific vehicle are necessary.

REFERENCES

1. Goldstein, H. "Classical Mechanics." Addison-Wesley Publishing Co., Inc. Reading, Massachusetts.
2. Magill, A. "Experimental Curves of System Resolution vs. Image-Motion." Fairchild Camera and Instrument Corporation.

Closure

SUMNER B. IRISH,
Moderator

THE scope of computational photogrammetry has been ably presented by the participants of the panel. It is indeed broad and challenging. The problems that face the workers in this field are basically three in number as follows:

1. The development of a suitable computer program which will not only truly represent the computational requirements of this type of photogrammetry in all its complexities, but which will also make use of the best computational techniques so that rapid solutions of complex problems can be effectuated.

2. A continuing recognition of computational significance for the various possible computer programs, so that round-off errors and the like will not creep into the solution of the photogrammetric problem, seriously deteriorating its usefulness, and

3. The investigation of the accuracy of the data inputs into the computer programs, and the developments of techniques, hardware, and related items which will provide a final accuracy compatible with the needs of the photogrammetric problems under consideration.

The moderator wishes to thank the panel members for their splendid presentations and their whole hearted cooperation with this undertaking.

*The Phoenix APR-HIRAN Test**†

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ABSTRACT: *Simultaneously flown Airborne Profile Recorder (APR) and HIRAN controlled aerial photography, covering an area 30 miles East-West by 35 miles North-South, was obtained over the Phoenix, Arizona, Test Area in 1959 with the RC-130A aircraft. Seven North-South and three equally spaced cross strips, flown at an altitude of 20,000 feet above sea-level, were used in the test. Vertical ground control in the North-West and South-East corners of the Phoenix Test Area was used to index the Mark-VI APR data, and a block adjustment was performed to bring the APR data on each strip to a common datum. In addition, two APR and HIRAN controlled strips, flown in the North-South direction at 36,000 feet above sea level, were used in the test. Each strip was bridged on a C-8 Stereoplanigraph and the photogrammetric data adjusted to ground-control, to APR, and to the given HIRAN control data. Vertical and horizontal ground-control, spaced at approximately 1-mile intervals in the cardinal directions, was used as check-points.*

The average of the RMS Errors for all the strips flown at 20,000 feet above sea-level was 7.82 feet on the vertical check points and 5.83 meters on the horizontal check-points, when APR and HIRAN was used to control the adjustment of each strip, compared to similar errors of 6.47 feet and 5.02 meters, when each strip was adjusted to ground-control. For the two strips flown at 36,000 feet above sea level, the relative accuracy of the APR control averaged 8.75 feet, and the average of the RMS Errors on the horizontal check-points was 7.21 meters when HIRAN was used to control the adjustment of each strip, compared to a similar error of 5.17 meters when the strips were adjusted to horizontal ground-control. The test results show that the bridging accuracies which can be achieved with Mark-VI APR and HIRAN control located generally in every stereo model of a flight strip are about the same as can be obtained with vertical ground-control in every other stereo model and horizontal ground-control spaced seven models apart along the flight line.

SEVERAL years ago, the U. S. Air Force took steps to develop an aircraft to replace the RB-50 type aircraft for long-range aerial mapping and charting missions. The result

* The information contained herein does not necessarily represent the official views of the Corps of Engineers of the Department of the Army.

† Presented at the 28th Annual Meeting of the Society, The Shoreham Hotel, Washington, D. C., March 14–17, 1962.

EDITOR'S NOTE.—Because of the request of the author all tables and graphs are grouped as far as practicable and possible.