1. The development of a suitable computer program which will not only truly represent the computational requirements of this type of photogrammetry in all its complexities, but which will also make use of the best computational techniques so that rapid solutions of complex problems can be effectuated.
2. A continuing recognition of computational significance for the various possible computer programs, so that round-off errors and the like will not creep into the solution of the photogrammetric problem, seriously deteriorating its usefulness, and
3. The investigation of the accuracy of the data inputs into the computer programs, and the developments of techniques, hardware, and related items which will provide a final accuracy compatible with the needs of the photogrammetric problems under consideration.

The moderator wishes to thank the panel members for their splendid presentations and their whole hearted cooperation with this undertaking.

*The Phoenix APR-HIRAN Test**†

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ABSTRACT: *Simultaneously flown Airborne Profile Recorder (APR) and HIRAN controlled aerial photography, covering an area 30 miles East-West by 35 miles North-South, was obtained over the Phoenix, Arizona, Test Area in 1959 with the RC-130A aircraft. Seven North-South and three equally spaced cross strips, flown at an altitude of 20,000 feet above sea-level, were used in the test. Vertical ground control in the North-West and South-East corners of the Phoenix Test Area was used to index the Mark-VI APR data, and a block adjustment was performed to bring the APR data on each strip to a common datum. In addition, two APR and HIRAN controlled strips, flown in the North-South direction at 36,000 feet above sea level, were used in the test. Each strip was bridged on a C-8 Stereoplanigraph and the photogrammetric data adjusted to ground-control, to APR, and to the given HIRAN control data. Vertical and horizontal ground-control, spaced at approximately 1-mile intervals in the cardinal directions, was used as check-points.*

The average of the RMS Errors for all the strips flown at 20,000 feet above sea-level was 7.82 feet on the vertical check points and 5.83 meters on the horizontal check-points, when APR and HIRAN was used to control the adjustment of each strip, compared to similar errors of 6.47 feet and 5.02 meters, when each strip was adjusted to ground-control. For the two strips flown at 36,000 feet above sea level, the relative accuracy of the APR control averaged 8.75 feet, and the average of the RMS Errors on the horizontal check-points was 7.21 meters when HIRAN was used to control the adjustment of each strip, compared to a similar error of 5.17 meters when the strips were adjusted to horizontal ground-control. The test results show that the bridging accuracies which can be achieved with Mark-VI APR and HIRAN control located generally in every stereo model of a flight strip are about the same as can be obtained with vertical ground-control in every other stereo model and horizontal ground-control spaced seven models apart along the flight line.

SEVERAL years ago, the U. S. Air Force took steps to develop an aircraft to replace the RB-50 type aircraft for long-range aerial mapping and charting missions. The result

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* The information contained herein does not necessarily represent the official views of the Corps of Engineers of the Department of the Army.

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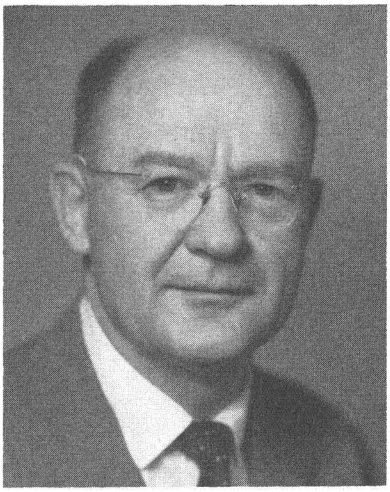
EDITOR'S NOTE.—Because of the request of the author all tables and graphs are grouped as far as practicable and possible.



Frank W. Masek

was a modified version of the Lockheed C-130A Hercules airplane, better known as the RC-130A Aircraft.[1] In addition to its capability for the procurement of various types of aerial photography, this versatile aircraft, with its electronic distance measuring equipment, can obtain profiles of the terrain, which can be converted into ground elevations, and it can determine, fairly precisely, the location of the nadir-point of each vertical aerial photograph, relative to known horizontal ground-control. The terrain profile information is obtained with the Airborne Profile Recorder (APR) and the horizontal positions are determined with HIRAN equipment. The function of the RC-130A aircraft is to obtain the photography and the airborne control data simultaneously.

The feasibility of obtaining the APR and HIRAN control data simultaneously was questioned by the Corps of Engineers, U. S. Army. Accordingly, when the Lockheed Aircraft Corporation, in the summer of 1959, flew several APR and HIRAN controlled photographic missions over the Phoenix, Arizona, Test Area as part of the acceptance tests of the RC-130A production models, the Army Map Service selected one of these missions for use in an operational test of the APR-HIRAN system as installed in the RC-130A aircraft.

The objective of this test, which was begun at Army Map Service in July, 1960, was to determine the accuracy obtainable under operating conditions with the APR and HIRAN control data, using the Army Map Service techniques of bridging and adjustment.

Figure 1 shows the flight line coverage for the Phoenix APR-HIRAN Test. The photography was flown at 20,000 and 36,000 feet above sea level. The lower altitude photography consisted of 10 strips, 7 North-South and 3 East-West, and the high-altitude photography consisted of 2 North-South strips, shown by dashed lines (see Figure 1). All of the prime vertical photography was controlled by APR and HIRAN and was taken with a 6-inch focal-length KC-1 camera in a stabilized mount. The prime vertical flights were planned for an average forward-lap of 56 per cent and an average side-lap of 15 per cent. However, the average side-lap on the lower altitude flights varied from 10 per cent between lines 1 and 2 to 30 per cent between lines 5 and 6. The large relief shadows on line 1, which indicated low solar altitude at the time of flight and the occasional zero side-lap between lines 1 and 2, caused no difficulty in the test, since this was a test to determine aerial triangulation accuracies only.

A positioning-camera is required to aid in tracking the radar profile on the prime vertical photography. Normally, a 35-mm. positioning-camera would be attached directly on the radar antenna, but because of the design characteristics of the RC-130A aircraft, such an arrangement is presently not economically feasible. Hence, as a temporary expedient for this test, a 12-inch focal-length K-38 camera, having a 9-×18-inch format, was installed in a separate mount in the alternate vertical camera station to serve as the spotting-camera for this test. Both the radar antenna and the K-38 spotting-camera were vertically stabilized and operated from a common vertical reference system.

The exposure interval of the spotting-camera was keyed to that of the prime vertical camera, but was set to fire at a 4 to 1 ratio relative to the exposure interval of the mapping camera.[2] As a result, not only were there four times as many APR photos as mapping photos, but the spotting-camera photos were larger than the survey photos. It is neither necessary nor desirable that the APR photos be as large as those obtained with the K-38 camera. A consideration of the problems involved in handling such a large volume of oversized prints immediately shows the undesirability of using the K-38 camera as a spotting camera on mapping projects requiring APR control.

Another undesirable feature of the separate mounting and separate stabilization of the K-38 spotting-camera was the uncertainty that the spotting-camera would always point precisely in the same direction as the radar beam. The purpose of the spotting-camera is to aid in locating on the survey photographs the ground area illuminated by the radar beam. Tests have shown that the radar beam spottings are more random with the K-38 installation than with the spotting camera attached directly to the radar antenna, or even with both the spotting-camera and the radar antenna fixed to the aircraft frame. Nevertheless, the K-38 installation was used in this test because of the predominantly flat terrain in the Phoenix Test Area. It was felt that in such terrain the uncertainty of an accurate positioning of the radar beam would not substantially reduce the accuracy obtainable with this APR system. Over rugged mountainous terrain, however, the positioning of the radar beam becomes more critical, and the spotting-camera installation used in this test would be unsatisfactory.

The APR set installed in the RC-130A aircraft is the Mark VI, manufactured in Toronto by the Canadian Applied Research Division of A. V. Roe. At the time this test began, it was known that the Mark VI APR set would not record signals over rugged mountainous terrain. This was obvious from an examination of the chart profiles and the terrain along the APR track, and was borne out by the results of this test, which showed vertical errors ranging up to 1,700 feet in the mountains near Phoenix.

An improved version of the Mark VI, with modified circuitry, is currently being tested by Army Map Service, using photography flown at an altitude of 30,000 feet above sea-level over the California mountains, from Los Angeles to Searles Lake to Paso Robles.

Prior to the receipt of the HIRAN data by Army Map Service, an evaluation of the accuracy of the HIRAN photo nadir positions was made by a private agency under contract to the Corps of Engineers. During this evaluation, 105 stereo models from the 20,000-foot altitude photography were scaled and leveled to ground-control to determine the UTM coordinates of the HIRAN nadirs. The results of this evaluation showed that the average error in the HIRAN nadirs was about six meters, and that the maximum error was about 16 meters. The results of the Army Map Service evaluation of the HIRAN nadirs agree in general with those of the contractor, but differ significantly for line 2. Investigations are being conducted to determine the cause of the discrepancies for this line. The important point is that the Army Map Service test plans called for the HIRAN data to be used in computing the scale for the vertical adjustment of the strip data, and it was desirable to have advance information on the reliability of the HIRAN data. The scale computation procedure would be the same on any mapping project involving HIRAN control, but in the latter case there would be no advance information on the reliability of the HIRAN data. The test results showed, however, that the vertical adjustment of Strip 2 was not affected by the type of discrepancy noted in the HIRAN control.

Figure 2 shows the extent of the Phoenix Test Area as originally surveyed in 1948. This is the control which was used to evaluate the accuracy of the APR and HIRAN control. The ground control is spaced at 1-mile intervals in the cardinal directions, generally along section line roads. The Phoenix Test Area has served its intended purpose quite well, since the time of its establishment. However, the postwar building boom has affected the metropolitan area of Phoenix perhaps as much as anywhere else in the United States. The city of Phoenix alone is rapidly approaching the 1-half million mark in population, reflecting a growth from 65,000 in 1940, according to census figures. The expansion of built-up areas, and the construction of new roads and realignment and widening of old roads, have all affected the photo-identifiability of the ground-control. Therefore, in order to determine the absolute and comparative accuracies obtainable with APR, HIRAN, and ground-control, it became necessary first to evaluate all the ground-control in the test area. The result of this evaluation is shown in Figure 2. Those points which were used in the test are shown with open-faced symbols, and those points which could not be identified in the stereo models (approximately 25 per cent of the available control in the area), are shown in solid black.

Figure 3 shows the 20,000-foot altitude flight line coverage for the test. Where the North-South flights crossed over the East-West flights, it was found that the terrain clearances as recorded by the APR equipment over these common areas differed by as much as 20 feet. These differences were removed or redistributed along the flight paths by a method of vertical block adjustment developed by C. C. Slama and W. H. Schwieder of AMS from a practical example of a method of least squares adjustment of correlation-free observations given by Tienstra.[4] The entire block of APR data was indexed to ground-control at the two points shown by large triangles, in the North-West and South-East corners of the test area. The remaining 19 tie-points are shown with circles enclosing small triangles, where secondary indexing was required, and with circles. The flight segments are numbered from 1 through 32, followed by the number of APR points in each segment. From the 32 flight segments and the clearance differences at the 19 tie-points, 13 condition equations were developed, and weights were assigned according to the number of APR points in each segment. Corrections for each segment were then computed and applied linearly to the APR clearances, which had been previously corrected for drift errors.

To make effective use of the HIRAN data during the adjustment phase of the test it became necessary first to locate the photograph nadir of each exposure. Two methods of determining the instrument nadir of each exposure during bridging were employed. Assuming the stereoplanigraph to be properly zeroed, the first method involved the summation of the *bx*'s and the *by*'s, while the second technique, called the ray-point method, involved traversing the *z*-column monoscopically a distance of 50 millimeters and determining the x and y coordinates for three points along one edge of each exposure at the various *z*-column readings. Using the data obtained by the second method, the point of intersection of three rays was computed by the point slope method familiar to surveyors. Differences were immediately noted in the x and y values for the instrument nadirs computed by each method.

The problem then was to determine which of the two methods produced the more accurate results in adjustment. Because of time limitations, the two methods were checked out on only two of the strips. The instrument nadirs were adjusted for the tip to tilt in each photogrammetric bridge to determine the respective photo-nadirs, which were then used with the corresponding HIRAN nadirs in the adjustment of the two strips. The limited check of the two methods on the short strips of this test indicated that the ray-point method of determining the photo-nadirs was more accurate than the summation method. For example, on Strips 6 and 7, the ray-point method produced resultant errors of 4.71- and 7.54-meters, respectively, on the ground-control, when the HIRAN control was held, compared to similar errors of 12.99- and 8.09-meters, using the method of summation of *bx*'s and *by*'s. On the remaining strips of the test, therefore, the ray-point method of determining the photo-nadirs was used. However, it is emphasized that the results of this 2-strip test did not thoroughly resolve the issue, and that further investigation of this problem is required, particularly on long flights.

All strips of photography were bridged on the Zeiss C-8 Stereoplanigraph, using the AMS undisturbed model method of bridging. All APR points, ground-control points, and other preselected image-points were read and recorded during the instrumentation phase of the test, along with the orientation data required for determining the photo nadirs.

Comprehensive photogrammetric adjustment tests were conducted. Some APR strips were adjusted on the UNIVAC, using quadratic adjustment equations, and were given auxilliary graphical corrections in an attempt to improve the results, but this method did not prove encouraging, either from a time standpoint or the results obtained. Ultimately, all APR-controlled strips were adjusted by the electronic computer, using the 6th degree adjustment equation now in use at AMS.[5]

A special vertical adjustment test was performed on one strip, using ground-control and varying the number of control bands and the degree of the adjustment equation. The results of this test are shown in Figure 4. When Strip 2 was adjusted to three bands of control, using a quadratic adjustment equation, the errors on the check points midway between the control bands averaged 20 feet. These errors were reduced significantly when a quadratic equation was used with seven bands of control, or control in every other stereo model. A still further reduction in the errors was noted with the use of a cubic adjustment equation and seven bands of control. Higher order adjustment equations were used with seven bands of control, but the results were about the same as with the cubic equation. Because of the results obtained on this special test, all of the lower altitude strip adjustments to ground-control were made with seven bands of control and cubic adjustment equations.

On the other hand, 6th degree equations were used to adjust all of the 20,000-foot altitude strips to the available APR control. Figure 5 shows the comparative vertical bridging accuracies obtained with the APR control and with the vertical ground-control. In obtaining these results, all APR points in areas of signal loss and some APR points in error by 10 feet or more were first deleted

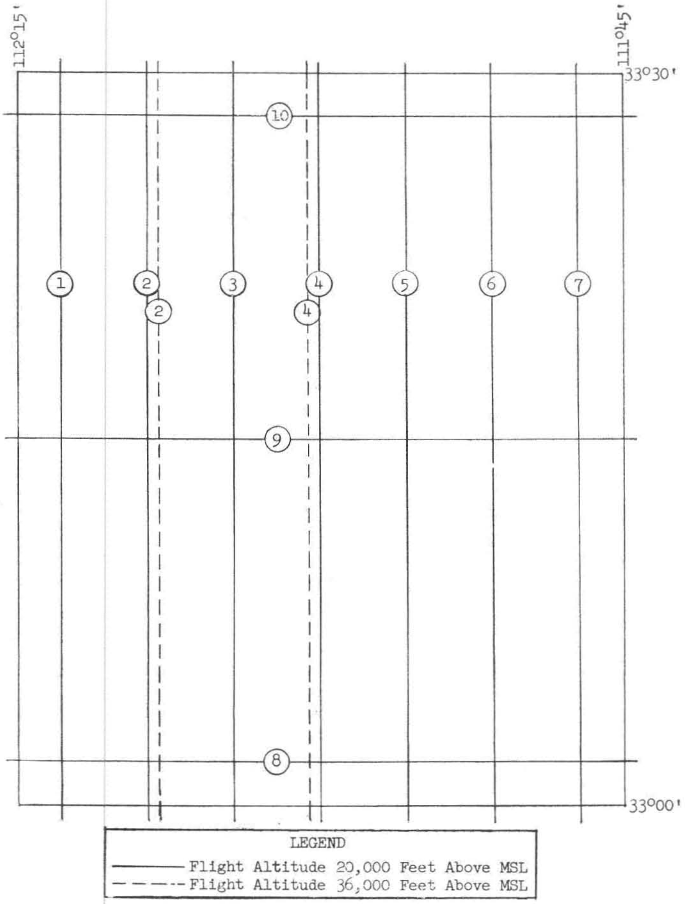


FIG. 1. Flight plan for Phoenix APR-HIRAN test.

from the solutions. In spite of these deletions, about 47 APR points per strip were used to adjust the strips. The test results indicate that the absolute vertical bridging accuracy, holding to vertical ground-control, is only about 1.5-feet better than that obtainable with the Mark VI APR control. On one strip, however, the results with APR were better than with ground-control. The minor superiority of the ground-control results over those of the APR was obtained only because every other stereo model was held to ground-control. It can reasonably be inferred from these test results that APR can show a marked superiority in bridging over that obtainable with ground-control, if the ground-control is spaced significantly farther apart than every two models.

Figure 6 shows a frequency distribution of the errors remaining in the 20,000-foot altitude APR control after using the APR data to adjust Strips 1 through 7. It shows that only a very small portion (9 per cent) of the APR points were in error by more than 20 feet. A substantial portion of these large errors occurred in the mountainous terrain. On the other hand, 55 per cent of the APR points were in error by not more than five feet. These small errors are a reflection of the flatness of the terrain. Still, there is a consistency in the results, which, in a sense, is a measure of the reliability of the Mark VI APR data.

All seven North-South strips flown at 20,000 feet were adjusted to three bands of horizontal ground-control spaced seven models apart, and to HIRAN control in every model, using quadratic adjustment equations in each case. The bridging accuracies, holding to ground control and to HIRAN control,

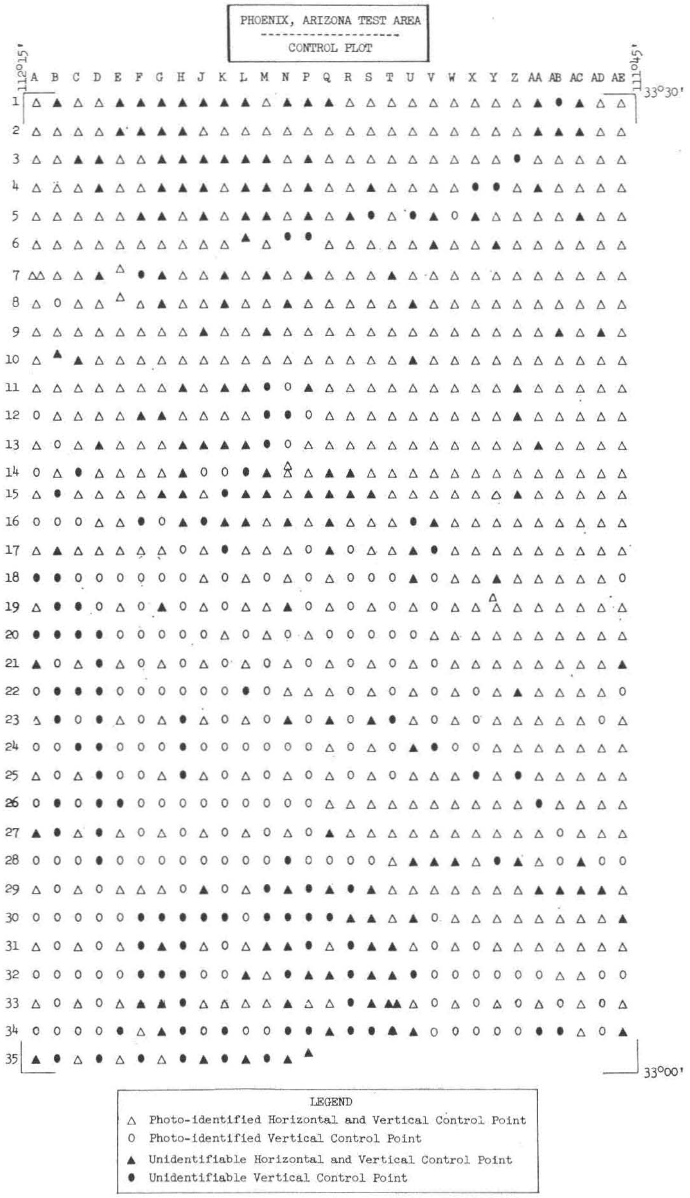


FIG. 2. Distribution of available control in Phoenix test area.

are shown in Figure 7 by the RMS errors on the ground-control used as check-points. Except for Line 2, the errors in the HIRAN nadirs, as determined by AMS and by the contractor, are in general agreement. Excluding Line 2, the bridging errors obtained with HIRAN control in every model are only slightly greater than the errors obtained by bridging with horizontal field-control spaced seven models apart. For all seven strips, the resultant errors on the check points averaged 5.84-meters, using HIRAN control, and 5.02-meters, holding to ground-control.

The two 9-model North-South strips flown at an altitude of 36,000 feet above sea level were first adjusted to vertical ground-control, using a 5th degree equation for Line 2, and a 6th degree equation for Line 4, in order to

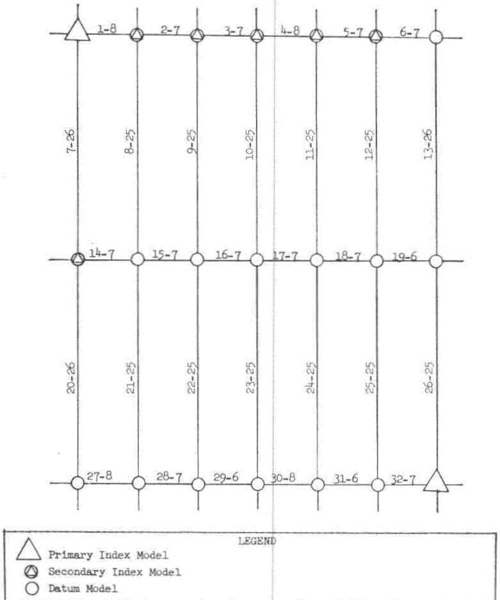


FIG. 3. Layout for block adjustment of APR data.

Strip Number	*Number of Control Bands*	*Degree of Adjustment Equation*	*Number of Check Points*	*RMSE on Check Points (feet)*
2	3	2	107	11.93
2	7	2	93	8.35
2	7	3	93	5.40

FIG. 4. Comparison of Vertical Bridging Accuracies obtained with quadratic and cubic adjustments of a single strip.

determine the "true" ground elevations of the APR points. Line 2 was held to 21 ground elevations, located in six different models of the strip, with 62 ground elevations being withheld as check points. Similarly, 26 ground elevations, distributed among seven models, were used to control the adjustment of Strip 4, and 37 additional elevations were withheld as check-points.

Following these adjustments, each strip was then readjusted to the APR chart elevations. The computed APR elevations from the readjusted strips were compared to the "true" APR elevations to determine the relative bridging accuracy obtainable with the Mark VI APR data measured from 36,000 feet above sea level. The results of bridging with vertical ground control and with APR are shown in Figure 8.

Both APR traces were obtained over terrain which was almost completely devoid of abrupt changes in elevation. The test results show that similar relative accuracies can be obtained over flat terrain with both the Mark VI and the Electronics Associates Model NBA-2 APR equipment.[2]

Figure 9 compares the horizontal bridging accuracies obtained with HIRAN control and with horizontal ground control for the two strips flown at 36,000 feet above sea level. It can be seen that the average of the resultant RMS errors on the horizontal check-points for these two strips was 7.21-meters, when the photogrammetric data were adjusted to the HIRAN control, and 5.17-meters, when the strips were adjusted to the horizontal ground-control.

	APR Control Held				*Vertical Ground Control Held*		
Flight Number	*APR Pts. Used in Solution*	*RMSE on APR Points (feet)*	*Elevation Points Checked*	*RMSE on Check Points (feet)*	*Elevation Points Used in Solution*	*Elevation Points Checked*	*RMSE on Check Points (feet)*
1	41	5.22	100	5.01	23	77	4.02
2	46	4.83	114	7.51	21	93	5.40
3	42	8.47	104	11.70	26	78	6.97
4	49	4.52	115	8.00	28	87	7.31
5	51	4.35	129	6.25	36	93	5.00
6	40	3.90	147	6.88	28	119	7.18
7	58	8.29	143	8.18	26	117	7.70
Average RMSE's		5.65		7.82			6.47

FIG. 5. Comparison of Vertical Bridging Accuracies obtained with Mark VI APR control and vertical ground control.

In summary, the test results with the 20,000-foot altitude photography show that, over predominantly flat terrain, 90 per cent of all adjusted terrain elevations derived from the Mark VI APR equipment were accurate, on the average, to 13 feet or less. Using the APR data as control, the average RMS error on the vertical check-points for the seven strips tested was 7.82-feet, compared to a similar error of 6.47-feet, obtained when the same strips were adjusted to vertical ground-control. Absolute bridging accuracies were not determined with the APR data obtained at 36,000 feet; however, relative accuracies averaging 13 feet for 90 per cent of the terrain elevations along the APR trace were obtained for the two high-altitude strips.

Error in Feet	*Percentage of APR Elevations*
0– 5	55.1
6–10	20.8
11–15	11.7
16–20	3.2
Over 20	9.2

FIG. 6. Distribution of APR Residual Errors when APR Data were used to adjust strips 1 through 7. (Flight altitude: 20,000 Feet.)

As for the horizontal results, average resultant standard errors of 5.83-meters for the seven 20,000-foot altitude strips, and 7.21-meters for the two high-altitude flights, were obtained when the HIRAN data were used to control the adjustments. These results

Flight Number	*HIRAN Control Held*			*Field Control Held*		
	Number of Check Points	*$RMSE_N$ (meters)*	*$RMSE_E$ (meters)*	*Number of Check Points*	*$RMSE_N$ (meters)*	*$RMSE_E$ (meters)*
1	58	3.47	4.91	52	3.10	3.17
2	65	3.26	238.89	57	2.78	4.77
3	56	2.40	3.08	48	3.16	3.68
4	72	6.02	4.25	67	3.03	5.33
5	102	3.05	4.59	95	2.67	3.18
6	126	3.68	2.94	119	3.61	2.91
7	125	5.90	4.69	119	4.63	3.64

FIG. 7. Comparison of horizontal bridging accuracies obtained with HIRAN and with horizontal ground control. (Flight altitude: 20,000 feet.)

Line Number	*RMS Errors (feet)*			
	Vertical Ground Control			*APR Control*
	Held in Solution	*Checked*	*All Points*	*Comparison of Final APR Elevations, APR versus Ground Control Solutions*
2	5.45	9.33	8.52	10.92
4	5.70	9.00	7.81	6.58

FIG. 8. Comparison of absolute and relative bridging accuracies obtained on two 9-model strips flown at 36,000 feet above sea level.

Flight Number	*HIRAN Control Held*			*Field Control Held*		
	Number of Check Points	*$RMSE_N$ (meters)*	*$RMSE_E$ (meters)*	*Number of Check Points*	*$RMSE_N$ (meters)*	*$RMSE_E$ (meters)*
2	61	4.42	4.49	56	4.12	3.30
4	54	5.85	5.60	48	3.86	3.30

FIG. 9. Comparison of horizontal bridging accuracies obtained with HIRAN and with horizontal ground control. (Flight altitude: 36,000 feet.)

compare quite favorably with similar resultant standard errors of 5.02-meters and 5.17-meters for each of the respective altitudes, when the adjustments were made to horizontal ground-control. On the average, 90 per cent of the horizontal check-points were in error by only 10 meters or less when the HIRAN control was obtained at 20,000 feet, and 12 meters or less, using the high-altitude HIRAN control.

In other words, the bridging accuracies which can be achieved with Mark VI APR and HIRAN control located, in general, in every stereo model of a flight strip are about the same as can be achieved with vertical ground-control located two models apart along the flight line, and with horizontal ground-control located about seven models apart.

The results of this test seem to indicate that in order to achieve optimum, and not merely tolerance, accuracies in mapping by photogrammetric methods, every model of a stereo bridge should be adequately controlled. It has been shown that APR and HIRAN can certainly fulfill this requirement.

Bibliographical Notes

1. Walls, J. Kermit, "The RC-130-A Aircraft—A New World Mapping System," PHOTOGRAMMETRIC ENGINEERING, Vol. XXVI, No. 3 (June 1960), pp. 395–401.
2. Slama, Chester C., "Evaluation of an APR System for Photogrammetric Triangulation of Long Flights," PHOTOGRAMMETRIC ENGINEERING, Vol. XXVII, No. 4 (September 1961), pp. 572–578.
3. Report, Final Technical "Precise Photogrammetric Orientation Data Determination of HIRAN Mapping System," U. S. Engineer Research and Development Laboratories, Ft. Belvoir, Virginia, Contract No. DA-44-009 Eng. 3483 (27 April 1960).
4. Tienstra, J. M., "Theory of the Adjustment of Normally Distributed Observations," pp. 125–128.
5. Nowicki, A. L. and Born, C. J., "Improved Stereotriangulation Adjustments with Electronic Computers," PHOTOGRAMMETRIC ENGINEERING, Vol. XXVI, No. 4 (September 1960), pp. 599–604.

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*Optimum Utilization of Airborne Sensors in Military Geography**

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(*Abstract is on the next page*)

AIRBORNE sensors can certainly be classed as tools of great potential value to the military geographer. However, optimum utilization of these tools and valid interpretation of sensor data for specific military geographic purposes can be assured only through the conduct of well-planned research and development programs. To illustrate some of the problems involved in such programs, I will discuss the role of airborne sensors in military terrain studies.

Terrain may be considered to be the aggregate of the physical characteristics of an area. Terrain can, therefore, be analyzed and described in terms of numerous component factors, such as slope, relief, distribution of topographic highs and lows, occurrence of steep slopes, soil type or grain size, soil

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