Stereophotogrammetry Applied to Hydraulic Analogue Studies of Unsteady Gas Flow

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ABSTRACT: *A description of the equipment, techniques, and results of data acquisition and reduction by stereophotogrammetry of the time-varying surface of a water-table analogy to unsteady two-dimensional compressible gas flow. While the partial admission turbine is discussed, the usefulness of the technique is equally applicable to a wide range of unsteady compressible fluid flow problems, as well as to other situations where the recording of time-variant surface irregularities is desired.*

INTRODUCTION

UNSTEADY flow conditions are inevitable in most applications of compressible fluids in information transmission, control, or energy conversion. Sometimes the unsteady effects are of secondary consideration, as for example the nozzle wakes in full-admission turbomachinery, or occur only periodically, as with "start-up" and "shut-down" of gas transmission in ducts and pipes. Frequently, however, unsteady operation is the normal mode and of significant effect, as in servo-valves, pneumatic voice transmission, and pulse-jet engines.

Because of the high speed of propagation of disturbances in a gaseous medium, of the order of 1,000 ft/sec, and the small size of many of the geometries of interest, it is very difficult, sometimes impossible, to instrument and record the pressure and density fluctuations in the two- and three-dimensional flow fields. For relatively small geometries, and where under unsteady flow conditions transonic flow velocities are possible, the insertion of instrumentation, however small, into the flow can unduly influence and contort authentic conditions.

OPTICAL EXPERIMENTS ON GAS FLOW MODELS

Optical techniques, such as Schlieren and interferometry, offer means of determining respectively density gradients and densities in the gas flow, but the use of these techniques in typical unsteady situations can be discouragingly difficult. Consider for instance the flow initiation and decay in a turbine-wheel blade-channel, as the channel passes through the flow from a single nozzle, as in the case of the partial-admission turbine. In an actual case, the high temperature and pressure of the working fluid results in nozzle exit velocities of 4,000 and 5,000 ft/sec. Corresponding blade-channel velocities relative to the stationary nozzle are perhaps 1,200 ft/sec. In order to avoid experimenting with higher temperature gases, one might choose to use ambient air as the test fluid, suitably scaling the other parameters so as to maintain geometric, Mach number and Reynolds number similitude. This choice would reduce fluid velocities to approximately 0.6 of the original hot-gas case. Even so, a relatively weak shock wave with a velocity of 1,400 ft/sec (pressure increase of 2.5, density increase of approximately 2) moves one inch in 6×10^{-5} sec, or 1/32 inch during a two microsecond photographic exposure, indicating some blurring and the need for relatively close fringe spacing in interferometric experiments. The relationship between density change and fringe shifts is given by

$$\rho_2 - \rho_1 = \frac{\lambda_{\text{vac}}}{L\left(\frac{n-1}{\rho}\right)_{\text{std}}}\,\epsilon$$

where

ρ = density of gas where subscript 1 refers to no flow and 2 with flow, lb/ft^3.
ϵ = number of fringes displaced.
L = thickness of test section, in.
λ_{vac} = wavelength of mercury vapor light in vacua $= 0.5\mu$ or 2×10^{-5} in.
$(n\text{-}1/\rho)_{\text{std}}$ = Gladstone-Dale constant $= 4.8 \times 10^4$ ft^3/lb for air

We can estimate the maximum fringe shift for our turbine example by assuming a test section zero-flow density of 0.21 lb/ft^3 (expanding atmospheric air adiabatically to 2.5 psia in the test section). For a test section thickness of $\frac{1}{2}$ inch, and for our shock of density increase of 2, the maximum fringe shift is four.

Since optical techniques give only density changes, measurements of two independent properties of the fluid will also be necessary. A static reference density is easy but transient velocities or pressures will require very responsive hot wire anemometry and piezoelectric pressure transducers while transient temperatures are probably immeasurable directly.

Since our measuring tool is the wavelength of light deflections and vibrations of even very small magnitude will render the optical approach useless. Undesired motion in an unsteady flow device, due either to the fluid motion itself or the apparatus causing the fluid motion, will be very difficult to avoid.

Furthermore, since the optical view will be obscured by any deposition on the test section walls, contaminants in the flow must be thoroughly removed should a compressor or heater be used. If atmospheric air is expanded to avoid fluid contamination and reduce pumping power, the air will have to be thoroughly dried to prevent condensation shocks from contorting the authentic flow field, and water condensation from obscuring the view.

The Hydraulic Analogy

In light of the foregoing difficulties of direct experimentation with the gaseous media, a study was made of the applicability of the hydraulic analogy to this class of problem.[1]* An analysis of the equations describing the frictionless, isentropic, two-dimensional flow of a perfect gas and the frictionless, shock-free, two-dimensional flow of shallow water on a plane surface indicates that, if the Froude number in the water is set equal to the Mach number in the gas, the water height is proportional to the gas density. The conditions of an actual unsteady flow experiment may, of course, violate the assumptions of the theoretical analogy; for example, friction is present in both gas and water cases, but frictional similitude and therefore frictional effects, i.e., boundary layer buildup, cannot be satisfied; shock and rarefaction waves and interactions of these with each other and with the boundary layer are inevitable.

* Numbers refer to the Bibliography.

Despite these limitations, however, the analogy has been shown to be a remarkably faithful replica of the gas flow for subsonic, transonic, and supersonic flow conditions as well as for specific nonsteady wave forms. For a complex unsteady flow, that of the partial-admission turbine, a good correlation has been shown between a necessarily simplified mathematical model of the flow and water table data taken along the channel centerline with point probes.[1]

Compared with the gaseous-optical measurement approach, the water table's primary advantage is an expansion of the time scale by the order of one-thousand. This comes about due to the Froude-Mach number similarity

$$F = \frac{V^2}{Lg}; \qquad M = \frac{V}{c}$$

where

F = Froude number
M = Mach number
V = fluid velocity
L = characteristic length
g = acceleration of gravity
c = local speed of sound

For Mach 1 similitude ($F=1$), a sheet of water 0.374 inch deep has a velocity of only 1 ft/sec compared with corresponding gas velocities of the order of 1,000 ft/sec. Thus phenomena on the water table can be observed with the unaided eye, or recorded with relatively simple equipment, whereas the gas case stretches the state of the art of available instrumentation to, or beyond, the possible.

The time scale and the availability of the working fluid permits easy and economical fabrication of scaled-up water tables whose geometry can be easily changed, enhancing systematic study of the influence of design variables.

A Water Table Simulating Partial-Admission-Turbine Gas Flow

The particular unsteady compressible flow situation under investigation at M.I.T. is the partial-admission turbine. The stereophotogrammetry experimental equipment and data acquisition and reduction techniques used in this project will be described,[2] but the usefulness of the approach is equally applicable to a wide range of unsteady, compressible fluid flow situations, as well as to other areas where the recording of time-varying surface irregularities is desired.

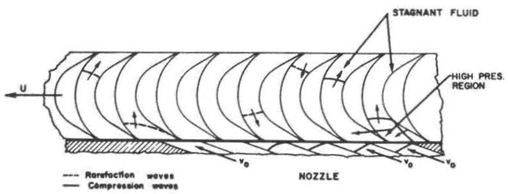

FIG. 1. Schematic of flow in partial-admission turbine.

Figure 1 gives a schematic of the flow geometry and the simplest model of the flow pattern in a partial-admission turbine. Figure 2 is a schematic of the water table. In operation the appropriate Froude number in the nozzle exits is established by the level of water in the supply basin and the height of the exhaust flow. The cart carrying the turbine blades is accelerated by the drive and cable to that constant velocity which satisfies the experimental velocity ratio desired. By the time the left-most blades are approaching the nozzles, the water level in the blade channel has reached a virtually stagnant condition. Previously reported experiments[1] were conducted using capacitive water-depth probes by which means plots of water-height variation versus time were obtained as the blade channels passed before the nozzle. While this technique proved satisfactory, the physical presence of the probe does affect the flow, and a great many runs are necessary to map the entire flow field in order to study two-dimensional flow effects and ultimately provide the basis for recommending blading design.

PHOTOGRAMMETRY

Solution of the "normal case" of photogrammetry is based on the geometric fact that the difference in respective parallax of the images of an object-point on two image-planes, parallel to the object-plane, but separated by a known distance, is given by the relation

$$P = \frac{fB}{H} \quad \text{or} \quad H = \frac{fB}{P} \tag{1}$$

where the parameters are defined in Figure 3 as follows:

P = parallax difference or total parallax $X_2 - X_1$
B = distance between lens axes (interocular)
f = distance from second principal plane of lens to image-plane
H = distance from first principal-plane of lens to object-plane

Thus all points on the same horizontal plane have the same total parallax.

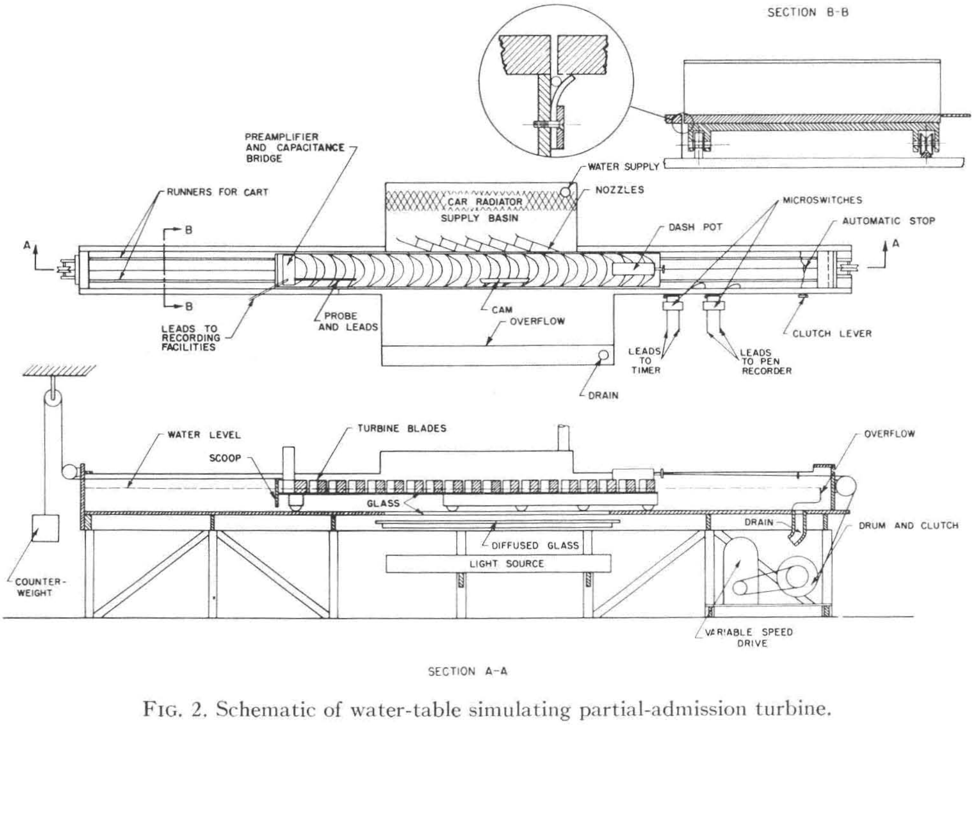

FIG. 2. Schematic of water-table simulating partial-admission turbine.

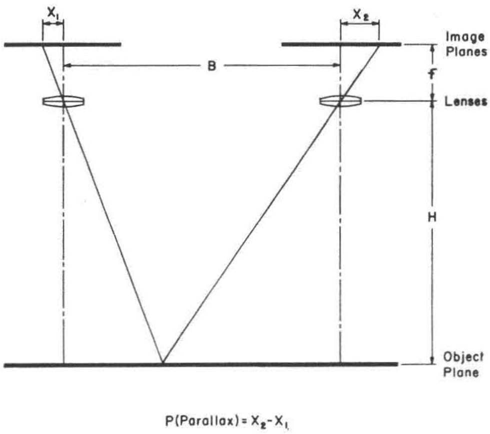

FIG. 3. Basic geometry of photogrammetry.

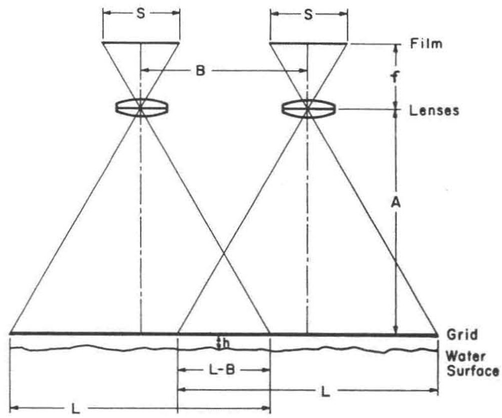

FIG. 4. Photogrammetry geometry including the influence of overlap and film-width on total parallax and the use of a reference grid.

The change in object distance with respect to parallax is given by

$$dH = -\frac{fB}{P^2}\,dP \qquad (2)$$

By defining a datum plane at the mean height of the undulating surface and taking H_0^2 as the lens height above the datum plane, and assuming that changes in H are small with respect to H_0, Equation 2 in finite difference form becomes

$$\Delta H = -\frac{H_0^2}{fB}\,\Delta P \qquad (3)$$

The constant H_0^2/fB, known as the vertical magnification, can be determined by measurement of the system parameters, or by photographing known height differences and measuring the resulting parallax.

Since parallax measurements depend on overlap, that portion of the object-plane which appears on both photographs, and since large parallax difference enhances accuracy of data reduction, large overlap and high total parallax conflict. From Figure 4, overlap is given by L-B or percentagewise $\eta\, L$ where

$$\eta = 1 - \frac{B}{L} \qquad (4)$$

and

$$L = \frac{SA}{f} \qquad (5)$$

where

S = film dimension parallel to line connecting both cameras

L = corresponding length of surface photographed

A = distance from the lenses first principal plane to a reference grid placed above the surface being photographed

The reference grid of Figure 4 serves several purposes:

(1) as a substitute reference surface for the water-table bottom which is not visible during actual runs.

(2) to permit the elimination, as sources of error of non-photogrammetric considerations such as surface irregularities in the wooden base of the table and systematic height differences occurring due to the motion of the turbine blade cart.

(3) as a coordinate system with which to identify specific points in the field of view.

(4) to facilitate parallel alignment of stereo-pairs. From Equation 1 and Figure 4

$$P = \frac{fB}{A + h} \qquad (6)$$

where h = distance from a reference grid to a point on the surface being measured.

Differentiating Equation 6 with respect to h gives

$$dP = -\frac{fBdh}{A^2 + 2hA + h^2} \qquad (7)$$

where dP/dh is constant for $h \ll A$. If all heights are measured from the grid, then the change in parallax is given by

$$P = P_{\text{grid}} - P_{\text{object}} \qquad (8)$$

$$\Delta P = \frac{fB}{A} - \frac{fB}{A + h} \qquad (8a)$$

or

$$h = \frac{A}{\dfrac{fB}{A\Delta P} - 1} \qquad (9)$$

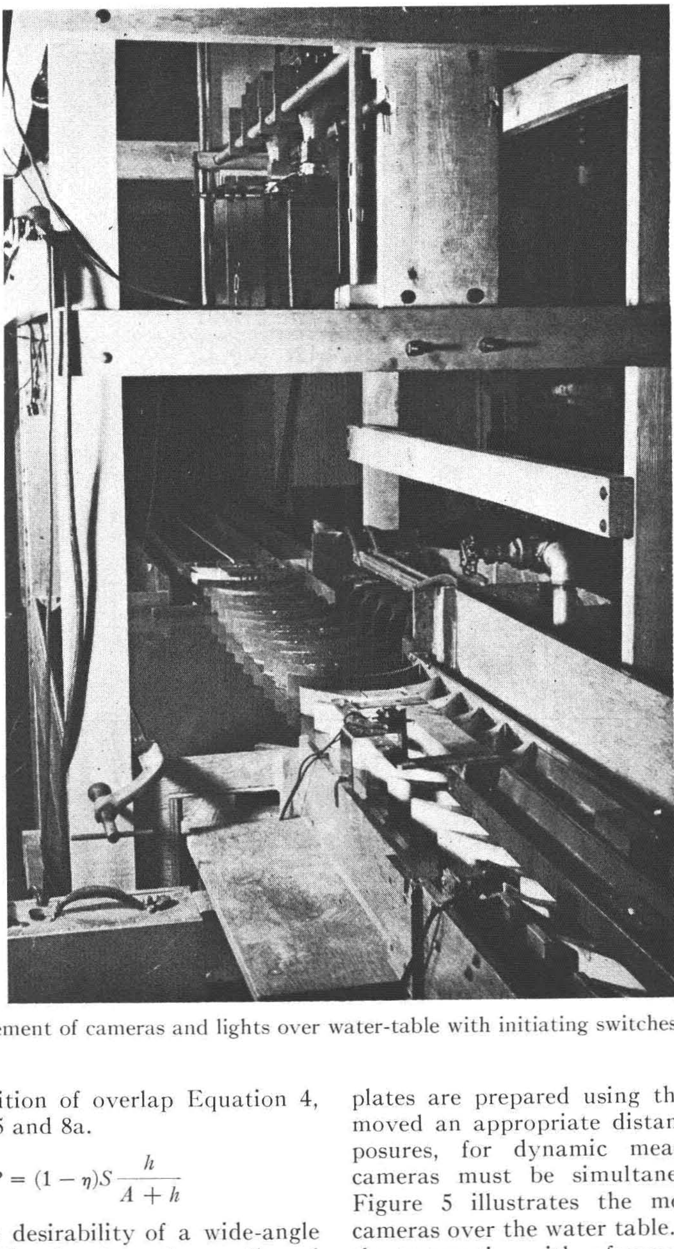

FIG. 5. Arrangement of cameras and lights over water-table with initiating switches in foreground.

From the definition of overlap Equation 4, and Equations 5 and 8a.

$$\Delta P = (1 - \eta)S \frac{h}{A + h}$$

establishing the desirability of a wide-angle lens and large film-size (to enhance S) and small object distance, to enhance parallax difference at constant overlap.

PHOTOSTEREO DATA ACQUISITION FOR HYDRAULIC ANALOGY MODELS

The equiheight, and therefore analogous equipressure contours of the undulating water surface, can be determined by photogrammetry in much the same manner as in topograhical mapping. As compared with the usual static subject where two photographic plates are prepared using the same camera moved an appropriate distance between exposures, for dynamic measurements two cameras must be simultaneously exposed. Figure 5 illustrates the mounting of the cameras over the water table. Figure 6 shows the test section with reference grid. The area of interest in Figure 6 is approximately 14×40 inches, the blade chord being 10 inches, a forty-fold magnification of the particular hot-gas turbine cascade of interest. Figure 7 gives the schematic arrangement of stroboscopic lights, cameras, synchronizing technique and initiating switch.

In experiments to date, two Wild T-2 phototheodolite cameras with 25°, 165 mm lenses (resolution approximately 30 lines per mm) were modified with new backs to accom-

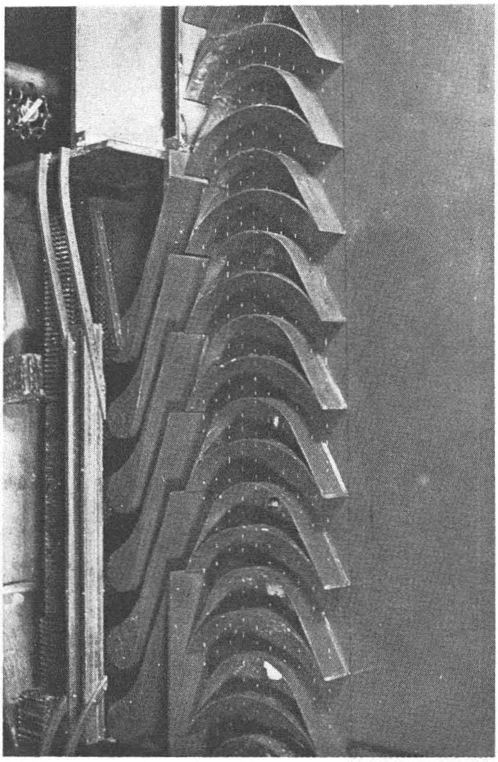

FIG. 6. Partial admission test section showing reference grid and sawdust shaker.

modate a nominal object distance of 3.2 feet, variable from 2 feet to the infinity object distance standard for the cameras. The plate size was 10 cm×15 cm, which together with the other characteristics of the commercial cameras, made it necessary to take several stereo-pairs to cover the entire field of interest.

Glass negative plates with a panchromatic emulsion of ASA 160 were used and stored horizontally to eliminate the possibility of film (or emulsion) dimensional change. The strobe lights were of 300 and 200 watt-seconds to permit reducing exposure time to 1/500 second, which, with a maximum water flow velocity of 2.1 feet per second caused some, but acceptable, blurring of the sawdust particles used to define the water surface. The table bottom was painted a dark red to enhance the contrast of the floating pale yellow sawdust. This particular combination of illumination conditions and film speed produced the best results. Faster exposures reduced light intensity and rendered underexposed pictures, while higher light intensities resulted in intolerable glare from the water surface.

PHOTOSTEREO DATA REDUCTION

The stereo-pair photographic plates (Figure A) were analyzed using a Wild ST-3 folding mirror stereoscope with stereometer (parallax bar) equipped with three- and eight-times binocular magnifiers as well as zero magnification lenses for rapid alignment of the stereo-pairs. With fairly good "stereo acuity" on the part of the operator, the stereometer was capable of providing individual parallax measurements reproducible to ±0.01 mm corresponding to water-heights of ±0.006 inch on the nominal depth of 0.38 inch.

Parallax errors are introduced in data reduction by misalignment of the stereo-pairs due either to rotation relative to each other or translations relative to the line defined by the principal-points of each camera, the intersection of the lens axis with the image-plane, and the image of each principal-point on the other camera's film. The principal points were determined by photographing plumb bobs hanging in the image-space orthogonal to the horizontal water-table bottom. An analysis of the two effects indicated errors due to translation insignificant even for translations of as much as one inch, while errors due to rotation could be important, a rotation of 0.02 radian causing an error in water-height of 0.03 inch or almost 10% the total water-depth. The reference grid facilitated parallel alignment of the stereo-pairs and eliminated the rotational error.*

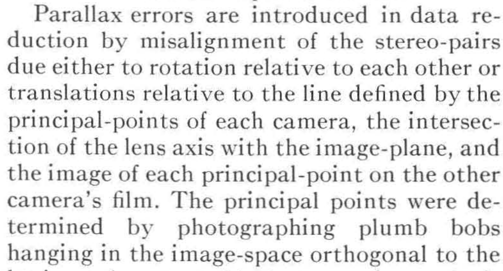

FIG. 7. Schematic of photogrammetric apparatus.

* EDITOR'S NOTE:—In response to a comment from the reviewer of this paper, the author submitted the following: "To avoid rotation of the stereo-pairs relative to each other, we desire to have on each stereo-pair the principal point of that camera and the image of the principal point of the second camera. In the absence of fiduciary marks on the first camera, and to obtain the image of the principal point of the second camera on plate No. 1, we use the plumb bobs. We were not using the coincidence of the nadir point and the principal point to determine the horizontality of the camera. We establish this independently."

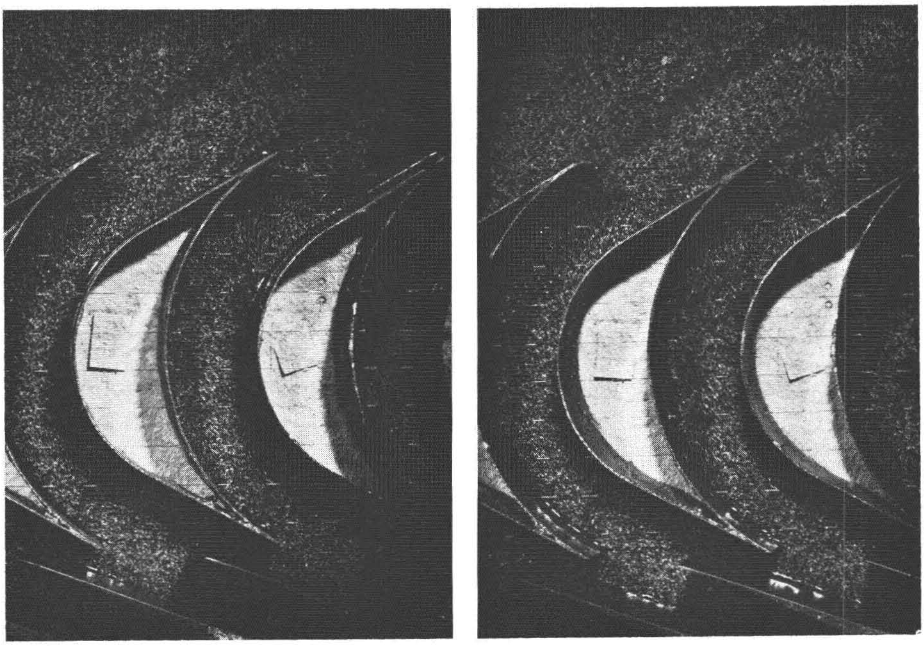

FIG. A. A stereo-pair of the photos used in the investigation. U (blade velocity) = 0.722 fps; V (nozzle velocity) 2.22 fps.; 5_h (scale of horizontal distance reduction) $\frac{1}{5}$.

Local distances between the grid and table bottom were established photogrammetrically by taking stereo-pairs with no water while identifying the bottom by means of sawdust particles. Then water-heights were determined from photostereo-pairs of active flow and the following relationship

$$h_w = K(\Delta R_{G-B} + R_w - R_G)$$

where

- h_w = water height
- R_w = the parallax reading on the water surface
- R_G = the parallax reading on the grid-point above the water's surface
- ΔR_{G-B} = the parallax difference between the grid and table bottom
- K = constant converting parallax difference to water-height difference for the particular photogrammetric system $-H_0^2/fB$ in Eq. 3

Lens distortion and/or slightly skewed lens-axes can cause "model warpage," an apparent parallax difference between points established to be on the same horizontal plane. For the experimental apparatus described, "warpage" sometimes contributed errors as large as 4% near the extremities of the overlap regions, when ΔR_{G-B} was taken as its averaged value over the entire area of interest. The grid makes possible the elimination of these "warpage" errors by providing point-by-point parallax differences between a discrete point on the water-surface and the grid intersection immediately above it.

TYPICAL RESULTS

Comparisons were made between dynamic water-heights measured using the photogrammetry system and the capacitive water-height probes. Water-height differences varied from zero to a maximum of 5%, with the electronic height measurement never smaller than the photographic indicating the presence of some small bias.

The over-all accuracy of the described system was within ± 0.02 inch on the mean water-height of 0.38 inch, or an over-all accuracy of $\pm 5\%$. The photogrammetric accuracy could undoubtedly be further improved by reducing the object distance (at the expense of requiring more stereo-pairs for the same area coverage), using the eight-times magnifier in the stereoscope (with concomitant increase of operator fatigue and data reduction time), and by providing more control points in the reference grid system.

Actually, for the purposes of hydraulic analogy evaluation, the present accuracy is quite acceptable. The validity of the water-table under transient shock conditions is probably only within $\pm 10\%$ of the corresponding gas case.[3] Primary interest is focused

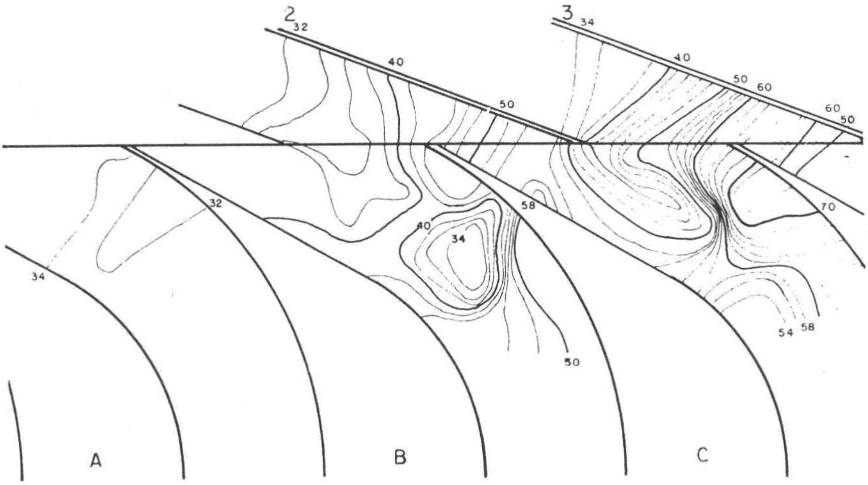

FIG. 8. Equiheight contours of water surface.

on height (or pressure) *ratios* rather than absolute values of height, and on the two-dimensional *shape* of the constant-height contours as influenced by blading geometry design. Figure 8 provides a small sample of a typical partial-admission turbine water-height contour map.[7] The turbine blade channel bounded by blades B and C has partially traversed the first nozzle bounded by surfaces 1 and 2. The nozzle Froude number (Mach number) is 2.26, wheel to nozzle-fluid velocity ratio 0.20, the nozzle exit water-height is 0.38 inch, and the contour interval is 0.02 inch with tenths of inches indicated by the heavier lines. Extensive data on the time-varying, two-dimensional flow-field in the partial-admission turbine will be presented in a forthcoming paper.

New Cameras

Neither the Wild cameras nor other commercially available cameras are intended specifically for the short object distance, large area coverage desirable for water-table, and other nontopographic mapping. Therefore, analyses were conducted on the camera attributes optimally suited to this work; specifications were prepared, and a pair of cameras have been designed around two war-surplus Metragon mapping-quality lenses of six-inch focal-length. The cameras, nearing completion, use 5×7 inch plate holders with a usable image-size of 4.75×6.75 inches. A single stereo-pair will provide an overlap region corresponding to an object size of 16×41 inches at an object distance of 42 inches.

Since overlap must be traded off against vertical magnification, see Equations 7 and 10, on the basis of over-all accuracy attained with the former set-up and the validity of the analogy for the tests of interest, the vertical magnification was reduced somewhat to 1 mm/0.8 or a practical resolution with the stereometer of ±0.01 mm corresponding to ±0.008 inch. Even with this decreased accuracy on individual parallax measurements, the over-all system accuracy will certainly improve over the present ±0.02 inch due to the use of only one stereo-pair and the elimination of the problem of matching the boundaries of several stereo-pairs. The horizontal magnification, the ratio of S to L, for the new cameras is $\frac{1}{8}$ compared to $\frac{1}{5}$ for the Wilds.

The speed of data reduction from the stereo-pairs, as well as alleviation of human fatigue and resulting errors, can be enhanced by use of a stereo plotting instrument, such as the Kelsh plotter. In order to use the Kelsh in the Photogrammetry Laboratory of the Civil Engineering Department at M.I.T., which has the added advantage of digital output for subsequent data processing, the ratio of base length B to object distance H must be greater than 0.30; compliance with this has been made.

In addition, provision has been made for accurately mounting the cameras in relation to each other and the object, changeable object-distance, and strobe synchronization.

Conclusion

The water-table, as the analogue of two-dimensional compressible gas flow, provides a relatively simple, inexpensive, flexible, direct means of qualitatively observing the influence of geometry on flow, providing an especially clear description of supersonic and nonsteady flow phenomena. The photogrammetric techniques described herein make possible *quantitative* measurements of the absolute pressures and pressure distributions in steady or nonsteady flow fields to an accuracy compatible with water-table accuracy, although enhanced accuracy could be achieved if desired. The optical technique avoids any direct influence on the flow, and can be adapted to use automated techniques of data reduction developed for topographic mapping.

ACKNOWLEDGEMENTS

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Tissot's Indicatrix and Photogrammetry

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IN 1881 the Frenchman M. A. Tissot provided certain theorems concerning the inevitable alteration of geometric relations which occurs on plane maps of the sphere or ellipsoid.[1] Since one of the problems of photogrammetry is the determination of the amounts by which geometric relations on the photograph differ from those on the object photographed, it is felt that a short review of Tissot's work will be of interest to photogrammetrists. Tissot's results are completely general and can be applied to aerial photographs, to radar images, or to camera lenses.

The concern is with transformations which take elements of one set into elements of a second set. This can be written as a pair of equations

$$u' = f_1(u, v)$$
$$v' = f_2(u, v)$$

The u', v' are to be interpreted as being plane coordinates on a piece of film (map, image) and u, v are coordinates used to describe locations on the original two-dimensional surface. In general it is assumed that the functions f_1 and f_2 are real, single-valued, continuous and differentiable functions of u and v in some domain, and that the Jacobian determinant

$$J = \frac{\partial u'}{\partial u}\frac{\partial v'}{\partial v} - \frac{\partial u'}{\partial v}\frac{\partial v'}{\partial v}$$

[1] M. A. Tissot, *Memoire sur la Representation des Surfaces* (Paris, Gautier—Villars, 1881). An application of Tissot's work in English can be found in: F. J. Marschner, "Structural Properties of Medium- and Small-scale maps," *Annals (Association of American Geographers)*, XXXIV, pp. 1–46.