*The Absolute Accuracy of Photogrammetry**

FREDERICK J. DOYLE,
Autometric Corporation
400 N. Washington St.
Alexandria, Va.

ABSTRACT: *Modern mapping systems propose to eliminate or reduce ground-control by providing auxiliary devices to record camera station position, altitude, and angular orientation. Formulas are developed to investigate the elevation accuracy obtainable under these conditions. It is shown that the feasible contour interval is three to eight times larger than that obtainable with complete ground-control.*

THE ability of a stereophotogrammetric mapping system to discriminate increments in elevation is expressed by the well-known formula:[1]

$$dh = \frac{H}{B} dP = \frac{H}{B} (dX_2 - dX_1) \tag{1}$$

in which:

dh = increment of elevation
H = flight altitude
B = length of airbase
dP = increment of parallax in ground units
dX_1, dX_2 = increments in projected image coordinates parallel to airbase

For near-vertical photography, the ground increment dX is equal to the image increment dx multiplied by the scale number.

$$dX = \frac{H}{f} dx \tag{2}$$

in which f is the camera focal-length. Now if the reasonable assumption is made that the precision of measurement is identical on both photos of the stereopair, i.e., $dx_1 = dx_2 = m_x$, the precision of a single observation for elevation may be expressed as

$$m_h = \frac{H}{f} \cdot \frac{H}{B} \cdot m_x \sqrt{2} \tag{3}$$

For any system, the limiting value of image measurement is a function of the linear resolution of the photography.[2] If the resolution is expressed as l lines per millimeter

$$0.6745\, m_x = \frac{1}{5} \frac{mm}{l}$$
$$m_x = 0.3\text{mm}/l \tag{4}$$

It is important to realize the physical meaning of Formula (3). It expresses the "mushiness" of the stereo-model. More precisely, it expresses the standard deviation of a single observation of the elevation of a point in the model. The standard deviation of the elevation can, in principal, be reduced by taking the mean of several observations, so that spot heights may be determined more precisely than the value given by these formulas. However, *if all other errors in the system are considered to be zero*, the contouring ability is directly related to this standard deviation since in the operation of contouring, each point is observed only once as it is passed. In order to meet the criterion that 90 per cent of elevations be correct within one-half the contour interval, National Map Accuracy Standards would require that

$$\text{c.i.} = 3.3\, m_h$$

Substituting Formula (3) in this relation yields

$$\text{c.i.} = 4.7 \frac{H}{f} \cdot \frac{H}{B} \cdot m_x \tag{5}$$

Formula (5) may be related to the usual concept of c-factor, defined as the ratio of flight altitude to contour interval.

[1] See, for example: Zeller, M., *Text Book of Photogrammetry*, H. K. Lewis, London 1952. Except for notation, Formula (1) is identical with Zeller's formula (25c) on p. 175.

[2] Gardner, I. C. The Optical Requirements of Airplane Mapping. *Bureau of Standards Journal of Research*, Vol. 8. "The probable error of a single setting is $\frac{1}{5}$ to $\frac{1}{6}$ of the distance between two lines which are just resolved."

* Prepared for presentation to the American Society of Photogrammetry Convention at St. Louis, Mo., September 10, 1962.

$$c = \frac{H}{\text{c.i.}} = 0.21 \frac{B}{H} \cdot \frac{f}{m_x} \tag{6}$$

Since the Base-Height ratio and the focal-length are geometric parameters, it is apparent that variations of c-factor among instruments are dependent essentially upon the capability of the instruments to exploit the resolution of the photography—subject to the condition of all other errors being zero.

To demonstrate the reasonableness of Formula (6), consider a Kelsh Plotter utilizing conventional six-inch focal-length wide-angle photography. Twenty lines-per-millimeter is probably an optimistic estimate of the average resolution which can be utilized. Then

$$m_x = 0.3/20 = 0.015 \text{ mm.}$$

and

$$c = 0.21 \times 0.6 \times 152/0.015 \approx 1300$$

Now the restriction imposed above—that all other error sources are negligible—is a serious one indeed. So far as elevation measurement is concerned, it implies that at least two horizontal-control points are available so that the stereo-model may be precisely scaled, and that at least five properly distributed elevation control points are available so that the stereo-model may be corrected to the sea-level datum, be properly levelled, and be cleared of any warping caused by errors in relative orientation. It is well-known that model errors caused by errors in both interior and exterior orientation can be compensated if sufficient ground-control is available. This really means that conventional photogrammetric measurements are *relative* rather than absolute.

In modern mapping systems there is a desire to eliminate or reduce the number of ground-control points by providing auxiliary data recorders which will indicate the exposure station coordinates and the angular orientation for each photograph. When mapping is performed in this way, compensating errors can no longer be counted upon, and photogrammetry becomes an *absolute* measuring technique. It is expedient to consider the effect which errors in the outer orientation will have upon the elevations of model points, and to compare the contour interval obtainable under these conditions with that obtained with the use of ground-control.

In such a case, the dX in Formula (1) must take into account not only the image measurement as given by Formula (2), but also the uncertainties in the exposure station coordinates and angular orientation. An expression which does this is:[3]

$$dX = dX_L + \frac{X}{H} dZ_L + \frac{XY}{H} d\omega - \frac{H^2 + X^2}{H} d\phi - Y d\kappa + \frac{H}{f} dx \tag{7}$$

in which

dX_L = displacement of exposure station in direction parallel to air-base

dZ_L = displacement of exposure station in altitude

$d\omega$, $d\phi$, $d\kappa$ = errors in roll, pitch, and yaw respectively

X, Y = projected coordinates of image-point referred to origin in nadir-point.

Now if it be assumed that the errors are equal at each of the exposure stations, and that no correlation exists among the errors, then the differentials in Formula (7) may be replaced by standard errors, the law of error propagation applied, and the results substituted in Formula (1). The effects of probable correlations can be eliminated by careful consideration of the error values which are eventually substituted in the final formula. The resulting expression for the standard error at a point in the model is

$$m_h^2 = \frac{2H^2}{B^2} \left[m_{X_L}{}^2 + \frac{X^2}{H^2} m_{Z_L}{}^2 + \frac{X^2 Y^2}{H^2} m_\omega{}^2 + \frac{(H^2 + X^2)^2}{H^2} m_\phi{}^2 + Y^2 m_k{}^2 + \frac{H^2}{f^2} m_x{}^2 \right] \tag{8}$$

Since this expression contains the X and Y coordinates of points, it is variable over the area of the stereo-model. In order to obtain an average value expressive of the whole model, Formula (8) may be integrated over the area of the model, and then divided by the area of the model. That is

$$\sigma_h{}^2 = \frac{\int_0^Y \int_0^B m_h{}^2 dX dY}{\int_0^Y \int_0^B dX dY}$$

For conventional wide-angle photography the limits of integration are

$$B = 0.6\ H$$

$$Y = 0.75\ H$$

[3] Except for differences in sign, and the inclusion of the final term, this formula is the same as formula (3–5), p. 130, in: Hallert, B., *Photogrammetry*, McGraw Hill, New York, 1960.

After integration and evaluation, the result is:[4]

$$\sigma_h^2 = 5.56\, m_{X_L}^2 + 0.67\, m_{Z_L}^2 + 0.125\, H^2 m_\omega^2 + 7.03\, H^2 m_\phi^2 + 1.04\, H^2 m_k^2 + 5.56 \frac{H^2}{f^2} m_x^2 \quad (9)$$

TABLE 1

EXPECTED STANDARD DEVIATIONS FOR CONTROLLED MAPPING SYSTEM AND PHOTOGRAMMETRIC INSTRUMENT

Error	*Symbol*	*USQ-28*[5]	*Wild A-7*
Camera position	m_{X_L}	5 feet[6]	1.3 feet[7]
Camera altitude	m_{Z_L}	10 feet[8]	1.3 feet
Roll	m_ω	30 seconds	15 seconds[9]
Pitch	m_ϕ	30 seconds	15 seconds
Yaw	m_k	180 seconds[10]	15 seconds
Image position	m_x	0.007 mm.	0.007 mm.

[4] The same approach may be applied to symmetrically convergent photography. However, since the stereo-model may not extend from nadir-point to nadir-point, the limits of integration on X must be from X_1 to X_2. Furthermore expression (2) must be replaced by

$$dX = \frac{(H \cos \alpha + X \sin \alpha)^2}{Hf} dx$$

in which α is the angle between the camera axis and the vertical. The use of the simple coefficients for vertical photography as given in Formula (7) is justified since any attitude measuring system will probably record pitch, roll, and yaw in a local vertical system rather than ω, ϕ, κ as they might be set in a stereo-plotting instrument.

[5] The USQ-28 system is not yet operational. The values listed are interpreted from: Robson, Walter M., *The AN/USQ-28 Mapping Survey Sub System*, presented to the American Society of Photogrammetry March 1962.

[6] The objective of USQ-28 is to provide a standard error of 10 feet in camera station positions. However the errors in adjacent air stations will certainly be correlated. After this correlation is removed, 5 feet is a reasonable estimate of the residual standard error in each station.

[7] The exposure station coordinates in the A-7 have a least reading of 0.01 mm. This value, multiplied by the photo scale gives the actual error.

[8] The error in exposure station altitude depends upon the barometric error and the radar altitude error, each of which will be about 10 feet. The barometric error, however, will be nearly equal at the two air stations.

[9] The angular orientation of the A-7 has a least reading of 0.01 cg $\approx$ 32 seconds.

[10] Swing (k) may be determined by relative orientation with a standard deviation of about 50 seconds. This would undoubtedly be done.

To demonstrate the application of the formula, a typical mapping mission may be assumed. The photography will be obtained with a 6 inch focal-length, 9×9 inch format, mapping camera capable of producing an AWAR of 40 lines-per-millimeter. Flight altitude will be 20,000 feet. Exposure-station coordinates, altitude, and angular orientation will be provided by auxiliary systems such as have been proposed for the USAF system AN/USQ-28. The map compilation will be performed in a first-order photogrammetric plotter similar to the Wild A-7. In such a case consideration must be given to whether the limiting accuracy is given by the data recorders or by the ability of the instrument to accept the data.

The expected value for the errors are given in Table 1.

It is apparent from Table 1 that the A-7 is a pretty good match to the USQ-28 system. It will be able to absorb all the inputs without contributing appreciably to the final map error.

The angular values in Table 1 may be converted to radians, and then the errors of the USQ-28 system may be substituted in Formula (9). It is instructive to evaluate the effect of each component error individually before taking the resultant. The values are listed in Table 2.

Now it is appropriate to compare the results in Table 2 with those which can be obtained with ground-control in the stereo-model. If the model is completely controlled, then the sole source of error is the 2.2 feet resulting from the image-resolution. Also it may be noted from Formula (8) that the errors resulting from error in the camera position, the correlated part of the error in camera altitude, and a major part of that due to pitch, are constant over the stereo-model. That is to

TABLE 2

ELEVATION ERRORS CAUSED BY RECORDING ERRORS OF THE USQ-28 SUBSYSTEM

Error	*Symbol*	*Magnitude*	*Effect*
Camera position	m_{X_L}	5 ft.	11.7 ft.
Camera altitude	m_{Z_L}	10 ft.	8.2 ft.
Roll	m_ω	0.000145 rad.	1.0 ft.
Pitch	m_ϕ	0.000145 rad.	7.7 ft.
Yaw	m_k	0.000242 rad.	4.9 ft.
Image resolution	m_x	0.007 mm.	2.2 ft.
Resultant elevation error	σ_h		17.1 ft.
Contour interval	c.i.	3.3 σ_h	56 ft.

TABLE 3

COMPARISON OF ELEVATION ACCURACY WITH AND WITHOUT GROUND CONTROL

Control	*Standard error*	*Contour interval*
Auxiliary data	17.1 ft.	56 ft.
One point	9.8 ft.	33 ft.
Two points	5.5 ft.	18 ft.
Complete control	2.2 ft.	7 ft.

say, they represent a shift in the datum for elevations, and could be removed if a single vertical-control point were present. Two control points at the X limits of the stereo-model would permit removal of the total error caused by camera altitude. These effects are summarized in Table 3.

It is apparent from Table 3 that auxiliary data, at the present state of accuracy, is not an adequate substitute for ground-control. The suitable contour interval is increased by a factor of 8 above that obtained with complete ground-control. The addition of a single control point reduces the factor to 5, and two control points reduce it to less than 3.

This investigation has been concerned only with a single stereo-model. Quite obviously the errors propagated from the auxiliary data can be reduced somewhat if the stereo-models are triangulated between reasonably spaced control-points. However, at the moment, no adequate means exists for utilizing all the auxiliary data in an aerial triangulation. In the future this may be accomplished by an analytical solution in which the auxiliary data are imposed as constraints with weights inversely proportional to their variances.

*"Accuracy and Precision in Photogrammetry"**

BERTIL P. HALLERT,
Inst. for Photogrammetri
Kungl Tekniska Högskolan
Stockholm 70, Sweden

ABSTRACT: *The main task of photogrammetry is to measure geometrical data with the aid of photographs. The measurements must be made with sufficient geometrical quality at lowest possible costs and within the shortest possible time.*

Since the quality of the measurements is of fundamental importance for the application of photogrammetry, the methods to determine this quality in actual cases and the terminology to express the results must be of great interest.

In photogrammetry as well as in other sciences of measurement, a certain confusion concerning the terminology for quality is frequently found. There are many possible interpretations of common expressions for quality and in most cases the reader does not know the real meaning behind expressions like "the results were obtained with an accuracy (precision) of . . . ." In this paper some points of view of a highly desirable standardization of the terminology for quality in photogrammetry are given.

INTRODUCTION

IN THE literature on measurements, not only concerning photogrammetry and geodesy, but other sciences as well, the geometrical quality of basic data and of final results is sometimes expressed in vague and unclear terms. In reading a paper or an advertisement, or when listening to an oral presentation on measurements or instruments, expressions of the following type will sooner or later appear:

The measurements have (have been made with, can be made with). Or the instrument has. . . .

An accuracy of . . . (for instance 1 micron), a precision of . . . (±1 micron).

Sometimes we also find expressions like: a reliability of . . . (10 feet), a geometrical quality of . . . , an uncertainty of . . . , or simply, an error of . . . , and in other cases accuracy within . . . , precision within . . . , accurate to . . . , precise to. . . .

All of these expressions may refer to one or more of the following proper concepts and

* Presented at St. Louis ACSM-ASP Convention, Sept. 10, 1962.