from Table 1, largely caused by misinterpretations, and only to a smaller degree by scale differences. Despite this, the loss of precision is rather small in comparison to the loss of precision in the adjustment for errors due to misinterpretations using plot sampling devices.

4. Conclusions

The field check is considered an essential part of any procedure of interpretation of aerial photographs. An adjustment for scale differences becomes possible if the altitudes of check plots are measured. The *t*-test decides upon the necessity of adjustment: If the differences in one and the scale adjustment factor $\overline{W}_j/\overline{W}$ are significant, adjustments have to be applied.

The standard error of the adjusted stratum area is considerably more influenced by misinterpretations than by scale variations.

The qualitative attribute "stratum" is transformed into a quantitative form in transect devices, and the adjustment can be made by regression analysis. Most suited for this treatment are transects which pass through the nadir. Both adjustments are done simultaneously in the field and in the computations. The lengths of strata along the transects measured on the photographs are correlated to the lengths measured on the ground. The standard error of the adjusted values is calculated from the standard error of the regression and the sampling error of all photo-transects. In case the correlation is close, the adjustment will influence only slightly the standard error of the adjusted proportion of area. This condition may be expected for most cases in practice.

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Strip-Triangulation with Independent Geodetic Control

SANJIB K. GHOSH,
Dept. of Geodetic Science,
Ohio State Univ., Columbus, Ohio

1. Introduction

The method of strip-triangulation with independent geodetic control as described by Dr. Brandenberger[1] has been creating further interest amongst the world's photogrammetrists. This has again been noticed in the recent paper by Mr. Colcord (pp. 117 to 127, PHOTOGRAMMETRIC ENGINEERING, March 1961).

The author investigated further possibilities of this method at the Ohio State University under the guidance of Dr. Brandenberger. Both aero-levelling and aero-polygon types of strip-triangulation were studied. In each case, again, particular experiments were carried out to investigate the precision obtained from such adjustments with different amounts of control in the strip. Adjustment formulas were freshly derived for each case separately. With a view to having a thorough comparative study, one and the same strip was used in all these cases. Finally an analysis of the residual errors (or, thereby, the precision obtained) was made and a preliminary conclusion was drawn therefrom.

In this paper the author discusses the different aspects of the entire project. He acknowledges the assistance in the computations received from his colleagues (particularly, Capt. (now Major) M. V. Jonah of the U. S. Army and Mr. K. C. Chopra, a graduate student from India) at the Ohio State University. Thanks are due to Dr. Brandenberger for his valuable guidance in the investigations.

1.2. PHOTOGRAPHIC DATA

The area of study is that used by Mr. Colcord, i.e., Laufen-Bauma, Switzerland. Photographs were taken with a Wild RC7a camera (with principal-distance of 100.26 mm.) from the flying-height of about 6,000 m. above ground. Glass diapositives were used in the observations. The portion of the strip considered was, however, a length of about 107 km., which is more than double what Mr. Colcord used in his studies.

.1	.9	.18	.31	.48	.61	.71	.84	.97	.107	.113	.123	.137	.146
.3	.7	.16	.30	.47	.60	.72	.85	.94	.106	.114	.121	.209	.151
.2	.8	.17	.29	.50	.59	.70	.83	.96	.105	.115	.122	.138	.148

FIG. 1. Planimetry of the Considered Points.

1.3. OBSERVATION DATA

a. Aero-levelling method of observation with a first-order stereo-plotting instrument (Wild A7), was made by Mr. E. W. Dickman at the Ohio State University for his thesis towards an M.S. degree in the year 1960.

b. Aero-polygon method of observation with a first-order stereo-plotting instrument (also Wild A7), was made by Mr. N. D. Sharma under the author's guidance while he was in India in the year 1960.

The same control points were observed in both cases as they were clearly identified with the help of the sketches obtained from the same source.

In both cases the model scale was 1:15,000, number of models being 36.

In each case, after correcting for the model connection errors, the machine system of co-ordinates of the control points was first transformed to the ground system (both systems being rectangular). These transformations were based on the two points of the controlling base in the first model of the strip. This was greatly helpful in the comparative study of the individual cases. The adjustments were made directly in terms of the ground data.

In each case utmost care was taken to get the best out of the first-order instru-

AUTHOR'S NOTE: *Aero-levelling* is the type of aerial triangulation where the geometry of exposure is reproduced separately in each model. With the knowledge of the flying-height of each exposure station (obtained from statoscope data or otherwise) the operator of the restitution instrument tries to make the model-datum follow the instrument-datum. This gives much less closing errors in the coordinates as compared with the situation in *Aero-polygon* (as will be evident from Table I) where the geometry of exposure for the strip as a whole is reproduced in the restitution instrument. This is done by using the principle of co-orientation of the individual photographs in the strip. Because the terrestrial-datum is a level surface and the instrument-datum is a plane surface, systematic discrepancies appear in the models.

ment used. Relative orientation of each model was so done that the residual Y-parallax at any point in the model was within ± 0.02 mm. In model-connection, three transfer-points (upper, central and lower) were used and the scale transference was carried out with the help of three points near about each nadir-point, in the usual way.

Although there are in all 176 points in the strip, only 42 of these were used, because of belief that this number (for which full ground-data are available) is sufficient for analyzing the different types of errors. The distribution of these points in the strip can be seen in Figure 1.

1.4. ADJUSTMENT

In this paper the author deals only with his studies of the planimetric coordinates (X and Y, i.e., Eastings and Northings). In a subsequent paper similar treatment of the elevations is planned.

The strip was adjusted in six different ways—three for each of aero-levelling and aero-polygon, as follows:

(a) Case with *two* bases—One (whose length and azimuth on ground are known) in each of the first and the last models of the strip.

(b) Case with *three* bases—One base in the first model, one in any of the central models, and one in the last model of the strip.

(c) Case with *four* bases—One base in each of the first and the last models and two other bases (evenly distributed) in two of the inner models of the strip.

An attempt has been made to present the description in such a manner that it may serve as a guide to anyone desirous of using the procedure derived and followed by us in the Ohio State University. The symbols and notations used by Mr. Colcord will also be used in this paper to make it convenient to the readers of his paper.

Lack of space may not permit publishing the complete tables of the data and computations. Only the relevant items will be presented in support of our report.

2. DERIVATION OF THE FORMULAS

2.1. CASE WITH TWO BASES

It is assumed that the following data are given:

1) Two distances D_A and D_E in each of the first and the last models, each base being as long as possible.

2) Two azimuths α_A and α_E of the two bases in plane rectangular grid system.

There need be no geodetic connection between the situations of these bases. The only essential condition is that they be expressed in the same system in terms of the same unit of measurement.

Considering a longitudinal section with Y equal to a constant, a falsified error-profile is obtained as caused by the errors of triangulation in the instrument and other reasons. The corrections assumed according to the laws of propagation of error for a reasonably short strip are expressed by

$$\Delta X = a_0 + a_1 X + a_2 X^2 \quad (1a)$$

$$\Delta Y = c_0 + c_1 X + c_2 X^2. \quad (1b)$$

Considering the origin of the error propagation at a point very close to the first base, i.e., D_A, we can make a_0 and c_0 equal to zero. This is particularly the case when the transformation of the coordinates is based on the two points of the first base. Thus we get

$$\Delta X = a_1 X + a_2 X^2 \quad (2a)$$

$$\Delta Y = c_1 X + c_2 X^2. \quad (2b)$$

Differentiating equation (2a) with respect to X, we get the scale correction

$$\frac{d\Delta X}{dX} = a_1 + 2a_2 X$$

as the general expression. Therefore scale corrections for the first and last bases are

$$\delta S_A = a_1$$
$$\delta S_E = a_1 + 2a_2X_E. \tag{3a}$$

By comparing the observed distances D_A and D_E with their true distances we get δs_A and δs_E. By solving the two equations (3a) we obtain the values a_1 and a_2.

Next by differentiating Equation (2b) with respect to X we get the azimuth correction

$$\frac{d\Delta Y}{dX} = c_1 + 2c_2X$$

as the general expression. Therefore azimuth corrections for the first base and last bases are

$$\delta\alpha_A = c_1$$
$$\delta\alpha_E = c_1 + 2c_2X_E. \tag{3b}$$

By comparing the observed azimuths (as derived in our case from the observed coordinates) with their given azimuths, we get $\delta\alpha_A$ and $\delta\alpha_E$. Next by solving the two Equations (3b) we obtain the values c_1 and c_2.

Since the observed X coordinates must be corrected also for the azimuth error and the observed Y coordinates also for the scale error, the correction equations finally assume the following forms:

$$\Delta X = a_1X + a_2X^2 + c_1Y + 2c_2XY$$
$$\Delta Y = c_1X + c_2X^2 + a_1Y + 2a_2XY. \tag{4}$$

Considering proper signs, since, particularly, a positive azimuth correction results in a negative Y correction and considering the coordinates of the assumed origin to be X_1 and Y_1 we arrive at the following general working equations:

$$\Delta X = a_1(X - X_1) + a_2(X - X_1)^2 + c_1(Y - Y_1) + 2c_2(X - X_1)(Y - Y_1)$$
$$\Delta Y = c_1(X - X_1) - c_2(X - X_1)^2 + a_1(Y - Y_1) + 2a_2(X - X_1)(Y - Y_1). \tag{5}$$

2.2. CASE WITH THREE BASES

Assume that the following data are given:

1) Three distances, D_A, D_M, and D_E, in each of the first, any central and the last models of the strips, each base being as long as possible.
2) Three azimuths, α_A, α_M, and α_E, of the bases in plane rectangular grid system.

With the same considerations on error-profile, etc., as in *para. 2.1* above, the corrections according to the laws of error propagation are assumed to be expressed by

$$\Delta X = a_0 + a_1X + a_2X^2 + a_3X^3$$
$$\Delta Y = c_0 + c_1X + c_2X^2 + c_3X^3. \tag{(1a)}$$

Considering the origin of error propagation to be close to D_A as in *para. 2.1* we can make a_0 and c_0 equal to zero. Then

$$\Delta X = a_1X + a_2X^2 + a_3X^3$$
$$\Delta Y = c_1X + c_2X^2 + c_3X^3. \tag{(2)}$$

Differentiating the expression for ΔX in Equations ((2)) with respect to X we get the scale correction

$$\frac{d\Delta X}{dX} = a_1 + 2a_2X + 3a_3X^2$$

as the general expression. Therefore the scale corrections for the first, middle and last bases are

$$\begin{aligned} \delta s_A &= a_1 \\ \delta s_M &= a_1 + 2a_2X_M + 3a_3X_M{}^2 \\ \delta s_E &= a_1 + 2a_2X_E + 3a_3X_E{}^2. \end{aligned} \qquad ((3a))$$

By solving these three simultaneous Equations ((3a)) we obtain the values a_1, a_2 and a_3.

Similarly, differentiating the expression for ΔY in Equations ((2)) with respect to X we get the azimuth correction

$$\frac{d\Delta Y}{dX} = c_1 + 2c_2X + 3c_3X^2$$

in the general case. Thus, for the first, middle and last bases,

$$\begin{aligned} \delta\alpha_A &= c_1 \\ \delta\alpha_M &= c_1 + 2c_2X_M + 3c_3X_M{}^2 \\ \delta\alpha_E &= c_1 + 2c_2X_E + 3c_3X_E{}^2. \end{aligned} \qquad ((3b))$$

By solving these three simultaneous equations we obtain the values c_1, c_2 and c_3.

Since each of the X and Y coordinates must be corrected for both scale and azimuth errors, the correction equations assume the following forms:

$$\begin{aligned} \Delta X &= a_1X + a_2X^2 + a_3X^3 + c_1Y + 2c_2XY + 3c_3X^2Y \\ \Delta Y &= c_1X + c_2X^2 + c_3X^3 + a_1Y + 2a_2XY + 3a_3X^2Y. \end{aligned} \qquad ((4))$$

Considering proper signs, etc., as we did in *paragraph 2.1*, we arrive at the following general working equations:

$$\begin{aligned} \Delta X = {} & a_1(X - X_1)^2 + a_2(X - X_1)^2 + a_3(X - X_1)^3 + c_1(Y - Y_1) \\ & + 2c_2(X - X_1)(Y - Y_1) + 3c_3(X - X_1)^2(Y - Y_1) \\ \Delta Y = {} & - c_1(X - X_1)^2 - c_2(X - X_1)^2 - c_3(X - X_1)^3 + a_1(Y - Y_1) \\ & + 2a_2(X - X_1)(Y - Y_1) + 3a_3(X - X_1)^2(Y - Y_1). \end{aligned} \qquad ((5))$$

2.3. CASE WITH FOUR BASES

As in *paragraphs 2.1 and 2.2*, let us assume that the following data are given:

1) Four distances D_A, D_B, D_D and D_E, evenly placed in the strip (at approximately equal distances apart), where D_A is placed in the first model and D_E is in the last model.
2) Four azimuths α_A, α_B, α_D and α_E corresponding to these bases in plane rectangular grid system.

With similar assumptions and considerations as in *paragraph 2.1* and *paragraph 2.2* we obtain the following for the first, second, third and last bases:

$$\begin{aligned} \delta s_A &= a_1 \\ \delta s_B &= a_1 + 2a_2X_B + 3a_3X_B{}^2 + 4a_4X_B{}^3 \\ \delta s_D &= a_1 + 2a_2X_D + 3a_3X_D{}^2 + 4a_4X_D{}^3 \\ \delta s_E &= a_1 + 2a_2X_E + 3a_3X_E{}^2 + 4a_4X_E{}^3. \end{aligned} \qquad (((3a)))$$

By solving these equations we obtain the values of a_1, a_2, a_3 and a_4. Also similarly,

$$\begin{aligned}
\delta\alpha_A &= c_1 \\
\delta\alpha_B &= c_1 + 2c_2X_B + 3c_3X_B^2 + 4c_4X_B^3 \\
\delta\alpha_D &= c_1 + 2c_2X_D + 3c_3X_D^2 + 4c_4X_D^3 \\
\delta\alpha_E &= c_1 + 2c_2X_E + 3c_3X_E^2 + 4c_4X_E^3.
\end{aligned} \quad (((3b)))$$

By solving these equations we obtain the values of c_1, c_2, c_3 and c_4.

Finally we arrive at the following general working equations:

$$\begin{aligned}
\Delta X = {} & a_1(X - X_1) + a_2(X - X_1)^2 + a_3(X - X_1)^3 + a_4(X - X_1)^4 \\
& + c_1(Y - Y_1) + 2c_2(X - X_1)(Y - Y_1) \\
& + 3c_3(X - X_1)^2(Y - Y_1) + 4c_4(X - X_1)^3(Y - Y_1) \\
\Delta Y = {} & - c_1(X - X_1) - c_2(X - X_1)^2 - c_3(X - X_1)^3 - c_4(X - X_1)^4 \\
& + a_1(Y - Y_1) + 2a_2(X - X_1)(Y - Y_1) \\
& + 3a_3(X - X_1)^2(Y - Y_1) + 4a_4(X - X_1)^3(Y - Y_1).
\end{aligned} \quad (((5)))$$

3. Computations

The coordinates of the control points were first transformed from the machine system to the ground system on the basis of the two points of the first base (in the starting model). These transformed coordinates are our observation data. The observed distances of the bases (D_A, D_B, D_M, D_D and D_E) are obtained from these observation data of coordinates; the observed azimuths are also obtained from them.

Simultaneous equations 3a and 3b in each case are easily solved, by using the principles of matrix computations, with a desk calculating machine.

The present study was made as for any normal case of aerial triangulation. Thus the effects of longitudinal tilt and torsion (lateral tilt) of the strip during observation on the planimetric coordinates were not considered—these being very negligible when the terrain is not mountainous (the ruggedness of the terrain in our case being less than 30% of the flying height), and when the tilts are not large (less than 5–6 grads). It is, however, intended to study these effects in our laboratory later on.

4. Results

A good general idea of the results obtained in the different cases can be formed from the closing errors and the residual errors (a highly modern scientist would like to call them discrepancies and not errors). The difference between the true ground values and the transformed values (called the observation data in *paragraph 3* above) of the coordinates of individual points give the closing errors (see Table I) as they are before the adjustments. The residual errors obtained from the difference between the true ground values and the adjusted values (different in different case of adjustment) are given in Table II. The mean square values of the residual errors are:

Aero-levelling		Aero-polygon	
Two bases	$m_X = \pm\ 9.3$ m.	Two bases	$m_X = \pm 41.5$ m.
	$m_Y = \pm\ 4.8$ m.		$m_Y = \pm 16.2$ m.
Three bases	$m_X = \pm\ 4.5$ m.	Three bases	$m_X = \pm 23.0$ m.
	$m_Y = \pm 12.7$ m.		$m_Y = \pm\ 4.4$ m.
Four bases	$m_X = \pm 26.0$ m.	Four bases	$m_X = \pm 11.6$ m.
	$m_Y = \pm\ 4.8$ m.		$m_Y = \pm\ 4.0$ m.

An analysis of these errors and other factors is given in the following paragraphs.

TABLE I

CLOSING ERRORS IN METES

Points	*Aero-levelling*		*Aero-polygon*		Points	*Aero-levelling*		*Aero-polygon*	
	X	Y	X	Y		X	Y	X	Y
1	−0.8	2.3	0.0	−0.1	84	−13.3	65.5	130.4	43.0
3	−1.2	1.3	−3.5	−0.4	85	−4.9	67.9	136.3	31.4
2	1.8	−0.7	0.0	0.1	83	4.5	72.5	142.0	21.0
9	−8.4	0.7	0.2	4.0	97	26.4	88.7	169.8	63.3
7	−0.1	0.6	5.1	−1.4	94	−13.1	83.8	162.1	44.0
8	0.3	0.6	3.6	−6.1	96	−6.4	93.3	173.6	36.2
18	−1.8	−1.6	18.6	0.5	107	−29.0	106.6	198.1	78.7
16	1.4	4.7	16.7	−2.1	106	−19.6	106.6	201.7	60.2
17	5.3	2.7	20.8	−7.0	105	−10.2	109.4	207.1	44.3
31	10.9	15.2	31.0	10.0	113	−31.9	118.9	230.5	89.5
30	5.0	8.8	38.7	−4.9	114	−23.8	121.8	244.8	70.0
29	6.3	12.4	34.1	−5.5	115	−15.2	127.9	243.7	55.1
48	−1.7	30.3	54.5	15.3	123	−48.0	136.9	273.3	105.2
47	3.8	25.3	57.8	3.6	121	−39.4	145.0	276.3	91.1
50	9.2	24.1	62.8	−8.7	122	−26.4	155.9	286.4	74.4
61	5.3	37.7	88.6	21.4	137	−65.0	161.7	310.1	116.8
60	5.5	39.4	85.0	10.3	209	−49.2	170.2	317.3	96.6
59	10.5	40.6	86.2	2.3	138	−42.2	180.3	327.0	85.3
71	−6.0	48.0	103.3	27.5	146	−76.8	179.9	331.1	128.3
72	−3.9	53.0	103.8	21.2	151	−67.4	190.5	345.5	112.3
70	6.9	51.3	113.5	7.0	148	−57.2	198.2	347.7	100.4

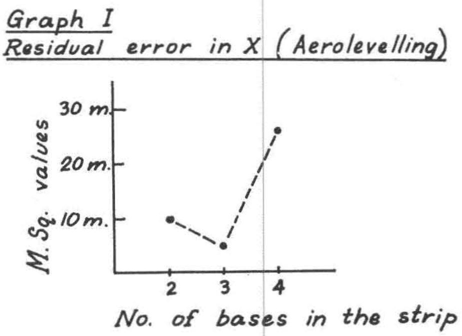
TABLE II

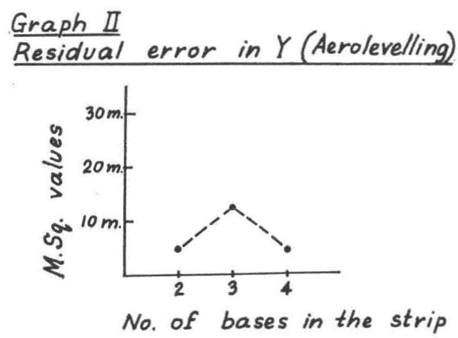
RESIDUAL ERRORS IN METERS

Points	*Aero-levelling*						*Aero-polygon*					
	Two bases		*Three bases*		*Four bases*		*Two bases*		*Three bases*		*Four bases*	
	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
1	−0.4	−0.9	−0.3	−0.9	−0.4	−0.9	0	0	0.0	0.1	0.0	0.0
3	1.7	−0.6	1.7	−0.6	1.8	−0.6	3	0	3.5	−0.5	3.7	0.5
2	−0.3	−0.9	−0.3	−0.9	−0.3	−0.9	0	0	0.0	0.0	0.0	0.0
9	9.0	3.8	9.5	4.7	10.7	6.2	1	−2	0.8	−7.0	3.4	1.1
7	2.6	2.8	2.8	3.4	4.5	3.2	−4	2	−3.3	2.3	−1.1	2.1
8	4.4	1.9	5.2	2.3	6.7	0.3	−2	5	0.7	−5.2	1.4	2.7
18	2.7	9.9	2.4	11.5	9.3	13.1	−14	3	−13.1	6.4	−5.0	9.0
16	2.6	3.2	3.4	5.3	9.4	3.8	−13	4	−9.5	5.0	−2.0	3.7
17	1.2	4.9	3.0	6.7	8.4	2.6	−15	7	−12.2	6.7	−5.0	2.2
31	−10.7	−1.0	−10.4	4.4	3.6	5.1	−22	−2	−16.8	2.5	−0.1	5.0
30	−1.2	5.9	0.6	10.8	13.7	7.6	−28	9	−22.1	11.9	−5.8	9.7
29	0.3	1.9	3.1	6.0	14.8	0.2	−19	6	−16.7	8.1	−1.7	2.6
48	0.1	−7.9	1.8	1.1	24.6	−0.9	−36	−3	−27.0	3.5	0.3	4.5
47	−1.1	−2.0	2.2	6.3	23.7	0.2	−38	−4	−27.7	8.3	−1.1	4.4
50	−2.2	1.9	3.2	10.1	24.7	−0.9	−40	10	−27.2	13.9	0.4	4.3
61	−11.8	−3.4	−8.6	10.8	25.7	5.2	−56	−3	−41.3	7.5	−0.7	7.2
60	−6.3	−3.7	−1.0	9.1	31.0	−0.5	−52	2	−35.1	9.2	4.4	3.8
59	−7.0	−3.7	−0.2	8.6	30.4	−4.5	−52	5	−33.9	10.2	4.9	0.7
71	−5.0	−3.8	−0.3	14.1	41.7	5.4	−59	−4	−39.3	9.1	10.2	7.7
72	−1.4	−5.8	5.4	11.5	46.1	−1.4	−56	−4	−35.1	6.1	14.5	−0.1
70	−6.1	−1.4	2.8	15.2	41.8	−2.0	−64	3	−40.5	10.3	8.6	−0.6
84	−6.5	−6.6	0.2	16.7	51.8	4.0	−69	−10	−41.8	6.9	18.8	5.0
85	−8.2	−4.7	0.3	17.9	50.9	1.2	−70	−6	−41.7	7.5	19.1	1.2
83	−11.0	−5.4	−0.2	16.7	48.8	−3.9	−72	−4	−41.9	7.0	19.0	−3.4
97	−7.8	−6.0	−1.0	24.1	64.0	5.1	−77	−30	−42.2	5.0	30.2	2.1
94	−8.9	−4.6	1.1	22.8	59.4	2.5	−76	−10	−41.7	6.6	27.5	0.6
96	−8.9	−6.1	3.4	21.5	61.6	−2.5	−77	−11	−42.0	4.5	28.7	−4.7
107	−14.5	−10.7	−4.5	22.5	62.9	0.5	−89	−20	−49.1	0.6	27.8	−2.7
106	−13.7	−6.6	−2.0	25.6	63.3	1.3	−88	−16	−48.9	3.6	27.6	−2.2
105	−14.2	−4.4	−0.7	27.1	63.0	0.4	−90	−12	−48.7	5.6	27.8	−2.7
113	−20.2	−6.9	−8.8	29.0	61.8	3.7	−100	−26	−56.7	−1.8	23.7	−5.6
114	−19.5	−3.3	−6.6	32.2	62.9	5.0	−107	−18	−54.0	4.2	26.7	−1.2
115	−18.8	−2.8	−4.4	32.2	63.9	3.2	−101	−16	−55.7	4.9	25.2	−2.2
123	−22.0	−2.3	−9.4	36.8	64.7	8.2	−111	−45	−64.8	1.1	18.6	−5.6
121	−20.3	−5.5	−6.8	32.0	66.2	2.6	−112	−24	−63.8	0.6	19.7	−3.9
122	−22.8	−7.4	−8.6	30.9	63.5	0.3	−112	−22	−64.6	1.5	19.2	−3.9
137	−19.9	−3.0	−6.2	37.5	68.9	7.7	−116	−24	−67.0	2.2	17.7	−1.6
209	−22.4	−3.6	−8.2	36.7	66.4	5.6	−118	−22	−67.7	4.0	17.1	−0.1
138	−21.2	−5.7	−6.6	34.6	67.6	3.1	−119	−21	−69.4	3.3	15.4	−1.0
146	−19.9	−5.9	−5.9	34.8	69.2	3.5	−116	−35	−66.3	1.0	18.6	−2.9
151	−19.8	−2.5	−5.8	38.3	69.3	8.0	−116	−24	−66.4	2.2	18.5	−1.7
148	−19.8	−6.0	−5.9	34.8	69.3	3.5	−116	−25	−66.1	1.1	18.8	−2.8

4.1. AERO-LEVELLING

A study of the mean square values of the residual errors in X and Y does not show any improvement in the results obtained by increasing the number of bases in the strip, as could be anticipated theoretically. This is apparent in graphs I and II. Actually the results are erratic. This, however, does not mean that the behaviour of the residual errors will be such in every strip.





It is found that the azimuth error during the observation (due to various reasons, e.g., errors in relative orientation, model connection, etc.) play a very important role in these cases. A special study was made to see how the azimuth-error was actually behaving during the observation of the strip. This was done on the basis of the observed coordinates of the individual points and comparing the resulting azimuths of various bases with those obtained from ground data. It was noticed to be purely of unsystematic nature.

A similar study of the scale-error of individual bases in the strip also shows the unsystematic behaviour of this error during observation.

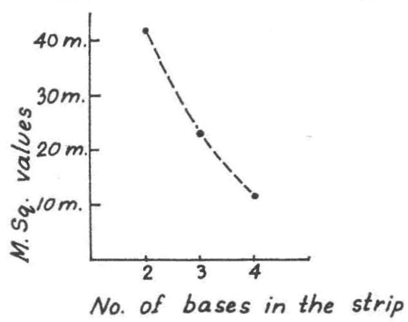
The above suggest that this adjustment procedure cannot be profitably utilized to obtain better results simply by increasing the number of bases in the strip while use is made of aero-levelling method of observation. This may not, however, be true for every strip of aero-levelling. Thus it is felt that a general conclusion along this line may be drawn only after a few more strips of aero-levelling are studied.

A study of the original machine coordinates shows discrepancies in the coordinates of the common points read in adjacent models (even after correcting for model connection errors). In certain cases discrepancies are large enough to cause an azimuth-error of 1′ minute (sexagesimal) between models, and these discrepancies are not systematic. This may give alarming propagation of error in a long strip. This shaky nature of observation may be due to various causes, e.g., the method of aero-levelling as used in this case, unsystematic and imperfect relative orientation of various models, defects in the photographic materials and the instrument used, and due to the personal error of observations.

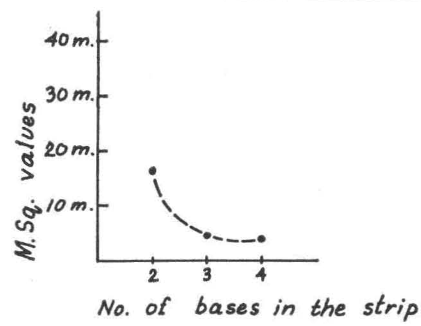
4.2. AERO-POLYGON

A study of the mean square values of the residual errors in each of the coordinates (X and Y) shows a definite improvement in the results obtained by increasing the number of bases, as could be anticipated theoretically. This may be further due to the systematic behaviour of the closing errors as can be expected generally in aero-polygon. These are clear from graphs III and IV.

Graph III
Residual error in X (Aeropolygon)



Graph IV
Residual error in Y (Aeropolygon)



4.3. GENERAL

The residual error in the X and Y of any point relative to those of the other points in the same model is similar in each of the six cases (three of aero-levelling and three of aero-polygon). Thus in both of the systems, aero-levelling and aero-polygon, each model individually can be considered as a rigidly fixed area, and similar in both systems. This is possible because, the author believes, each model was oriented (relatively) with meticulous care. This further shows that the differences in the residual errors of the points in the same model are due to reasons beyond the scope of the operator or the computer (one who adjusts the strip)—they may be due to errors inherent in the photography or the ground control.

It is observed that in both aero-levelling and aero-polygon, in the case with two bases, the residual errors are proportional to the closing errors. This is not true, however, in the cases with three or four bases. It may not be true with more bases also.

The adjustment can be simpler and more convenient if the transformation of the coordinates from the instrument system to the ground system (before adjustment) is done with respect to only the two points of the base in the first model. This, however, presupposes that the two points of the first base are correctly identified in the model and the ground coordinates are flawless, apart from other considerations like good photography, good instrument, etc. In that case the values of a_1 and c_1 become zero, we have thus to handle fewer computations and also can save a considerable amount of time.

Considering that the strip triangulated is more than 100 km. in length and the picture scale is 1:60,000, the residual errors in each case are within normal limitations of any type of aerial triangulation.

Colcord's concluding remarks are quite appropriate. We find that this method of strip adjustment (i.e., with independent geodetic control) has a future, especially in areas with sparse geodetic control. However, the authors believe that where more precision is desired, it will be worth while to use aero-polygon with a greater number of bases than just two in the strip. If the strip is very long, it might be convenient to handle the situation by breaking the long strip into parts and treating them as two or more separate consecutive strips. If aero-levelling has to be resorted to, it is not necessary to have more than two bases.

It is intended later on to publish the studies in full (including the minute details of each of the operations) as an Ohio State University publication.

Reference

(1) Brandenberger, A. J., 1959; "Strip Triangulation With Independent Geodetic Controls; Triangulation of Strip Quadrangles." Publication of the Ohio State University, Columbus, Ohio, U.S.A.