*Mapping with a Strip Camera**

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ABSTRACT: *A stereoscopic strip camera might have some advantages over a frame camera as part of an automatic mapping system as it produces one long photograph instead of a number of separate ones; this paper considers the possibility of its use in this way, and develops the necessary equations for height and distance measurements with a strip camera. These equations are first derived for vertical photography and straight and level flight at constant speed; and then they are elaborated to take into account changes of aircraft altitude, attitude and speed. It is concluded that, given certain positional and attitude information, the strip camera could be used for mapping.*

1. INTRODUCTION

THE technical advances in photogrammetry since the end of the War have been very great—so much so that the present generation of optical-mechanical plotters and other instruments have made automatic a large part of the work involved in transforming a series of photographs into a map. This process of automation has not ended, for already work is being done on sophisticated machines which will rectify a photograph or form part of a map automatically. If this trend continues we may soon see fully automatic mapping systems in use.

An automatic system usually operates more efficiently as a continuous process than as an intermittent one. This made the author think of using a strip-camera instead of a frame-camera: a camera whose roll of film becomes one long picture rather than a series of separate exposures. However, all available publications (see (Bibliography) said that this could not be done, that it was impossible to use a strip-camera for mapping. The present paper grew out of the author's doubts about the accuracy of this statement; and it sets out to show that although to use the strip-camera with conventional plotting systems may not be feasible, it is by no means impossible to use such a camera as part of an automatic mapping system.

The analysis in this paper first develops the basic relations for making height and distance measurements from vertical continuous-strip photography, with the photographic aircraft flying straight and level at uniform speed. The paper continues with the consideration of the effects of speed, altitude and attitude change on these relations: these effects are dealt with separately as it is felt to be beyond the scope of this preliminary work to consider any cross effects that might arise. It is assumed that a scanning system connected to an electronic computer and a plotter would be used, though no attempt is made to consider the design of such units. It is also assumed that continuous information of aircraft speed and orientation would be available, perhaps on magnetic tape. Ways of obtaining this information will be discussed in a later section. Throughout the paper, except in one case, no attempt is made to assess the magnitudes of the various error terms involved.

The approach to the analysis of strip-camera photography given here contains several concepts which do not seem to have been used previously. The most important of these is the use of a parallel-line geometry as opposed to the similar-triangle geometry used for frame-cameras. Attempts to use the latter approach on strip-camera problems have led to errors in past publications. Other points are the

* In the 1961 contest for the Bausch & Lomb Photogrammetric Award for the best paper on photogrammetry by a College Student, this paper was given the Graduate Award.

NOTE: While great care was exercised by the Author, the Printer and me, a few errors are to be expected. If discovered, please inform the Author—*Editor* importance of using a fixed and known film speed throughout, and choosing a datum for height measurement which is related to the aircraft and not directly to ground-control.

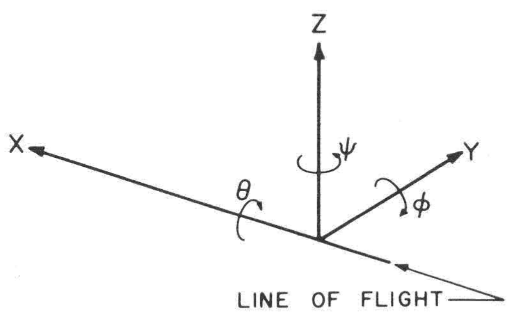
This paper, therefore, does not try to show that an automatic mapping system based on a strip-camera would necessarily be better than one based on a frame-camera; neither does it attempt a detailed feasibility study of such a system—such would have to be the subject of a far lengthier paper. Instead it is a preliminary study showing that not only is it possible to use a strip-camera for mapping work (contrary to all previous statements) but that its use might have some very real advantages.

2. Notation and Sign Convention

- A, B positions of objects A and B
- a, b positions of images of A and B
- c camera constant, equal to $f \tan \alpha$
- d longitudinal image shift from datum
- D distance between two objects
- D_F fore-and-aft distance between two objects
- D_L lateral distance between two objects
- f camera focal-length
- H aircraft height from datum
- h object height from datum
- l film datum displacement
- q longitudinal distance between images on film
- r lateral distance of image from film quarter-line
- t time, exposure time
- V aircraft speed
- v film speed
- α lens angle with vertical
- β angle at camera subtended by object and the vertical
- δ blurring
- θ roll
- ϕ pitch
- ψ yaw

Positive Directions

All expressions in this paper are algebraic, so that all lengths are assumed to be measured in the same units.



3. Aircraft Flying Straight and Level

3.1 Height Measurement

In the following analysis, the aircraft is assumed to be flying straight and level at constant speed with no pitch, roll or yaw.

Let the ground speed of the aircraft be V and the speed of the film in the camera be v. (Figure 1) Then the time between recording the two images of A is

$$t = \frac{2H \tan \alpha}{V}$$

and the distance between the images on the film is

$$2c = vt = \frac{v}{V} 2H \tan \alpha$$

If the film speed is correctly adjusted for image motion compensation, then

$$\frac{v}{V} = \frac{f}{H} \tag{1}$$

$$\therefore \; c = f \tan \alpha \tag{2}$$

Now let us suppose that the film speed is such that it is correct for a datum plane at a height H below the aircraft, and that we want to know the height h of an object above the plane. Note that the datum plane we have specified is related to the aircraft and not to any terrestrial datum. The film speed must be so adjusted

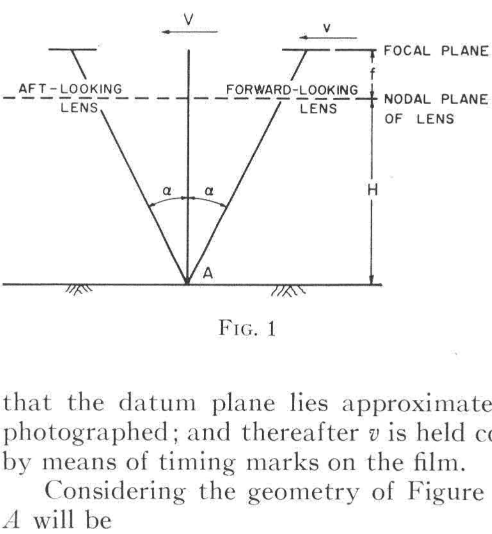


FIG. 1

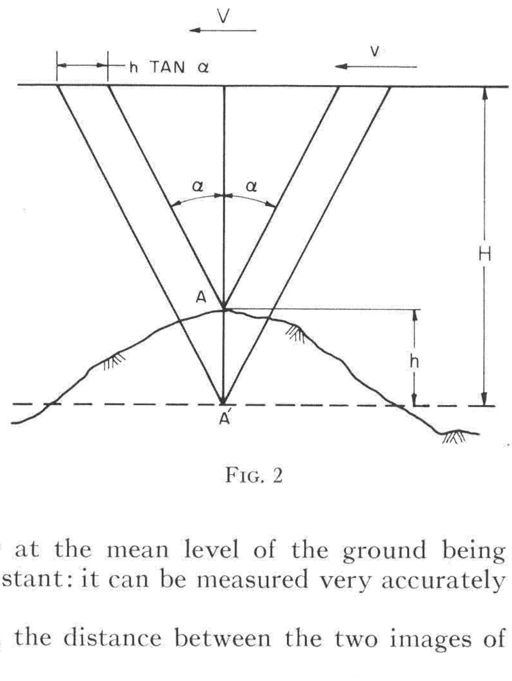


FIG. 2

that the datum plane lies approximately at the mean level of the ground being photographed; and thereafter v is held constant: it can be measured very accurately by means of timing marks on the film.

Considering the geometry of Figure 2, the distance between the two images of A will be

$$2(c - d) = \frac{2v}{V}(H - h) \tan \alpha$$

and using Equations (1) and (2) we obtain an expression for the height

$$h = \frac{Vf}{vc} d \tag{3}$$

where d is the change in position of each image of A due to A being above the datum plane, as shown in Figure 3. The expression may also be written:

$$h = \frac{Hd}{c} \tag{3a}$$

Both expressions apply equally well to the height of objects not on the line of flight of the aircraft. The accuracy of the height measurement clearly depends on the

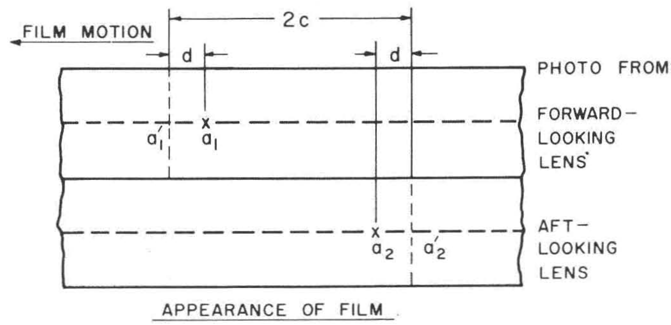


FIG. 3

accuracy with which we know V. Ways of measuring V are discussed at the end of the paper. The measurement of d is straightforward and simple.

3.2. DISTANCE MEASUREMENT—AIRCRAFT FLYING STRAIGHT AND LEVEL

In conventional aerial photography, image displacements due to relief radiate outwards from the nadir of the photograph, whereas in strip-camera photographs the equation for fore-and-aft image displacement is different from that for lateral displacement. Hence lateral and longitudinal distance measurements will be considered separately.

Consider two objects A and B with heights h_a and h_b, a distance apart $D=\sqrt{D_F^2+D_L^2}$ where D_F and D_L are the distance apart of the objects along and perpendicular to the line of flight.

If the objects are both on the datum plane, time between recordings of A and B would be

$$t = \frac{D_F}{V}$$

$$\therefore\ l = \frac{v}{V} D_F$$

so longitudinal distance between objects on film is

$$q = l + (d_b - d_a)$$

$$\therefore\ D_F = \frac{V}{v}\left[q + (d_a - d_b)\right] \tag{4}$$

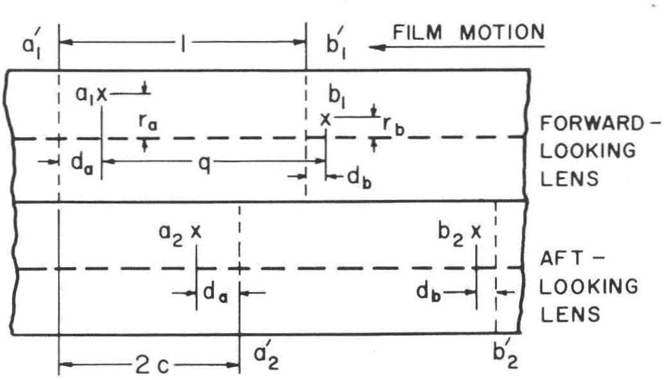


FIG. 4

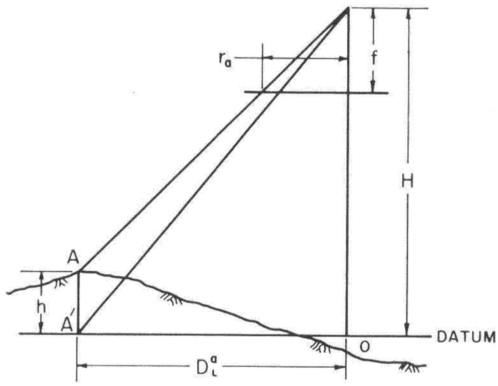


FIG. 5

The lateral displacement of the object is obtained from the usual geometrical relationships of conventional frame-camera photography.

$$\frac{r_a}{D_L{}^a} = \frac{f}{H - h_a}$$

$$\therefore \quad D_L{}^a = \frac{1}{f}(H - h_a)r_a = \frac{V}{v}\left(1 - \frac{d_a}{c}\right)r_a \tag{5}$$

so the lateral distance from A to B in the direction of B is given by

$$\therefore \quad D_L = D_L{}^b - D_L{}^a$$

$$= \frac{V}{v}\left[\left(1 - \frac{d_b}{c}\right)r_b - \left(1 - \frac{d_a}{c}\right)r_a\right] \tag{6}$$

Hence total distance between objects is

$$D = \sqrt{D_F{}^2 + D_L{}^2}$$

$$= \frac{V}{v}\sqrt{[q + d_a - d_b]^2 + \left[\left(1 - \frac{d_b}{c}\right)r_b - \left(1 - \frac{d_a}{c}\right)r_b\right]^2} \tag{7}$$

Note that this expression depends only on photographic measurements, the known film speed and the aircraft speed. We have the interesting and important result that distance measurements depend not at all on the height of the aircraft.

For rectification of images (to produce a planimetric map):

$$\left.\begin{array}{l} \text{Fore and aft image shift is } d \\ \text{Lateral image shift is } \dfrac{rh}{H} = \dfrac{rd}{c} \end{array}\right\} \tag{8}$$

These complete the necessary relationships needed for mapping assuming that the aircraft flies straight and level at constant speed. In the following sections we shall consider how various aircraft velocity and positional errors—translational and rotational—affect and modify the basic Equations (3) to (8).

4. Error Due to Lack of Full Image-Motion Compensation

Unless the aircraft is flying very fast or very low, or the terrain beneath is very rugged, the blurring of the image due to lack of complete synchronization will be negligible. For

$$\text{speed of film} \quad v_f = \frac{f}{H} V$$

$$\text{speed of image} \quad v_i = \frac{fV}{(H - h)}$$

$$\therefore \text{ relative speed} = fV\left(\frac{1}{H-h} - \frac{1}{H}\right) = \frac{fV}{H}\left(\frac{h/H}{1 - h/H}\right)$$

If the effective exposure time is t, then the movement of the image across the film, i.e. the blurring, is

$$\delta = \left(\frac{h/H}{1 - h/H}\right)\left(\frac{V}{H}\right)tf \tag{9}$$

On Figure 6, h/H is plotted against $(V/H)tf$ for various values of δ. This graph could be used to determine limiting heights or speeds for a given acceptable value of δ.

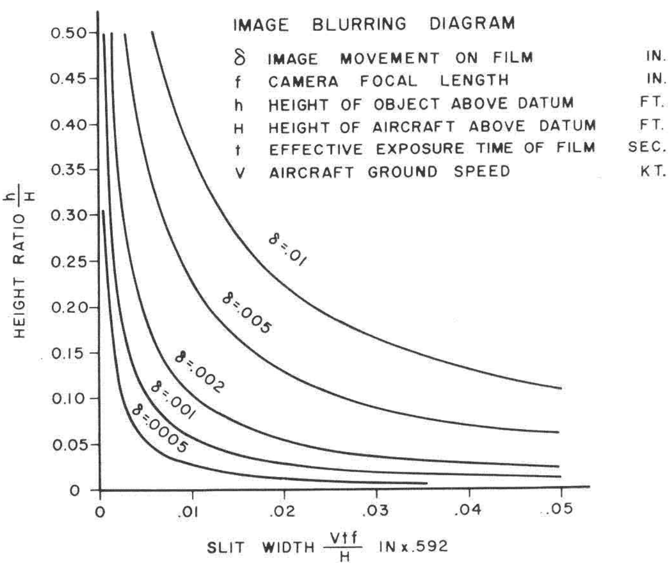


FIG. 6

As an indication of the magnitude of the error, let us consider the fairly extreme example of an aircraft flying at 3,000 ft with a speed of 300 kt over an object 300 ft high. Take the camera focal-length as 12 in. and the effective exposure time as 1/100 sec. Then from Figure 6 the blurring is seen to be just over 0.002 ins.

It can be seen that if the camera is used at normal mapping speeds and altitudes, the image will not be measurably blurred for height variations of the order of one-tenth the aircraft altitude, and accurate measurements can still be made for even greater variations in height.

5. The Effect of Variation of Aircraft Forward Speed

The most convenient approach to this problem is to consider the aircraft speed $V = V(t)$ as a variation of speed about an average value V_A. H is based on this, and so is the film velocity v. In the following analysis, a number of running average speeds are assumed to have been calculated.

5.1. EFFECT ON h

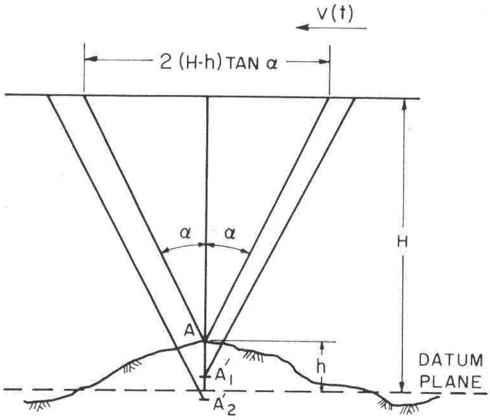


FIG. 7

Let V_a be the mean velocity between the two exposures of the images of the object A. Then distance between images on the film is

$$2(c-d) = 2(H-h)\tan\alpha \cdot \frac{v}{V_a} = \frac{2c}{f}(H-h)\frac{v}{V_a}$$

$$\therefore\ d = c\left(1 - \frac{(H-h)v}{fV_a}\right)$$

$$= c\frac{V_A}{V_a}\left(\frac{V_a}{V_A} - 1 + \frac{hv}{fV_A}\right) \quad \text{as} \quad H = f\frac{V_A}{v}$$

$$\therefore\ h = f\frac{V_a}{v}\left[\frac{d}{c} - \left(1 - \frac{V_A}{V_a}\right)\right] \tag{10}$$

and if $V_A = V_a = V$ we get the uncorrected relation

$$h = \frac{fVd}{vc}$$

also,

$$V_a = \frac{1}{\tau}\int_{t_A}^{t_A+\tau} V(t)dt$$

$$= \frac{v}{2(c-d)}\int_{t_A}^{t_A+\tau} V(t)dt$$

where

$$\tau = \frac{2(c-d)}{V}, \qquad \text{i.e. time between exposures of images}$$

5.2. EFFECT OF FORWARD SPEED VARIATION ON D_F

Let:

mean speed over distance D_F $= V_m$

average speed between the two shots of images A and B $= V_a,\ V_b$

speeds when taking forward shots of A and B $= V_a',\ V_b'$

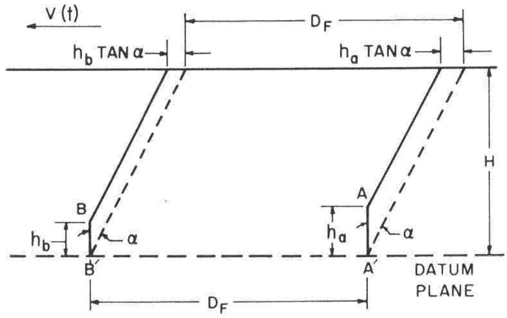


FIG. 8

Figure 8 and Figure 4 give the necessary geometry. Film distance between images of A and B is

$$q = v\left[\frac{D_F}{V_m} - \frac{h_a \tan\alpha}{V_a'} + \frac{h_b \tan\alpha}{V_b'}\right]$$

Using Equation (10) and putting $\tan\alpha = c/f$,

$$q = \frac{vD_F}{V_m} - \frac{V_a}{V_a'}\left[d_a - c\left(1 - \frac{V_A}{V_a}\right)\right] + \frac{V_b}{V_b'}\left[d_b - c\left(1 - \frac{V_A}{V_b}\right)\right]$$

$$\therefore\ D_F = \frac{V_m q}{v} + \frac{V_m V_a}{vV_a'}\left[d_a - c\left(1 - \frac{V_A}{V_a}\right)\right] - \frac{V_m V_b}{vV_b'}\left(d_b - c\right)\left(1 - \frac{V_a}{V_b}\right)\Big]$$

$$= \frac{V_m}{v}\left(q + \frac{V_a}{V_a'}\left[d_a - c\left(1 - \frac{V_A}{V_a}\right)\right] - \frac{V_b}{V_b'}\left[d_b - c\left(1 - \frac{V_A}{V_b}\right)\right]\right) \qquad (11)$$

If we put $V = V_m = V_A = V_a = V_a' = V_b = V_b'$ in this expression, we come back to Equation (4). If alternately we were to write

$$V_A = V$$
$$V_m = V + \delta V_m$$
$$V_a = V + \delta V_a$$
$$V_a' = V + \delta V_a'$$
$$V_b = V + \delta V_b$$
$$V_b' = V + \delta V_b'$$

we would get a rather formidable expression for D_F in terms of variations of velocity:

$$D_F = \frac{V}{v}\left(1 + \frac{\delta V_m}{V}\right)\left\{q + \frac{\left(1 + \frac{\delta V_a}{V}\right)}{\left(1 + \frac{\delta V_a'}{V}\right)}\left[d_a - c\frac{\delta V_a}{V}\left\{\frac{1}{1 + \frac{\delta V_a}{V}}\right\}\right]\right.$$
$$\left. - \frac{\left(1 + \frac{\delta V_b}{V}\right)}{\left(1 + \frac{\delta V_b'}{V}\right)}\left[d_b - c\frac{\delta V_b}{V}\left\{\frac{1}{1 + \frac{\delta V_b}{V}}\right\}\right]\right\} \qquad (11a)$$

which again reduces to Equation (4) for constant velocity.

No attempt has been made to simplify this expression by, for instance, eliminating second-order terms, as this would not be valid unless a systematic error analysis were made. Such an analysis was considered beyond the scope of this paper.

6. The Effect of Variation of the Height of the Aircraft

6.1. Effect on measurement of h

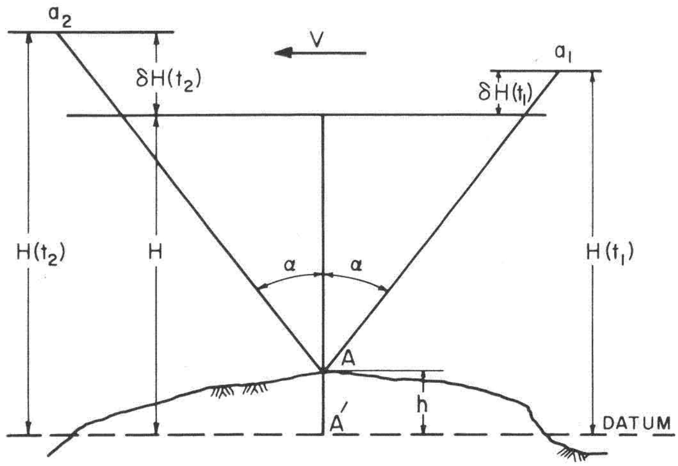


Fig. 9

Suppose the forward and aft shots of the object A are taken at times t_1 and t_2. Then horizontal distance between shots is

$$(H(t_1) + H(t_2) - 2h) \tan \alpha$$

so film-motion between shots is

$$2(c - d) = \frac{v}{V}(H(t_1) + H(t_2) - 2h) \tan \alpha$$

$$= \frac{v}{V}(2H - 2h + \delta H(t_1) + \delta H(t_2)) \tan \alpha$$

where $\delta H(t_1)$ and $\delta H(t_2)$ are the variations from the standard height H.

$$\therefore \ H(c - d) = c(H - h + \tfrac{1}{2}\delta H(t_1) + \tfrac{1}{2}\delta H(t_2))$$

$$\therefore \ h = H - \frac{H(c - d)}{c} + \tfrac{1}{2}\delta H(t_1) + \tfrac{1}{2}\delta H(t_2)$$

$$= \frac{Hd}{c} + \tfrac{1}{2}(\delta H(t_1) + \delta H(t_2))$$

$$= \frac{fV}{cv} d + \tfrac{1}{2}(\delta H(t_1) + \delta H(t_2)) \qquad (12)$$

6.2. Effect of variation of height on D_F

Reference should also be made to Figure 4 for geometrical relationships. Film distance between the forward-looking images of A and B is

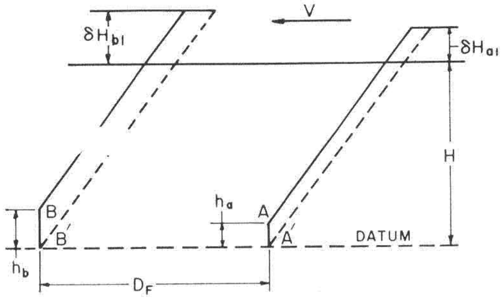


FIG. 10

$$q = \frac{v}{V}\left\{D_F + \frac{c}{f}[\delta H_{a_1} - \delta H_{b_1}] - \frac{c}{f}[h_a - h_b]\right\}$$

Substituting from Equation (12),

$$q = \frac{v}{V}\left\{D_F - \frac{V}{v}(d_a - d_b) + \frac{c}{2f}(\delta H_{a_1} + \delta H_{a_2} + \delta H_{b_1} + \delta H_{b_2})\right\}$$

$$\therefore \; D_F \simeq \frac{V}{v}[q + d_a - d_b] - \frac{2c}{f}\delta H_m \tag{13}$$

where δH_m is the running average height variation. This is normally assumed to be small and smooth.

6.3. EFFECT OF VARIATION OF HEIGHT ON D_L

Lateral displacement γ_a of the image of A on film is given by

$$\frac{r_a}{D_L{}^a} = \frac{f}{(H - h_a + \delta H_{a_1})}$$

$$\therefore \; D_L{}^a = \frac{1}{f}(H - h_a + \delta H_{a_1})r_a$$

$$= \frac{V}{v}\left(1 - \frac{d_a}{c}\right)r_a + \frac{r_a}{2f}(\delta H_{a_1} - \delta H_{a_2})$$

similarly

$$D_L{}^b = \frac{V}{v}\left(1 - \frac{d_b}{c}\right)r_b + \frac{r_b}{2f}(\delta H_{b_1} - \delta H_{b_2})$$

And so total lateral distance between objects is

$$D_L = D_L{}^b - D_L{}^a$$

$$= \frac{V}{v}\left[\left(1 - \frac{d_b}{c}\right)r_b - \left(1 - \frac{d_a}{c}\right)r_a\right]$$

$$+ \frac{1}{2f}[(\delta H_{b_1} - \delta H_{b_2})r_b - (\delta H_{a_1} - \delta H_{a_2})r_a] \tag{14}$$

7. The Effect of Lateral Velocity of the Aircraft (Crabbing)

Suppose the aircraft is flown as before with a forward speed of V, but that it now has a superimposed lateral velocity $V_2(t)$ which varies with time. Then the height-measurement and fore-and-aft distance measurement formulae given in Sections 3.1 and 3.2 still hold, but the lateral distance measurement formulae must be modified to include the drift. This merely adds a term

$$\int_{ta}^{t_b} V_L(t)dt$$

to the original expression, so that the modified lateral distance formula is now

$$D_L = \frac{V}{v}\left[\left(1 - \frac{d_b}{c}\right)r_b - \left(1 - \frac{d_a}{c}\right)r_a\right] + \int_{ta}^{t_a+\tau} V_L(t)dt \qquad (15)$$

As an approximation, we could let

$$\int_{t}^{t+\tau} V_L(t)dt = \tau(V_L)_m$$

where $(V_L)_m$ is the mean velocity. Or, putting $\tau = q/v$,

$$\int_{t}^{t+\tau} V_L(t)dt \simeq \frac{q(V_L)_m}{v}$$

8. The Effect of Roll

Rolling of the aircraft probably produces the most awkward of the correction relations needed for the strip camera. Suppose the aircraft rolls by a small angle $\delta\theta$. We shall first consider the effect of roll on lateral distance measurements.
Effective height of camera above object is, from Figure 11,

$$\begin{aligned} H_{\text{eff}} &= (H - h) \sec\beta \cos(\beta - \delta\theta) \\ &= (H - h)[\cos\delta\theta + \tan\beta \sin\delta\theta] \end{aligned}$$

From the geometry of the figure,

$$r/f = \frac{(H - h)\sec\beta \sin(\beta - \delta\theta)}{H_{\text{eff}}} = \frac{(\tan\beta\cos\delta\theta - \sin\delta\theta)}{(\cos\delta\theta + \tan\beta\sin\delta\theta)}$$

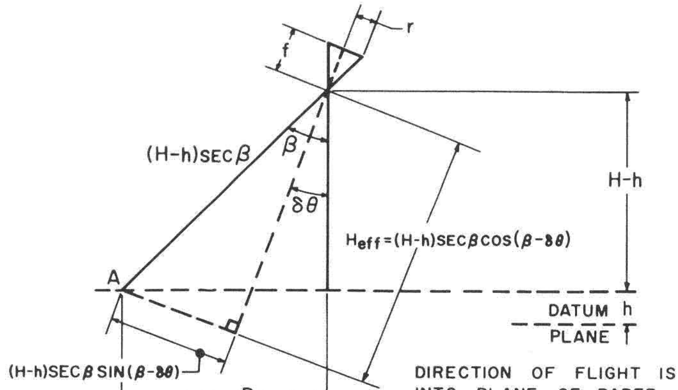


Fig. 11

$$\therefore \tan\beta = \frac{f\sin\delta\theta + r\cos\delta\theta}{f\cos\delta\theta - r\sin\delta\theta}$$

By geometry,

$$\frac{D_L}{(H-h)} = \tan\beta$$

$$\therefore D_L = (H-h)\left[\frac{f\sin\delta\theta + r\cos\delta\theta}{f\cos\delta\theta - r\sin\delta\theta}\right] \tag{16}$$

and if $\delta\theta \to 0$, this equation reduces to the vertical photography equation

$$D_L = (H-h)\frac{r}{f}$$

We obtain the necessary relationships for finding height and fore-and-aft distances by considering the optical geometry in a plane through the camera vertical and the line of flight, inclined at an angle $\delta\theta$ to the horizontal. Figure 12 is a view on this plane.

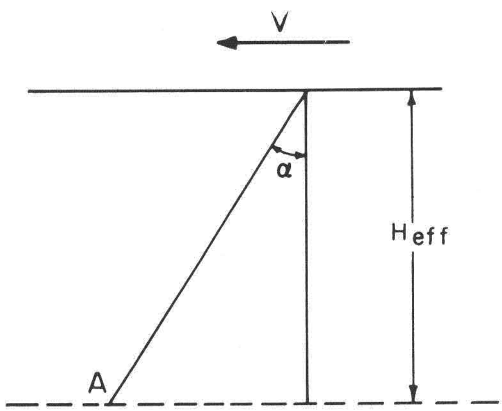


FIG. 12

The time elapsed between recording the image and the moment when the aircraft is directly above A is

$$t = \frac{\tan\alpha}{V}(H-h)(\cos\delta\theta + \tan\beta\sin\delta\theta)$$

The distance the film moves in this time is, putting $\tan\alpha = c/f$,

$$c - d = \frac{v}{V}\frac{c}{f}(H-h)(\cos\delta\theta + \tan\beta\sin\delta\theta)$$

and writing

$$\tan\beta = \frac{D_L}{(H-h)}$$

$$c - d = \frac{v}{V}\frac{c}{f}\left[(H-h)\left(1 - 2\sin^2\left(\frac{\theta}{2}\right)\right) + D_L\sin\theta\right]$$

So fore and aft shift of image position due to roll is

$$\Delta = \frac{vc}{Vf}\left[2(H - h)\sin^2\left(\frac{\theta}{2}\right) + D_L \sin\theta\right] \tag{17}$$

Note that for a positive (clockwise) roll and a positive (port) D_L, images due to the forward-looking lens are advanced along the film while those due to the aft-looking lens are retarded. Equation (17) can be used in conjunction with Equations (3) and (4).

9. The Effect of Pitch

Let us consider the effect on an image photographed by the forward-looking lens of a small increase of incidence $\delta\phi$.

The aft shift in the position of photography (i.e. the image will be recorded earlier) will be

$$(H - h)(\tan(\alpha + \delta\phi) - \tan\alpha) = (H - h)\frac{(f + c)\tan\delta\phi}{(f - c\tan\delta\phi)}$$

Hence image for-and-aft shift due to pitch is

$$\Delta = \frac{v}{V}\,\frac{(H - h)(f + c)\tan\delta\phi}{(f - c\tan\delta\phi)} \tag{18}$$

From this and Equations (3) and (4), h and D_F may be found. For lateral image position, distance from object to camera becomes (see Figure 13)

$$(H - h)\sec(\alpha + \delta\phi) = \frac{(H - h)\sec\alpha}{(\cos\delta\phi - \tan\alpha\sin\delta\phi)}$$

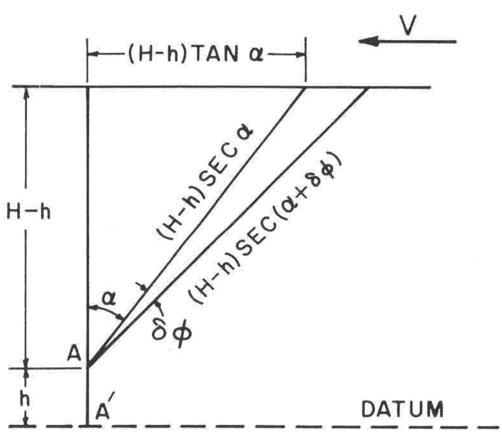


Fig. 13

Now,

$$r/D_L = \frac{f(\cos\delta\phi - \tan\alpha\sin\delta\phi)}{(H - h)}$$

$$\therefore\ D_L = \frac{(H - h)r}{(f\cos\delta\phi - c\sin\delta\phi)} \tag{19}$$

10. The Effect of Yaw

The shift in position at which the aircraft takes a photo of A due to yaw $\psi(t)$ can be seen from Figure 14 to be

$$(D_L)_{\text{app}} \sin \psi(t) + (H - h) \frac{c}{f} (1 - \cos \psi(r))$$

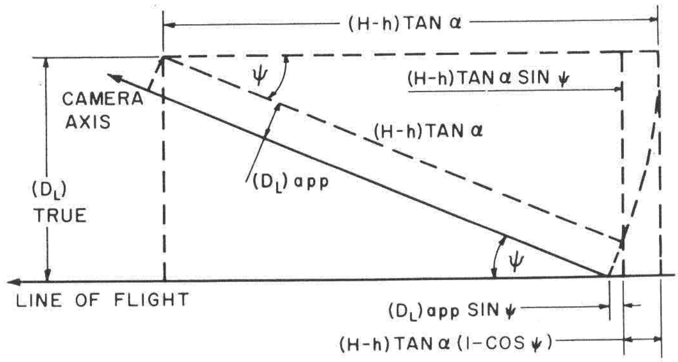


Fig. 14

where $(D_L)_{app}$ is the apparent offset of the object from the L.O.F. Hence longitudinal shift of image on the film from the zero-yaw position is

$$\frac{v}{V}\left[(D_L)_{\text{app}} \sin \psi(t) + (H - h) \frac{c}{f} (1 - \cos \psi(t))\right]$$

the yawed image lateral offset is

$$r_y = \frac{(D_L)_{\text{app}} f}{(H - h)}$$

which we can measure from the photograph. Hence the longitudinal image shift on the film due to yaw is

$$\Delta = \frac{v}{V} \frac{(H - h)}{f} [r_y \sin \psi(t) + c(1 - \cos \psi(t)] \qquad (20)$$

This expression may be used with Equations (3) and (4) to find h and D_F.

D_L may be found from the equation:

$$(D_L)_{\text{true}} = (H - h) \tan \alpha \sin \psi(f) + (D_L)_{\text{app}} \cos \psi(t)$$
$$= \frac{(H - h)}{f} (r_y \cos \psi(t) + c \sin \psi(t)) \qquad (21)$$

11. Methods of Determining the Position and Orientation of the Aircraft

In the preceding work it is assumed that we have available a continuous reference to:

1. time
2. aircraft forward ground-speed
3. aircraft lateral ground-speed
4. aircraft height

5. angle of roll
6. angle of pitch
7. angle of yaw.

It would also be useful for mapping work to have a continuous positional reference available.

In general, this information would be recorded on multi-channel tape. A clock would provide synchronising markings on both tape and film; these markings could also be used to measure the film-speed v.

There are basically four ways of providing information for the above requirements.

1. VISUAL OBSERVATIONS

These can be used for continuous information on heading, roll, pitch and yaw, using the sun or the horizon as references.

2. RADIO METHODS

Vortac, Hiran or a miniature H.F. form of the Decca system are suitable for positional and velocity information, which they are capable of giving very accurately. They cannot, however, give angular information. A Doppler system is also suitable and utltimately will be more useful than the other methods as it needs no control station for its operation. However, distance measurements are obtained by integration so that errors are cumulative and an initial position has to be known.

3. INERTIAL METHODS

Based on the use of integrating rate gyros and stabilised platforms, inertial guidance systems currently used for missile and submarine guidance should become available for other uses. These can be highly accurate; but like the Doppler system they need an initial positional reference from which to work and errors are cumulative. The very low drift rates of some systems make them suitable even for height measurements.

4. BAROMETRIC METHODS

A sensitive altimeter or statoscope can give the necessary height variation information for the equations of Section 6. Given suitable terrain and suitably modifying the readings to allow for meterological conditions, it has been claimed (24) that the height of aircraft can be given to a mean accuracy of about 5 ft. over a distance of many miles from a known control point.

Many papers have been written on the different ways of locating and orientating a photographic aircraft, and already there is much equipment on the market. However, this paper seeks to do no more than suggest ways in which the information necessary for the use of a strip camera for mapping could be obtained.

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*Close-Range Photogrammetry— A Useful Tool in Motion Study**

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AT PRESENT, three basic techniques are being used in industrial methods study. The first and most popular in the United States is the classical "stop-watch and personal observation" means of studying time and motion. Hundreds or even thousands of cycles of a specific working process are sometimes required in the analysis.

The second technique involves the use of high-speed motion pictures. An advantage is that a permanent record of the motion is made which can be reviewed at any time. Nevertheless, this method has failed to gain much popularity because it offers little advantage, with respect to cost, over personal observation.

The cyclegraph or so-called point-light process is the third possibility. This technique has given some aid to industry in analyzing motions and improving methods. Light patterns are made on photographic film in a still camera from lamps attached to an operator's hands or arms. However, a continuous path of light leaves much to be desired in the

* Submitted in the competition for the 1962 Bausch and Lomb Photogrammetric Award.