for one crew, a relief crew was always available to take off on a second sortie immediately after the aircraft had landed, had been refuelled and had been checked by the engineers. Thus full advantage was taken of the good photographic weather during the very short season.

During the 1958 season, due to exceptionally good flying weather, Kenting Aviation crews achieved remarkably high production by flying 40,000 line miles of photography and simultaneous A.P.R., covering some 115,000 square miles, an area equal to about half the size of France. During the next three years the remainder of the area was completed; this included reflights over some areas where excessive snow conditions made the quality of the photography unacceptable. Only a very small line mileage of these reflights now remains to be completed.

To date, mapping the Canadian Arctic has been attempted only on a reconnaissance scale. This new vertical photography is part of the second step in the Federal Government's plan for providing the necessary base maps, essential to the planned development of the Canadian north.

Preliminary surveys carried out by the Geological Survey of Canada in the Queen Elizabeth Islands in 1955, showed favourable structures which pointed to the possibility of an accumulation of oil and gas as well as coal and gypsum. A Shoran/A.P.R. control network, also initiated by the Federal Government, has been built up during the last decade, over much of the area as the first step in the vast program for recording the natural resources. Hunting also contributed to this phase by establishing Shoran control stations over an area of 40,000 square miles of Baffin Island in 1957. The resulting grid of control points together with the latest photography plus the A.P.R. coverage means that the various elements needed for proceeding with mapping the Arctic now exist and the large job of compilation can now be carried on.

Position Determination of Artificial Clouds in the Upper Atmosphere

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(*Abstract is on next page*)

Introduction

Artificial clouds formed by rocket-borne chemical releases have proven to be an important tool in the study of the upper atmosphere. The position, as a function of time, of a set of recognizable points on the cloud is of fundamental importance in these endeavors. This information yields the size, growth, spatial orientation, and drift of the cloud, and these data provide a basis for the study of winds, turbulence, and diffusion in the upper atmosphere. The positions of these cloud-points are found by a method of triangulation based on photographic data. For the sake of clarity this paper will first delineate the various coordinate systems employed, the transformations among them, and the spatial intersection of lines necessary for triangulation, and lastly discuss the corrections that arise when this formal procedure is applied to the physical situation.

Coordinate Systems

TERRESTRIAL COORDINATES

The position of a point near the surface of the earth may be specified in any of several coordinate systems, each having its own merits. Four such systems are employed.

(a) The *geographic system* consists of the familiar longitude, latitude, and altitude above sea-level.

(b) The *geocentric system* is a set of Cartesian coordinates whose origin is at the center of the earth. Its axes pierce the earth's surface at the geographic north pole, the intersection of 90° east longitude and the equator, and the intersection of the Greenwich meridian and the equator. For convenience, this latter axis is designated the "Greenwich geocentric axis."

(c) A *topocentric system* is a set of Cartesian coordinates whose origin is placed at a fixed point on the surface of the earth at an observation station. The axes are directed outward along a line from the center of the earth through the station, to the north, and to the east.

In addition, the position of a line from the topocentric origin at an observation station to a cloud-point may be expressed in azimuth (angle east of north) and elevation (angle above the topocentric horizon).

(d) A *station-pair system* is a set of Cartesian coordinates whose origin is placed at the point where a line from the center of the earth perpendicularly intersects a line connecting two observation stations. (These observation stations are also the origins of two topocentric coordinate systems.) Two axes of the station-pair system are directed along these two aforementioned lines. The third lies, of course, perpendicular to them.

Sets of direction cosines between these four types of coordinate systems can be calculated after at least two observation stations have been established in longitude, latitude, and altitude. With these sets of cosines, transformations among these four types of coordinate systems are possible.

CELESTIAL COORDINATES

In addition to these four terrestrial systems, the customary celestial coordinates, right ascension and declination, are used to specify the fixed position of a star on the celestial sphere. A line from the center of the earth to a particular star can then be described by either the celestial coordinates of the star or the earth-fixed geocentric coordinates. However, because of the earth's rotation relative to the celestial sphere, it can be seen that the geocentric coordinates of this line to the star are not constant, but vary with time. Hence, a transformation from celestial coordinates to geocentric coordinates is a function of time.

The celestial sphere is intersected at its north and south poles by the earth's axis of rotation, and the equator of the earth and the equator of the celestial sphere are coplanar. Because of this orientation, the geocentric axis that pierces the surface of the earth at the north pole remains intersected in the north pole of the celestial sphere, despite the earth's rotation. The two other geocentric axes intersect the celestial sphere at its equator, and the right ascensions of these points of intersection change with an angular rate equal to that of the earth's rotation.

A knowledge of the angle between the Greenwich geocentric axis and the line from the geocentric origin to the First Point of Aries completely specifies the angular orientation of the two systems at a certain time. This angle, the Hour Angle of the First Point of Aries, is tabulated daily in the annual *American Ephemeris and Nautical Almanac*. New angular orientations are then obtained from the elapsed time and the earth's rate of rotation. Thus, the directional cosines between the celestial and geocentric systems are dependent upon the initial angular orientation specified by the Hour Angle of the First Point of Aries at the reference date and the time that has elapsed since that date.

These celestial-terrestrial transformations are utilized in the determination of the azimuth and elevation of a star as seen from

ABSTRACT: *A computer-oriented procedure is developed for calculating the position of artificial clouds in the upper atmosphere by a method of triangulation. The data used are obtained from photographs made from two or more observation stations. Each photograph contains not only the cloud, but also a background of stars. From the relative positions of these stars and the cloud, the angular position of a line-of-sight from each station to the cloud is determined. The intersection of any two lines-of-sight locates the cloud. The data are corrected for atmospheric refraction, oblateness of the earth, precession, and certain anomalies of the photographic equipment.*

an observation station. The procedure is as follows:

(a) The right ascension and declination of the star is found in standard tables [e.g., Becvar, 1959], thereby specifying the position, in celestial coordinates, of the line from the geocentric origin to the star.

(b) The celestial coordinates of the line to the star are transformed into the earth-fixed geocentric coordinates for the time at which the photograph was made.

(c) A final transformation from geocentric to the appropriate topocentric system yields the azimuth and elevation of the line-of-sight from the station to the star for the particular time of the photograph.

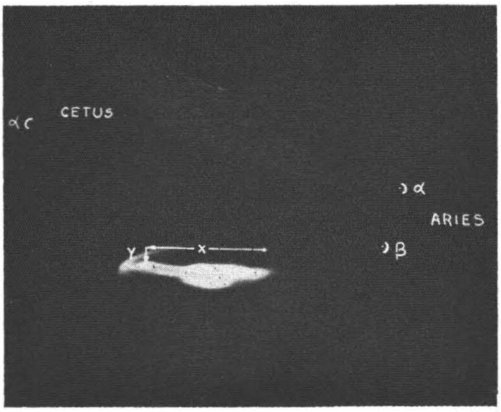

FIG. 1. Typical artificial cloud photograph from an observation station showing the star background and the location of a cloud-point.

Calculation of Coordinates of a Line from an Observation Station to a Point on the Cloud

The first step in a triangulation procedure between two stations is to find the azimuths and elevations of lines-of-sight from the two stations to the point on the cloud. (The photographic equipment utilized at each station is described in detail in an article by H. D. Edwards.) At each station, the azimuth and elevation of the camera axis, which intersects the center of the fiducial grid, is found from the calculable azimuths and elevations of stars visible in the background on the photographs of the cloud. As shown in Figure 1, the axis then provides a reference line from which the azimuths and elevations of lines to any number of points on the cloud can be calculated.

Spherical Triangles

From each observation station, spherical triangles can be constructed in the manner shown in Figure 2a. This same spherical triangle is shown in projection in Figure 2b. Capital letters refer to angles on the surface of the sphere and small letters refer to angles (shown as sides in the projection) subtended at the center of the sphere by the sides of the triangle.

The sphere in this case is a hypothetical sphere of lines of azimuth and elevation centered at an observation station. Let point Z in the Figure 2a be the intersection of the topocentric vertical with the sphere, point S be the intersection of a line to a star with the sphere, and point X be the intersection of the camera axis with the sphere. Consequently, A is the difference in the azimuths of the star and the camera axis, and b and c are the co-elevations of the star and the camera axis respectively. Angle a can be calculated from the relation

$$\tan a = d/f, \tag{1}$$

where d is the distance from the center of the

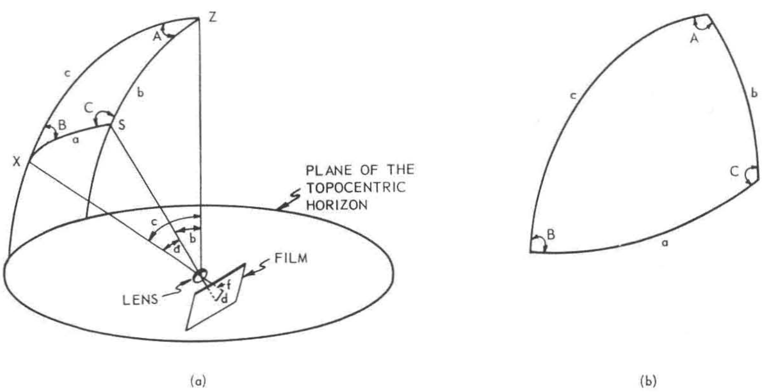

FIG. 2. Realistic and projected views of a spherical triangle on the azimuth-elevation hemisphere about an observation station.

photograph's fiducial grid to the star image measured on the film, and f is the focal-length of the camera.

AZIMUTH AND ELEVATION OF CAMERA AXIS

Specifically, several such spherical triangles are used to determine the azimuth and elevation of a camera axis. These triangles are shown in projection in Figure 3. The known values of these spherical triangles are:

$$D = (\text{azimuth of star } A) - (\text{azimuth of star } B)$$

$$a = 90° - (\text{elevation of star } A)$$

$$b = 90° - (\text{elevation of star } B)$$

$$e = \arctan\left(\frac{\text{length from center of fiducial frame to star } A \text{ image}}{\text{focal length}}\right)$$

$$f = \arctan\left(\frac{\text{length from center of fiducial frame to star } B \text{ image}}{\text{focal length}}\right)$$

Then solving these triangles:

a, b, D determine B, d

d, e, f determine F

F, B, determine C

a, e, C determine c and E, where

$$c = 90° - (\text{elevation of camera axis})$$

$$E = (\text{azimuth of camera axis}) - (\text{azimuth of star } A)$$

Actually, this calculation of camera axis azimuth and elevation could be accomplished using only one star; however, this would assume that the fiducial grid vertical is perfectly aligned with the topocentric vertical. As will be discussed later, this is not always the case.

AZIMUTH AND ELEVATION OF A CLOUD POINT

Only one additional spherical triangle is necessary to calculate the azimuth and elevation of a specific point on the cloud. This triangle utilizes the previously calculated azimuth and elevation of the camera axis and subtended angles calculated by (1) from film measurements of the cloud point.

Calculation of Intersection of Lines of Sight

The azimuth and elevation of one point on the cloud identifiable from two stations establish two lines in space, one from each station, whose intersection will be the location of the point in space. The cloud features are rather nebulous (cf. Figure 1); consequently, the two lines-of-sight to the feature will not, in general, intersect. However, the point of closest approach locates the cloud point.

After transforming the topocentric azimuths and elevations of these two lines of sight to the station-pair coordinate system between the stations, a first approximation of these two lines-of-sight to this point of closest approach is found in the following way:

a) A plane is passed through the line-of-sight from station A and a line connecting stations A and B.
b) A second plane is passed through the line-of-sight from station B and the line connecting stations A and B.
c) A third plane through the line connecting the stations A and B is then formed which bisects the angle between the other two planes.
d) Finally the two lines-of-sight are projected onto this third plane, forming a triangle with the line connecting the stations. The apex of this triangle is a first approximation to the position of the cloud point.

Since the effect of any error present in the azimuth and elevation of the line-of-sight on cloud-point position location is magnified as the distance from station to cloud increases, the shorter of the two lines-of-sight will furnish the more accurate contribution to the intersection. Thus, a second approximation of the closest intersection can be found by weighting the azimuths and elevations in favor of the station closest to the cloud as follows:

a) The lengths of the two lines from the stations to the apex of the triangle are calculated.
b) A new "third plane" is found which divides the angle between the first two planes in the ratio of these two lengths.
c) The apex of the triangle formed by the lines-of-sight projected on this new

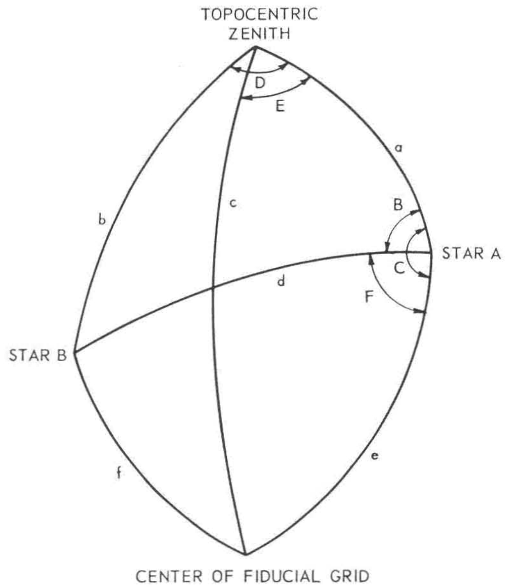

FIG. 3. Spherical triangles used in calculating camera axis azimuth and elevation.

plane is then taken as the final position of the point.

The coordinates of the cloud-point thereby determined are in a station-pair coordinate system. Successive coordinate transformations from this system to the geocentric system and then to the geographic system put the position of the cloud-point in the familiar longitude, latitude, altitude coordinates. Many of the protuberances on the cloud persist over several minutes and can serve as cloud-point positions. Successive photographs record a time history of the changing position of the cloud-points. Plots of these positions yield cloud size and spatial orientation, and from the time rate of change of these cloud-point positions, cloud growth and wind velocities are calculable.

All of the foregoing calculations have been programmed for solution by a digital computer. Further development is being made on the program so that horizontal wind velocities can be calculated from long thin clouds without recognizable features for positioning cloud points.

CORRECTIONS

The remainder of this paper will discuss various corrections that are applied to the foregoing procedure. Most of these corrections were not mentioned previously for the sake of clarity, but are used whenever applicable.

a) OBLATENESS OF THE EARTH

Geographic longitude and latitude, the coordinates used to locate the positions of the observation stations on the earth, are obtained from measurements based on the assumption that the local gravitational vertical passes through the center of the earth. Since the figure of the earth is to a first approximation that of an oblate spheroid, such an assumption is somewhat in error. As a result of this, the topocentric vertical differs slightly from the local gravitational vertical. No use is made of the gravitational vertical in this procedure.

The calculations of the geocentric coordinates of the observation stations require a knowledge of the geocentric latitude (the angle formed at the center of the earth by an earth radius to the observation station and the equatorial plane) rather than the geographic latitude. This transformation from geographic to geocentric latitude is calculated from the following relation derivable from the equations of an ellipse.

$$\Phi' = \tan^{-1}(K_1 \tan \Phi) \tag{2}$$

where

$$\Phi' = \text{geocentric latitude}$$

$$\Phi = \text{geographic latitude}$$

$$K_1 = \left(\frac{\text{polar radius}}{\text{equatorial radius}}\right)^2$$

Since the earth is assumed to be symmetric about its polar axis, the geographic and geocentric longitudes are the same.

Due to the oblateness, the earth radius is now a function of the geocentric latitude. The following relation, also derivable from the equations of an ellipse, is used to determine the radius:

$$R = \frac{R_p}{(1 - K_2 \cos^2 \Phi')^{1/2}} \tag{3}$$

where

$$R_p = \text{polar radius}$$

$$K_2 = 1 - \left(\frac{\text{polar radius}}{\text{equatorial radius}}\right)^2$$

The following values of the polar and equatorial radii are used [Kuiper, 1954]:

equatorial radius = 6,378.099 Km.

polar radius = 6,356.631 Km.

Thus $K_1 = 0.99327954$ and $K_2 = 0.0067205$.

b) PRECESSION

The axis of rotation of the earth does not remain precisely in a fixed orientation with the stars. Its small precession has the effect of continuously changing the celestial coordinates of the stars. This time rate of change of either right ascension or declination is small enough to allow the approximations

$$RA_{t_2} = RA_{t_1} + \left.\frac{d(RA)}{dt}\right|_{t} (t_2 - t_1) \quad (4)$$

and

$$\mathrm{Dec}_{t_2} = \mathrm{Dec}_{t_1} + \left.\frac{d(\mathrm{Dec})}{dt}\right|_{t_1} (t_2 - t_1) \quad (5)$$

where t_2 is any time later than some reference time, t_1. The determination of the above time derivatives is a standard inclusion in most astronomy texts [e.g., Chauvenet, 1960]. A star catalog tabulates stellar coordinates for a certain epoch (reference date). With (4), (5), and the elapsed time, the precessed coordinates at any date can be computed.

c) CAMERA TILT

In the construction and installation of the photographic equipment, it is difficult to align the vertical axis of the fiducial plate with the topocentric vertical to an accuracy better than a few tenths of a degree. This tilt about the camera axis has a significant effect on the final results, particularly for measurements made near the edge of the film. The magnitude of the camera tilt is calculated at the same time that the azimuth and elevation of the camera axis are found. (The calculated azimuth and elevation of the camera axis are independent of camera tilt since only distances between the stars and the center of frame, rather than actual grid coordinates, are used.)

To determine the magnitude of the tilt, the calculated slope of a line connecting the two star-images on the film plane (cf. Figure 3) is compared with the slope of this line as measured by the tilted grid coordinates. The difference between these slopes is the angle of camera tilt. In subsequent calculations, wherever grid coordinate measurements are used, a rotation of coordinates through the tilt angle is performed.

d) FOCAL-LENGTH

The focal-length of each camera is used to determine, from length measurements, the angular separation between cloud-points or stars. Because these angular separations are used extensively in the preceding spherical triangles, the focal-length is needed to an accuracy greater than the nominal values quoted by the lens manufacturer. To achieve this, the focal-length of each camera is calculated separately from star data.

Focal-length is determined from the measured distance between two star-images on the film and the known angular separation between them. Again, spherical triangles are utilized. Referring to Figure 3, the angular separation, d, of the two stars can be calculated from a, b, and D. Then, the plane triangle on the film as shown in Figure 4a is solved for G', the angle between the star-images measured from the center of frame. The lengths f', e', and d' are used in this calculation. G' and the angular separation of the stars, d, are then used to evaluate a second spherical triangle shown in Figure 4b. Here,

$$e = \arctan \frac{e'}{(\text{focal length})}, \quad (6)$$

$$f = \arctan \frac{f'}{(\text{focal length})}, \quad (7)$$

and $G = G'$, since the plane of the film is normal to the camera axis. The solution of this triangle yields the true value of the focal length.

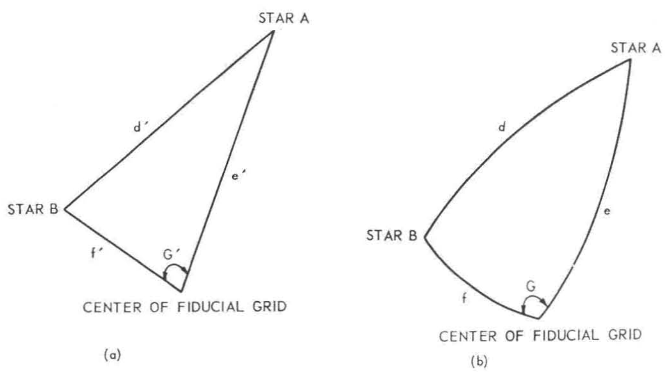

FIG. 4. Plane and spherical triangles used to determine camera focal-length.

e) FILM SHRINKAGE

The film shrinks nonuniformly between the time of exposure and analysis. To minimize the effect of this shrinkage, a fiducial grid of fine lines is recorded on the film at the time of exposure. With the spacing of the fiducial lines known from the construction of the grid, the displacement of an image is obtained by measuring its distance to the nearest fiducial line, thereby confining the error of shrinkage to a small interval.

f) ATMOSPHERIC REFRACTION

The position of an extraterrestrial source as seen by a terrestrial observer must be corrected for the apparent displacement caused by atmospheric refraction. This refraction affects both the position of the stars and the artificial cloud.

Because the star-images recorded on the film have undergone refraction, this effect must be taken into consideration before star-image position is used in the calculation of the azimuth and elevation of the camera's axis. The refracted star positions are corrected in the calculations by an approximation to standard stellar refraction tables.

These tables do not strictly apply to the apparent displacement of the artificial cloud because it is not entirely beyond the atmosphere. It is, however, above the denser portions of the atmosphere. Consequently, the cloud position undergoes nearly the same refraction as the stars. As a first approximation, the stellar refraction correction is applied to cloud points also.

g) FIDUCIAL PLATE REFRACTION

In passing from the camera lens to the film, light must pass through a glass fiducial plate which is in contact with the film. The plate is approximately 5 mm thick; consequently, it can cause a considerable light ray deviation due to refraction, particularly for images near the edge of the film. A refraction correction factor, C, is calculated from the focal-length, index of refraction, and thickness of the plate. The corrected coordinates have the form

$$x' = x(1 + C/r) \tag{8}$$

$$y' = y(1 + C/r) \tag{9}$$

where r is the distance from the fiducial grid center:

$$r = (x^2 + y^2)^{1/2}$$

this correction, like the camera tilt correction, is applied in the calculations whenever grid coordinates are used.

CONCLUSION

The cloud-points, whose positions are determined by the foregoing procedure, are actually rather nebulous bulges on the cloud. Each observation station sees a slightly different view of the cloud; consequently, the apparent center of a particular protuberance chosen from the data from one station may not be precisely the same as that chosen from another station. Immediately after the release of the chemical, however, all stations record the cloud-image as a sharply defined point. The error in the placement of the center is a minimum at that time. Therefore, the accuracy with which this release position can be determined is indicative of the errors within the procedure under discussion.

The various values of a release height obtained from all possible pairs of observation stations consistently agree within 0.05 per cent, viz., 50 meters in a typical height of 100 km. The agreement in longitude, latitude, and slant range is of the same order of magnitude.

Measurements of image displacement on the film cannot be made to an accuracy better than 0.005 cm, i.e., an angular error of 0.00028 radians. At typical slant ranges of 160 km, this introduces an error in position of approximately 45 meters.

Therefore, the spread in the values of release height obtained from the intersections of the various lines of sight is commensurate with the error inherent in film measurement.

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