The Adjustment of Area Computations from Sampling Devices on Aerial Photographs

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ABSTRACT: *Misinterpretations and scale variations are the two main sources for biased errors. The paper proposes a way for calculating the adjustment due to scale variation. It deals with the various influences of adjustments due to misinterpretation and to scale variation on the combined standard error. A comparison of plot sampling with transect sampling favored the latter because of its better and more efficient statistical effect with regard to the combined standard errors.*

1. INTRODUCTION

ADJUSTMENT becomes necessary if checking has revealed biased errors caused by mistakes of measurements or through interpretation.

Two kinds of bias occur mainly in computations of areas of strata in sampling on photographs, namely: 1) Bias due to misinterpretation and; 2) Bias due to scale variation.

Misinterpretations can be reduced in many cases by careful selection and training of the interpreters, continual checking of work and restriction of the stratification to a minimum of easily determined strata, but their occurrence must always be expected. Checking the interpretation must be part of the standard procedure of any method of area sampling from photographs.

Bias due to scale differences is a consequence of the natural features of aerial photographs. The relief displacement produces an over-estimate of areas above datum plane and vice versa. This bias can be neglected if differences of altitude in the area, relative to the flying height, are small or if the strata are not correlated with topographical position. All photographs are assumed to be truly vertical in the following study.

2. PHOTO PLOT SAMPLING (*plot device*)

Plot sampling on aerial photographs is the most common device.

The proportion of stratum P_j of the total area equals the proportion of plots interpreted as belonging to stratum j $(=k_j)$ of all plots $(=k)$:

$$P_j = \frac{k_j}{k} \cdot \qquad (1)$$

The error of the proportion of area is computed according to the error formula for binomial distributions because qualitative attributes are concerned:

$$S_{P_j} = \frac{\sqrt{P_j(1 - P_j)}}{k} \cdot \qquad (2)$$

INTERPRETATION ADJUSTMENT FOR PLOT DEVICE

The necessity of adjustment is determined by the results of a subsampling. A fraction of the photo plots is checked in the field for stratum determination. The selection may be proportional (every n'th plot) or disproportional (every n'th plot of stratum j).

The number of misinterpretations in the photo-sampling is indirectly deducted from the number of misinterpretations discovered by the field-check. The checked plots have a relatively high weight because the number of plots which can be checked in the field has to be kept small on account of costs. It has to be assumed that the determinations of the stratum, and the location of the ground positions of the plots, are without error. The realization in practice of this assumption sets a high technical standard of checking.

The mathematical technique of the interpretation adjustment is well known (Forestry Handbook, ASP PHOTOGRAPHIC INTERPRETATION MANUAL). The example given below corresponds to the conditions in the Uthai Thani forest in Thailand. There are two strata:

Stratum 1, the tropical evergreen forest, important through the occurrence of valuable commercial tree species and situated predominantly at low and medium altitudes.

Stratum 2, this covers mainly higher altitudes with forests of species of small commercial value.

5,600 photo-plots are interpreted of which 1,680 plots are classed as "Stratum 1" and 3,920 as "Stratum 2." The proportion of Stratum 1 of the total area is:

$$P_1 = \frac{k_1}{k} = \frac{1680}{5600} = 0.300.$$

The standard error of P_1 is calculated by formula (2) as ± 0.012 at a 95% probability. The area assessment of the photo interpretation is therefore:

$$P_1 = 0.300 \pm 0.012(= \pm 4.0\%).$$

150 plots of Stratum 1 and 50 plots of Stratum 2 are selected for checking in the field (disproportionate sub-sample). Misinterpretations are discovered in three plots of Stratum 1 and in two plots of Stratum 2.

The adjusted area (adj. *i.*) P_1 of Stratum 1 is calculated by multiplying the proportion of stratum of the photo interpretation (P_j) with the proportion of field check plots (n_{j1}) determined in the field as Stratum 1 ($= P_j p_{j1}$). The sum of the products calculated for all strata gives the required "adjusted" proportion of area of Stratum 1:

$$(\text{adj. } i.)P_1 = \sum_1^M P_j p_{j1} \tag{3}$$

where

$$p_{j1} = \frac{n_{j1}}{n_j} \quad \text{and} \quad M = \text{number of strata.}$$

The standard error of this adjusted area proportion is a combined error of the standard error of the photo interpretation and of the ground check, and is calculated as follows:

$$S^2{}_{(\text{adj. } i.)P_j} = \sum_1^M \frac{P_j^2 p_{j1}(1 - p_{j1})}{n_j} + \frac{1}{k}\left(\sum_1^M P_j p_{j1}^2 - \left(\sum_1^M P_j p_{j1} \right)^2 \right). \tag{4}$$

The following table gives the basic data and the computations for the adjustment:

Stratum	Photo-plots		Check-plots			Adjustment		Error calculation		
	k_j	P_j	n_j	n_{j1}	n_{j2}	p_{j1}	P_jp_{j1}	$P_jp_{j1}^2$	$\frac{P_j^2p_{j1}}{n_j}$	$\frac{P_j^2p_{j1}^2}{n_j}$
$j=1$	1,680	0.30	150	147	3	0.98	0.294	0.28812	0.000588	0.000576
2	3,920	0.70	50	2	48	0.04	0.028	0.00112	0.000392	0.000016
$\sum_1^2$	5,600	1.00	200	—	—	—	0.322	0.28924	0.000980	0.000592

The proportion of area adjusted for errors due to misinterpretation is therefore

$$(\text{adj. } i.)P_1 = 0.322$$

and the error of this expression is according to (4):

$$S^2_{(\text{adj. } i.)P_1} = 0.000980 - 0.000592 + \frac{0.28924 - 0.10368}{5600}$$

$$S_{(\text{adj. } i.)P_1} = \pm 0.02052.$$

The standard error of the adjusted stratum proportion at 95 per cent probability level is therefore ± 0.04104 $(=\pm 12.75\%)$.

The relatively few misinterpretations increased the standard error from $\pm 4\%$ to almost $\pm 13\%$. This shows clearly the degree to which misinterpreted checked plots influence the results of sampling, and reduce the value of information on stratum areas obtained from photographs.

SCALE ADJUSTMENT IN PHOTO PLOT SAMPLING

Plot sampling devices provide the stratum area as projected on the photographs· Required is the stratum area projected on a map. The bias caused by differences of scale can be eliminated in various ways.

Aldrich, R. C. (1955) and Hartmann, F. J. (1947) recommend elimination of this bias by locating plots at precise map intervals.

The ASP MANUAL OF PHOTOGRAPHIC INTERPRETATION mentions a method by scale adjustment: "The direct measurements may be adjusted by factors based on the relative elevations of the points sampled" (p. 469).

The following symbols are used in the description of the mathematical calculations of this method:

H_0 = Flying height a.s.l.
h_{0i} = Altitude of a sample plot i a.s.l.
f = Length of focus
RF_d = Representative fraction in datum plane $=f/(H_0-h_d)$
RF_i = representative fraction of sample plot $i=f/(H_0-h_{0i})$.

The aerial photographs are assumed to be truly vertical and the flying-height a.s.l. is considered constant in the quoted example. Scale differences caused by major variations of flying-height can be eliminated by adjustment if the flying-height is included as variable in the scale adjustment factor.

The quotient of the two representative fractions can be written as

$$\frac{RF_d}{RF_i} = \frac{H_0 - h_{0i}}{H_0 - h_d}.$$

The factor

$$\left(\frac{H_0 - h_{0i}}{H_0 - h_d}\right)^2$$

represents the weight with which each photo plot has to be weighed according to its altitude.

The following formula is used for weighing any plot which is transferred from the air-photo projection to the orthogonal projection:

$$w_i = \left(\frac{H_0 - h_{0i}}{H_0 - h_d}\right)^2.$$

The fraction $(RF_d/RF_i)^2$ may also be expressed by parallaxes (P), measured on the photo, and air-base (B), measured on the ground. Hence,

$$w_i = \left(\frac{P_d \cdot B_i}{B_d - P_i}\right)^2.$$

This version for w_i was developed by Rogers, E. J. (1962).*

The proportion of area of stratum j, adjusted for scale differences, is therefore:

$$(\text{adj. } c.)P_j = \frac{\sum_1^{k_j} w_i}{\sum_1^{k} w_i}. \tag{5a}$$

It is not necessary to obtain the altitudes of all plots and subsampling is sufficient. In this way formula (5a) is transformed:

$$(\text{adj. } c.)P_j = P_j \frac{\frac{1}{n_j}\sum_1^{n_j} w_i}{\frac{1}{n}\sum_1^{n} w_i} = P_j \frac{\bar{w}_j}{\bar{w}}. \tag{5}$$

The control of the interpretation requires the location of check plots in the field. This makes possible obtaining the altitude of a number of plots with an aneroid barometer. A second possibility is the assessment of altitude by measurement of parallax differences, if the photograph containing the plots includes a distinguishable point of known altitude. This requirement will easily be met in all cases where useful maps are available.

The following example is again the Uthai Thani forest in Thailand. The altitude of all 200 check plots has been measured. The discovered misinterpretations are disregarded for the moment in order to consider the influence of the scale adjustment on proportion of stratum and accuracy of area assessment alone.

The photo-scale is 1:15,000. The flying-height a.s.l. is 2,700 meters ($=H_0$) and the altitude a.s.l. of the datum plane is 400 meters ($=h_d$).

The following table gives the distribution of checked plots in altitude classes. The mean weight of each of the eight altitude classes is nominated as w_g.

* Unpublished, personal communication of June 4, '62.

Altitude of check plots a.s.l. in m.	w_g	w_g^2	n_{1g}	n_{2g}
g = 200–250	1.15768	1.34022	9	—
250–300	1.11145	1.23532	27	—
300–350	1.06616	1.13670	59	4
350–400	1.02182	1.04412	30	8
400–450	0.97842	0.95731	20	9
450–500	0.93596	0.87602	5	15
500–550	0.89444	0.80002	—	10
550–600	0.85386	0.72908	—	4
			150	50

$$\sum_{g=1}^{g=8} n_{1g} w_g = 158.23451 \qquad \sum n_{1g} w_g^2 = 167.33082$$

$$\sum n_{2g} w_g = 47.64422 \qquad \sum n_{2g} w_g^2 = 45.57237.$$

The formula (5) has to be altered for the calculation of the adjustment because the sub-sample in this case is disproportional and the w_i-values are grouped.

$$\bar{w}_j = \frac{\sum_1^8 n_{jg} w_g}{n_j}$$

and

$$\bar{w} = \frac{\sum_1^M \bar{w}_j k_j}{k} = \frac{\bar{w}_1 k_1 + \bar{w}_2 k_2}{k}$$

$$\bar{w}_1 = 1.0549$$

$$\bar{w}_2 = 0.9529$$

$$\bar{w} = 0.9835.$$

The scale adjustment factor for the proportion of area of Stratum 1 is therefore:

$$\frac{\bar{w}_1}{\bar{w}} = 1.0726.$$

The proportion of area of Stratum 1, adjusted for differences of scale, is:

$$(\text{adj. } c.) P_1 = 1.0726 \cdot 0.3 = 0.32178.$$

The scale adjustment factor $\bar{w}_j/\bar{w}$ has the errors of $\bar{w}_j$ and $\bar{w}$. The error of $\bar{w}_j$ is calculated as:

$$S_{w_j}^2 = \frac{\sum_{g=1}^{g=8} n_{jg}(w_g - \bar{w}_j)^2}{n_j(n_j - 1)} = \frac{\sum_{g=1}^{g=8} n_{jg} w_g^2 - \frac{\left(\sum_{g=1}^{g=8} n_{jg} w_g\right)^2}{n_j}}{n_j(n_j - 1)} \cdot \qquad (6a)$$

The calculation of the error of $\bar{w}$ has again to allow for the disproportionate sub-sample:

$$\bar{w} = \frac{\bar{w}_1 \cdot k_1 + \bar{w}_2 \cdot k_2}{k}.$$

Hence

$$(\text{adj. } c.)P_j = \frac{\dfrac{\bar{w}_1}{\dfrac{\bar{w}_1 k_1 + \bar{w}_2 k_2}{k}}}{} \cdot \frac{k_1}{k} = \frac{\bar{w}_1 \cdot k_1}{\bar{w}_1 k_1 + \bar{w}_2 k_2}.$$

The mean standard error of the scale adjustment is approximately according to the propagation of errors:

$$S^2_{(\text{adj. } c.)P_j} = ((\text{adj. } c.)P_j)^2 \left\{ \frac{k_j^2 S_{w_j}^2 + \bar{w}_j^2 S_{k_j}^2}{(\bar{w}_j \cdot k_j)^2} + \frac{\sum_1^M (k_j^2 S_{w_j}^2 + \bar{w}_j^2 S_{k_j}^2)}{\left(\sum_1^M \bar{w}_j k_j\right)^2} \right\}.$$

The proportion of area estimate of the example is at 95 per cent probability:

$$(\text{adj. } c.)P_1 = 0.32178 \pm 0.01485 (= \pm 4.6\%).$$

The proportion of area of the predominantly lowland tropical evergreen forest (Stratum 1) is, therefore, underestimated by 7.3% as a result of scale differences due to relief, or by 5,500 ha compared with a total forested area of the Uthai Thani forest of 250,000 ha.

The example shows the importance of scale adjustment. The maximum deviation of altitude above and below datum plane is 8.7% of the flying height. The errors can be substantial even in areas with small deviations, if the strata correlate to topographical positions. This is the case in most forest areas of the world. Forest types frequently change according to ecological conditions, which are strongly affected by altitude, exposition and inclination. In agriculturally cultivated regions, forest is, by human activity, restricted to inaccessible areas of high altitude.

It is advisable to obtain the necessary information on altitudes when field checks are made, if areas are assessed from aerial photographs, and if the sample plots are distributed without regard to scale differences. The scale adjustment, in comparison to the interpretation adjustment, shows one agreeable feature: The adjustment only slightly increases the standard error. This error in the present example is $\pm 4.0\%$ for photo-sampling. The scale adjustment changes the proportion of area considerably ($P_1 = 0.3$; (adj. $c.$)$P_1 = 0.32178$). The standard error, however, increases only to $\pm 4.6\%$. Even much more extreme differences of altitudes than in the example would not produce conditions in which the accuracy of the area assessment, and the value of the information obtained, would be greatly reduced by the error of the scale adjustment factor.

COMBINED ADJUSTMENT IN PHOTO PLOT SAMPLING

The control of area assessment by photo plot sampling almost always requires adjustments of both interpretation and scale. In this case it is necessary to combine both adjustments to one procedure.

The double-adjusted proportion of area is then:

$$(\text{adj. } i.c.)P_1 = \frac{\sum_1^M P_j p_{j1} \bar{w}_{j1}}{\bar{w}} = \frac{P_1 p_{11} \bar{w}_{11} + P_2 p_{21} \bar{w}_{21}}{\bar{w}}. \quad (7)$$

Therefore it is necessary to obtain the mean weights of the checked plots ($\bar{w}_{11}$ and $\bar{w}_{21}$) which were, in the field, determined as Stratum 1 plots.

Altitude of the check plots a.s.l. in m.	n_{11g}	n_{12g}	n_{22g}	n_{21g}
g = 200–250	9	—	—	—
250–300	27	—	—	—
300–350	59	—	4	—
350–400	28	2	7	1
400–450	19	1	8	1
450–500	5	—	15	—
500–550	—	—	10	—
550–600	—	—	4	—
	147	3	48	2

Thence:

$$\sum n_{11g}w_g = 155.21245 \qquad \sum n_{11g}w_g^2 = 164.28527$$
$$\sum n_{21g}w_g = 2.00024 \qquad \sum n_{21g}w_g^2 = 2.00143$$

The three check plots in Stratum 1, which were misinterpreted on the photographs, (n_{12}), belong to altitude classes 350–400 and 400–450. The two misinterpreted plots in Stratum 2 are in the same altitude classes. Thence, from the sum values:

$$\bar{w}_{11} = 1.055867 \quad \text{and}$$
$$\bar{w}_{21} = 1.000120,$$

and further:

$$(\text{adj. } i.c.)P_1 = \frac{0.3\cdot 0.98\cdot 1.055867 + 0.7\cdot 0.04\cdot 1.000120}{0.983488}$$
$$= 0.34418.$$

The combined adjustment alters the result of the non-adjusted photo-sampling in this case by 11.5%. The area of the commercially valuable Stratum 1 in the Uthai Thani forest is 75,000 hectares (ha) according to pure photo-sampling. The adjustment of the two biases results in a stratum area of 86,045 ha.

The mean standard error of this double-adjusted proportion of area is according to the propagation of error

$$S^2_{(\text{adj. } i.c.)P_1} = ((\text{adj. } i.c.)P_1)^2 \left[\frac{\sum_1^M S^2_{P_j p_{j1} w_{j1}}}{\left(\sum_1^M P_j p_{j1} \bar{w}_{j1}\right)^2} + \frac{S_{\bar{w}}^2}{\bar{w}^2} \right]. \tag{8}$$

This expression, in the example of only two disproportionately sampled strata, can be transformed:

$$S^2_{(\text{adj. } i.c.)P_1} = ((\text{adj. } i.c.)P_1)^2 \left(\frac{\bar{w}_{11}^2 S^2_{P_1 p_{11}} + \bar{w}_{21} S^2_{P_2 p_{21}} + (P_1 p_{11})^2 S_{w_{11}}^2 + (P_2 p_{21})^2 S_{w_{21}}^2}{(P_1 p_{11} \bar{w}_{11} + P_2 p_{21} \bar{w}_{21})^2} + \frac{\bar{w}_1^2 S_{P_1}^2 + \bar{w}_2^2 S_{P_2}^2 + P_1^2 S_{w_1}^2 + P_2^2 S_{w_2}^2}{(P_1 \bar{w}_1 + P_2 \bar{w}_2)^2} \right).$$

If this case is at 95% probability ($t \sim 2.0$):

$$tS_{(\text{adj. } i.e.)P_1} = \pm\ 0.04290 (= \pm 12.5\%).$$

The absolute standard error has been somewhat increased above the error of the proportion of area adjusted because of misinterpretation, but the relative standard error has become slightly smaller. This indicates that the error of the adjustment due to scale differences has a very small effect on the total error. The misinterpretations provide the greatest source of error and influence on the combined standard error. This fact is not limited to the above example and is typical for area estimates from photo plot sampling devices. The standard error of the scale adjustment factor will always play a subordinate rôle in comparison with that of misinterpretations, because of the relatively small span of variation between w_g-factors (in extreme cases 0.65–1.4, but mostly smaller).

3. Transect Devices

The quantitative attribute "stratum" is transformed into a qualitative attribute if the area assessment is made by sampling with transect devices. A fixed number $(=k)$ of transects is distributed in a random or systematic manner on the aerial photographs. The transect may be equal $(=l)$ or of unequal $(=l_i)$ length. The length (X_i) along each transect of the particular stratum is measured and the proportion of area of the stratum is then calculated from the total length of the stratum and the total length of all transects:

$$P = \frac{\sum_1^k X_i}{\sum_1^k l_i}. \tag{9}$$

If all transects are of equal length then l_i becomes a constant factor $(=l)$ and expression (9) becomes:

$$P = \frac{\sum_1^k X_i}{k \cdot l} = \frac{\overline{X}}{l}. \tag{9a}$$

The check of the photo-interpretation in the field is possible without restrictions if the ground is flat and the photographs are free of scale differences due to tilt and variations of flying height. The check-transects are located in the field, walked along the compass-bearing and the lengths of the various strata are measured.

In hilly country only the starting and the end point of the transect can be located accurately. The line between the two points as a result of displacement due to relief, will become a more or less irregularly curved line in the field which can be located and checked only with great difficulty. In extreme cases the radial displacement can be of such magnitude that there is only a loose correlation between the photo-measurements and the measurements which shall be made in the field-check.

These deviations can be eliminated if the transects are located along lines which pass through the nadir, or if the nadir itself is the starting point of each transect. In this case radial point displacements only occur along the line and do not result in deviations from the line. This makes possible carrying out the ground check and also eliminating the sources of bias in transect sampling which were mentioned.

A disproportional ground check is not possible, because each transect contains not only a single stratum, but may contain several or all strata. A representative frac-

TABLE 1
TRANSECT-SAMPLING (CHECK-TRANSECTS)

Photograph	*Transect 1*			*Transect 2*			*Transect 1 + 2*		
	Length of stratum on the photograph x_{j1}	*Length of transect on the ground* l_{0j1}	*Length of stratum on the ground* z_{j1}	*Length of stratum on the photograph* x_{j2}	*Length of transect on the ground* l_{0j2}	*Length of stratum on the ground* z_{j2}	*Length of stratum on the photograph* x_j	*Length of transect on the ground* l_{0j}	*Length of stratum on the ground* z_j
$j=$ 1	30.0	28.9	25.0	30.0	29.2	21.0	60.0	58.1	46.0
2	30.0	29.0	27.0	30.0	28.1	27.1	60.0	57.1	54.1
3	30.0	27.9	26.3	30.0	27.9	26.1	60.0	55.8	52.4
4	22.0	29.8	15.0	12.7	29.5	8.5	34.7	59.3	23.5
5	27.0	29.0	28.5	28.0	29.0	20.0	55.0	58.0	48.5
6	16.0	31.0	15.9	20.0	29.5	15.3	36.0	60.5	31.2
7	30.0	27.0	23.0	10.5	29.9	15.5	40.5	56.9	38.5
8	8.5	31.0	12.5	28.0	27.5	27.5	36.5	58.5	40.0
9	—	30.0	—	—	30.0	3.5	—	60.0	3.5
10	15.0	31.0	16.5	—	31.5	6.5	15.0	62.5	23.0
11	—	30.6	3.5	—	31.6	—	—	62.2	3.5
12	—	30.5	3.5	—	30.9	0.5	—	61.4	4.0
13	—	30.5	2.5	15.5	30.0	15.5	15.5	60.5	18.0
14	7.5	30.6	6.2	21.0	30.1	20.0	28.5	60.7	26.6
15	30.0	27.8	22.8	5.0	31.5	3.5	35.0	59.3	26.3
16	—	31.0	—	6.0	30.9	1.5	6.0	61.9	1.5
17	28.0	28.0	23.5	27.0	28.4	25.0	55.0	56.4	48.5
18	4.0	30.5	3.9	—	30.0	6.5	4.0	60.5	10.4
19	22.0	30.3	20.5	14.0	29.8	12.3	36.0	60.1	32.8
20	—	31.5	5.9	—	31.0	—	—	62.5	5.9
21	30.0	28.3	20.0	4.0	31.1	4.0	34.0	59.4	24.0
22	—	31.0	1.5	—	31.9	—	—	62.9	1.5
23	—	31.5	0.5	—	30.6	6.5	—	62.1	7.0
24	30 0	27.9	21.8	4.0	30.5	8.5	34.0	58.4	30.3
25	14.0	30.0	7.0	13.0	30.0	6.5	27.0	60.0	13.5
26	—	31.0	—	—	29.5	—	—	60.5	—
27	13.0	29.8	10.5	28.5	30.0	25.3	41.5	59.8	35.8
28	—	30.6	5.5	—	29.4	—	—	60.0	5.5
29	12.0	30.5	9.5	—	29.5	1.5	12.0	60.0	11.0
30	12.0	30.4	11.5	15.0	30.0	12.1	27.0	60.4	23.6
31	—	32.2	—	—	30.6	6.0	—	62.8	6.0
32	13.0	30.3	10.0	18.0	30.0	13.0	31.0	60.3	23.0
33	—	27.0	6.3	—	31.1	1.5	—	58.1	7.8
34	30.0	30.0	26.3	30.0	28.5	22.0	60.0	58.5	48.3
35	9.0	29.5	10.1	3.0	30.3	7.0	12.0	59.8	17.1
36	20.0	29.8	14.2	21.0	29.5	14.0	41.0	59.3	28.2
37	10.0	29.9	5.3	—	30.3	5.0	10.0	60.2	10.3
38	—	31.0	4.3	—	30.2	—	—	61.2	4.3
39	—	31.3	2.3	—	30.0	3.0	—	61.3	5.3
40	—	30.9	4.3	—	30.1	1.5	—	61.0	5.8
							Σ 907.2	2,398.2	846.1

tion $(=n)$ of the photo-transects is selected and the lengths of the strata are measured in the field. It is impossible to determine in each individual case whether any deviations of lengths of strata measured in the field from the lengths measured on the photograph are caused by differences of interpretation or by scale differences, because both kinds of bias may be superimposed.

A simple example illustrates the combined adjustment in photo-transect sampling.

The area is covered by 800 $(=M)$ aerial photographs of a mean scale of 1:30,000. Two transects $(=n_j)$ of 30 mm. each and starting from the nadir are interpreted on each photograph. The mean length on the ground of the transect is 1 km. Only two strata are distinguished (forest and non-forest). The length of forest along transect i on photograph j is called X_{ij}. The number of single observations is therefore $M \cdot n_j$ $(=1{,}600)$. The selection of two transects per photograph imposes a restriction, and the standard error has to be determined by analysis of variance. Forty photographs $(=m)$ of the 800 $(=5\%)$ are selected for ground checking. The check transects have two functions. First, the correlation between the length of strata on the photograph (x_{ij}) and the length on the ground (z_{ij}) is investigated by regression analysis; second, the mean length of transect on the ground $(=l_0)$ is derived from the 80 transects.

As a first step the mean length of the stratum forest $(\overline{X})$ is computed from the photo-transect sampling and the precision of this information is calculated. The sums for these calculations are given in Table 2. It is:

$$\overline{X} = \frac{\sum_1^M \sum_1^{n_j} X_{ij}}{M \cdot n_j} = 11.2169 \text{ mm.}$$

and

$$S_{\bar{X}}^2 = \frac{\sum_1^M \sum_1^{n_j} (X_{ij} - \overline{X}_j)^2}{M^2 n_j} = \frac{\sum\sum X_{ij}^2 - \dfrac{(\sum\sum X_{ij})^2}{M \cdot n_j}}{M^2 \cdot n_j} = 0.02953.$$

The proportion of the stratum "forest" in the photo-sampling is:

$$P = \frac{\overline{X}}{l} = \frac{11.216875}{30} = 0.3739.$$

The standard error is at a 95% probability level $(t=1.96)$:

$$tS_P = \frac{tS_X}{l} = \frac{0.3368}{30} = \pm\ 0.0112 \qquad (=\pm 3.0\%).$$

The proportion of the stratum forest adjusted for misinterpretation and scale differences is:

$$(\text{adj. } i.c.)P = \frac{\bar{z} + b(\overline{X} - \bar{x})}{\bar{l}_0}. \tag{10}$$

The regression between z_{ij} and x_{ij} is calculated as follows: The single values of the calculations are given in Table 1 and the sums in Table 2. The single values of the field-measurements have been transformed to photo-scale. The variances and covariances inside blocks must be computed for the calculation of b because of the restriction mentioned earlier. From the data of Table 2:

$$\bar{z} = \frac{\sum\sum z_{ij}}{m \cdot n_j} = 10.57625 \text{ mm.}$$

$$\bar{x} = \frac{\sum\sum x_{ij}}{m \cdot n_j} = 11.34000 \text{ mm.}$$

$$\bar{l}_0 = \frac{\sum\sum l_{0ij}}{m \cdot n_j} = 29.97750 \text{ mm. and}$$

TABLE 2

SUMS FOR THE COMPUTATION OF THE ADJUSTMENTS IN PHOTO TRANSECT SAMPLING

Photo transects	*Check transects*		
$\sum_{1}^{800}\sum_{1}^{2} X_{ij} = 17,947.0$	$\sum_{1}^{40}\sum_{1}^{2} x_{ij} = 907.2$	$\sum\sum z_{ij} = 846.1$	$\sum\sum l_{oij} = 2,398.2$
$\sum_{1}^{800}\sum_{1}^{2} X_{ij}^2 = 434,485.25$	$\sum\sum x_{ij}^2 = 21,228.54$	$\sum\sum z_{ij}^2 = 15,421.43$	$\sum\sum l_{oij}^2 = 71,994.70$
$\sum_{1}^{800}\left(\sum_{1}^{2} X_{ij}\right)^2 = 793,370.25$	$\sum(\sum x_{ij})^2 = 38,295.34$	$\sum(\sum z_{ij})^2 = 28,543.83$	$\sum(\sum l_{oij})^2 = 143,902.30$
$M = 800$	$m = 40$	$\sum\sum x_{ij}z_{ij} = 17,601.85$	
$n_j = 2$	$n_j = 2$	$\sum(\sum x_{ij}\sum z_{ij}) = 32,506.50$	

$$b = \frac{\sum\sum (x_{ij} - \bar{x}_j)(z_{ij} - \bar{z}_j)}{\sum\sum (x_{ij} - \bar{x}_j)^2} = \frac{\sum\sum x_{ij}z_{ij} - \dfrac{\sum(\sum x_{ij} \sum x_{ij})}{n_j}}{\sum\sum x_{ij}^2 - \dfrac{\sum(\sum x_{ij})^2}{n_j}}$$

and from this

$$(\text{adj. } i.c.)P = \frac{10.57625 + 0.648094(11.216875 - 11.34000)}{29.97750}$$

$$= \frac{10.49653}{29.97750} = 0.350147.$$

The adjusted proportion of stratum forest is therefore 6.4% less than the proportion determined from the photographs.

The error of this combined expression is derived from three components:

a) the standard error of X
b) the standard error of the regression, and
c) the standard error of l_0.

The standard error of X has been mentioned earlier. It is included in the standard error of the regression, which represents the standard error (adj. *i.c.*)$\overline{X}$:

$$S^2_{(\text{adj. } i.c.)\bar{X}} = \frac{s_{zx}^2}{m} + (\overline{X} - \bar{x})^2 \frac{s_{zx}^2}{\sum\sum (x_{ij} - \bar{x}_j)^2} + b^2 S_X^2,$$

where

$$s_{zx}^2 = \frac{\sum\sum (z_{ij} - \bar{z}_j)^2 - b\sum\sum (x_{ij} - \bar{x}_j)(z_{ij} - \bar{z}_j)}{m - 1}.$$

This expression is calculated from the data of Tables 1 and 2:

$$S^2_{(\text{adj. } i.c.)\bar{X}} = 0.100755.$$

The precision of (adj. *i.c.*) $\overline{X} = 10.49653$ mm. is ± 0.64147 ($\pm 6.11\%$) at 95% probability level.

The standard error of the lengths of transects measured in the field is calculated as:

$$S_{l_0}^2 = \frac{\sum\sum (l_{0ij} - \bar{l}_{0j})^2}{m^2 \cdot n_j} = 0.0136094.$$

The adjusted proportion of area is composed of the quotients of (adj. *i.c.*) $\overline{X}$ and $\bar{l}_0$. Both values have an error. The error of the quotient is approximated by the formula:

$$S_P^2 = (P)^2 \left(\frac{S^2_{(\text{adj. } i.c.)\bar{X}}}{((\text{adj. } i.c.)\overline{X})^2} \right) + \frac{S_{l_0}^2}{\bar{l}_0^2}. \qquad (11)$$

The precision of the adjusted proportion of area is therefore:

$$(\text{adj. } i.c.)\ P = 0.35015 \pm 0.02158 (= \pm 6.16\%).$$

The adjusted proportion of area differs considerably from the value which was determined from photo-sampling (−6.4%). This difference is without doubt, as seen from Table 1, largely caused by misinterpretations, and only to a smaller degree by scale differences. Despite this, the loss of precision is rather small in comparison to the loss of precision in the adjustment for errors due to misinterpretations using plot sampling devices.

4. Conclusions

The field check is considered an essential part of any procedure of interpretation of aerial photographs. An adjustment for scale differences becomes possible if the altitudes of check plots are measured. The t-test decides upon the necessity of adjustment: If the differences in one and the scale adjustment factor $\overline{W}_j/\overline{W}$ are significant, adjustments have to be applied.

The standard error of the adjusted stratum area is considerably more influenced by misinterpretations than by scale variations.

The qualitative attribute "stratum" is transformed into a quantitative form in transect devices, and the adjustment can be made by regression analysis. Most suited for this treatment are transects which pass through the nadir. Both adjustments are done simultaneously in the field and in the computations. The lengths of strata along the transects measured on the photographs are correlated to the lengths measured on the ground. The standard error of the adjusted values is calculated from the standard error of the regression and the sampling error of all photo-transects. In case the correlation is close, the adjustment will influence only slightly the standard error of the adjusted proportion of area. This condition may be expected for most cases in practice.

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Strip-Triangulation with Independent Geodetic Control

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1. Introduction

The method of strip-triangulation with independent geodetic control as described by Dr. Brandenberger[1] has been creating further interest amongst the world's photogrammetrists. This has again been noticed in the recent paper by Mr. Colcord