TABLE 3

COMPARISON OF ELEVATION ACCURACY WITH AND WITHOUT GROUND CONTROL

Control	*Standard error*	*Contour interval*
Auxiliary data	17.1 ft.	56 ft.
One point	9.8 ft.	33 ft.
Two points	5.5 ft.	18 ft.
Complete control	2.2 ft.	7 ft.

say, they represent a shift in the datum for elevations, and could be removed if a single vertical-control point were present. Two control points at the X limits of the stereo-model would permit removal of the total error caused by camera altitude. These effects are summarized in Table 3.

It is apparent from Table 3 that auxiliary data, at the present state of accuracy, is not an adequate substitute for ground-control. The suitable contour interval is increased by a factor of 8 above that obtained with complete ground-control. The addition of a single control point reduces the factor to 5, and two control points reduce it to less than 3.

This investigation has been concerned only with a single stereo-model. Quite obviously the errors propagated from the auxiliary data can be reduced somewhat if the stereo-models are triangulated between reasonably spaced control-points. However, at the moment, no adequate means exists for utilizing all the auxiliary data in an aerial triangulation. In the future this may be accomplished by an analytical solution in which the auxiliary data are imposed as constraints with weights inversely proportional to their variances.

*"Accuracy and Precision in Photogrammetry"**

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ABSTRACT: *The main task of photogrammetry is to measure geometrical data with the aid of photographs. The measurements must be made with sufficient geometrical quality at lowest possible costs and within the shortest possible time.*

Since the quality of the measurements is of fundamental importance for the application of photogrammetry, the methods to determine this quality in actual cases and the terminology to express the results must be of great interest.

In photogrammetry as well as in other sciences of measurement, a certain confusion concerning the terminology for quality is frequently found. There are many possible interpretations of common expressions for quality and in most cases the reader does not know the real meaning behind expressions like "the results were obtained with an accuracy (precision) of . . . ." In this paper some points of view of a highly desirable standardization of the terminology for quality in photogrammetry are given.

INTRODUCTION

IN THE literature on measurements, not only concerning photogrammetry and geodesy, but other sciences as well, the geometrical quality of basic data and of final results is sometimes expressed in vague and unclear terms. In reading a paper or an advertisement, or when listening to an oral presentation on measurements or instruments, expressions of the following type will sooner or later appear:

The measurements have (have been made with, can be made with). Or the instrument has. . . .

An accuracy of . . . (for instance 1 micron), a precision of . . . (± 1 micron).

Sometimes we also find expressions like: a reliability of . . . (10 feet), a geometrical quality of . . . , an uncertainty of . . . , or simply, an error of . . . , and in other cases accuracy within . . . , precision within . . . , accurate to . . . , precise to. . . .

All of these expressions may refer to one or more of the following proper concepts and

* Presented at St. Louis ACSM-ASP Convention, Sept. 10, 1962.

terms which are found in various textbooks and other publications on statistics and theory of errors for measurements:

Standard deviation (referred to one single measurement or to the mean of several repeated or replicated measurements), standard error, probable error, standard error of unit weight, root mean square error, mean error, average error, discrepancy, root mean square value of discrepancies, closing error, standard closing error, variance, sigma, two-or-three sigma, the range, the semi-interquartile range, various kinds of confidence limits on certain and different confidence levels, tolerance limits, maximum errors, maximum deviations, etc., etc.

It is certainly not easy to find one's way through this jungle of expressions and to interpret correctly quality information as given by the terms: accuracy of . . . , precision of . . . , accurate to . . . , and precise to. . . . It is not easy either to interpret the magnificent expression "exact results of medium accuracy" which is given in a good modern textbook of photogrammetry.

In many cases the consequences of such inexact expressions might not be too serious. But sometimes it is of basic importance to know what the author really means. In particular, for investigations of the error propagation from the actual observations, and for the determination of tolerances for instruments and procedures, it is absolutely necessary to know the real meaning of the quality expressions.

It is for instance not satisfactory to characterize the geometrical quality of an instrument with the expression that it has an accuracy of one micron because there are different opinions as to what such an expression really means. For a definite reason an investigation concerning the interpretation of this expression was made among prominent scientists in the field of technology in Europe, many of whom are responsible for teaching. Out of twenty persons: as to what an "accuracy of one micron" means to them:

Five interpreted the expression as *standard deviation of one measurement.*

Eleven understood it to mean *maximum error.*

One proposed its meaning to him as *probable error* but proceeded to define this concept incorrectly.[1]

Three said they did not know. Theirs were doubtless the best answers.

[1] It should be noted that the "probable error" is not most probable to occur.

Also in this country a number of scientists were asked about their interpretation. A very common answer is that an "accuracy of one micron" means "probable error."

In summary, the existing jungle of concepts and terms for geometrical quality of measurements needs to be cleared, and some kind of standardization is doubtless highly desirable. This has also been noted by the International Society for Photogrammetry who made some general statements in the resolutions from the London Congress in 1960. The resolutions 2a and 4 from Commission II are as follows:

> "All information on accuracy should be expressed in clear and well-defined terms. This information should be combined with data about its reliability, e.g. by giving the number of redundant observations."
>
> "In order to compare the results of different theoretical and practical investigations into instruments and methods, it is suggested that the observations are adjusted according to the method of least squares."

A committee was also appointed for further investigations into this difficult and controversial problem.

What can now be done?

First it is obvious that a close cooperation between statistics, geodesy, photogrammetry, and other measuring sciences is very desirable. This has also been proposed. It is probably impossible to find a terminology which will satisfy all of these branches completely because of the great variety of the measuring methods and the basic data. In geodesy, for instance, the basic data-angles and distances are generally directly measured and the basic measurements can often be repeated or replicated arbitrarily, for the purpose of correction of possible regular or systematic errors before further computations, and in order to increase the geometrical quality of the averages.

In photogrammetry, the basic data are the image coordinates, which always are functions of at least nine parameters, namely the elements of the interior and exterior orientation, which, however, can compensate each other to a certain extent. In addition there are regular errors, for instance radial and tangential distortion, affinities, etc. The imaging procedure usually takes place during a very short time and cannot be replicated. Consequently regular or systematic errors of the image coordinates must play a very important role in the basic photogrammetric data. Calibration procedures of cameras and instru-

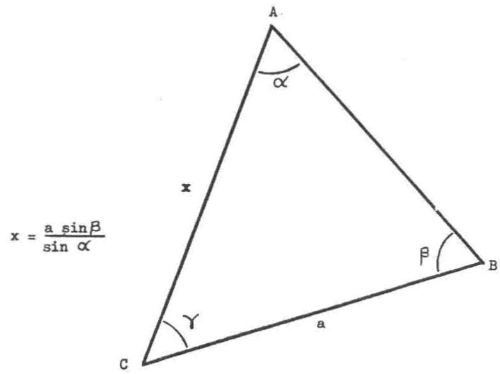

FIG. 1.

ments are therefore of fundamental importance for the photogrammetric procedure but must also be made under real operational conditions in addition to laboratory work.

Regular or systematic errors do not lend themselves to "probabilistic" treatment and are therefore not particularly observed in statistics. In fact, the determination of the geometrical quality of calibration procedures is given very little treatment in the statistical literature, and the terminology is not entirely satisfactory for the photogrammetric problems.

In order to survey the opinion among American photogrammetrists concerning possible terms to be used, a draft containing some *existing* concepts and terms has been distributed with a request for critical remarks and positive proposals. The answers show very different opinions and clearly indicate that there is a real need for a well-defined terminology. The work which is being performed in this respect within the American Society of Photogrammetry and within the International organizations is therefore very important. Close cooperation is obviously much desired.

In this paper only two terms will be further discussed, namely *accuracy* and *precision* because these are so widely used and seem to be of particular importance.

Clear definitions are given in the statistical literature of the two concepts. The book—"A Dictionary of Statistical Terms"—by Kendall and Buckland, published in 1957, contains the following definitions:

> *Accuracy* in the general statistical sense denotes the closeness of computations or estimates to the exact or true values.
>
> *Precision* is a quality associated with a class of measurements and refers to the way in which repeated observations conform to themselves.

A similar definition has been given by H. C. Mitchell in a paper: "Precision and Accuracy," published in *Military Engineer*, Nov.–Dec., 1950. This paper refers to another publication: "Definitions of Terms Used in Geodetic and Other Surveys," also by H. C. Mitchell and published by the Coast and Geodetic Survey in 1948 as a special publication No. 242. As an illustration of the two concepts, measurements and checks of the angles in a triangle will be shown, see Figure 1.

Assume the side x of the triangle ABC to be computed from the measured side a and the three angles α, β and γ:

$$x = \frac{a \sin\beta}{\sin\alpha}$$

If the geometrical quality of the measured data is known, the geometrical quality of x can be determined from the special law of error propagation. For the determination of the geometrical quality of the angles, two principles can be utilized namely:

1. Replicated[2] or repeated measurements of each angle and statistical study of the deviations between the individual measurements and the average. This gives the *precision*.
2. Comparison between the sum of the three angles with the condition that this sum shall be 180° or 200^g. The *accuracy* can be determined from the discrepancy.

1. THE PRECISION

The measurements of each angle are replicated or repeated an arbitrary number of times n and the averages of the measurements are to be used as the final angles. The precision of the measurements of the angles and of the averages can then be determined as follows: The measurements of the angle α have given the following results:

Measured Angles	*Corrections to each angle with respect to the average*	
α_1	$+v_1' = \bar{\alpha}$	$v_1' = \bar{\alpha} - \alpha_1$
α_2	$+v_2' = \bar{\alpha}$	$v_2' = \bar{\alpha} - \alpha_2$
$\vdots$	$\vdots$	$\vdots$
α_n	$+v_n' = \bar{\alpha}$	$v_n' = \bar{\alpha} - \alpha_n$
$\bar{\alpha} = \dfrac{\sum \alpha}{n}$		$\sum v' = n\bar{\alpha} - \sum \alpha = 0$

[2] *Replicated* measurements are made at one place and at one period of time.
Repeated measurements are made at different places and at different periods of time. (Kendall and Buckland)

The precision of the measurements of the angle α can now, according to the definition, be determined from the conformity of the observations to themselves i.e. from the corrections v_1' to v_n'. According to well-known formulas the *standard deviation of* ONE *observation is*

$$s_\alpha = \sqrt{\frac{\sum v'^2}{n-1}}$$

The *standard deviation of the* AVERAGE is then found as

$$s_{\bar{\alpha}} = \frac{s_\alpha}{\sqrt{n}} = \sqrt{\frac{\sum v'^2}{n(n-1)}}$$

A very important consequence of this principle is that the *standard deviation of the average decreases with the square root of the number of the observations*, and that at least *theoretically* the standard deviation of the average can be arbitrarily decreased, or that the precision of the average can be arbitrarily increased, in repeating or replicating the observations a sufficient number of times. This increase is in practice limited by the fact that not all errors may show up in the deviations between the average and the individual observations.

Errors in the centering of the theodolite and signals or side refraction may affect the individual angles with equal amounts and will therefore affect the averages also. Such errors may be interpreted as correlations. The centering of signals and theodolites, and the leveling of the instruments etc., are in fact also measuring operations which never can be made entirely free from errors. Even if the nature of these errors may be of irregular character, the effect upon the angles can be of constant or systematic character. Therefore, even if the individual angles are measured an infinite number of times it cannot be expected that the sum of the angles will become exactly 180° or 200^g. A *discrepancy e* in this condition is to be expected according to the formula:

$$e = \bar{\alpha} + \bar{\beta} + \bar{\gamma} - 180°$$

2. THE ACCURACY

Disregarding possible influence of spherical excess, the discrepancy e can be regarded as an indication of the *accuracy* of the measured angles because it denotes the closeness of the computation $(\bar{\alpha}+\bar{\beta}+\bar{\gamma})$ to the corresponding exact or true value 180°.

For determining a formal expression for the accuracy according to usual procedures, the discrepancy is distributed to the measured data i.e. the angles $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$. Assuming the geometrical quality of the angles to be mutually equal—that is they are regarded as having the same weights—the discrepancy is divided into three equal parts $e/3$ and these are interpreted as the error of each angle. The correction[3] to each angle is consequently $-e/3=v$. The sum of the squares of the corrections is

$$\sum v^2 = \frac{e^2}{3}$$

There is only one redundant angle or one degree of freedom in this adjustment, and the standard error of unit weight becomes consequently according to usual procedures

$$s_0 = \sqrt{\frac{\sum v^2}{3-2}} = \frac{e}{\sqrt{3}}$$

This is the expression for the basic *accuracy* of each of the three angles, obtained after an adjustment of the discrepancy in the condition, in fact, according to the method of least squares.[4] More sources of errors are included in this determination of the geometrical quality (the accuracy) of the angles than from repeated measurements only, which gives the precision of one measurement and of the average.

The precision expressed in terms of standard deviation of the average may be regarded as the limiting value of the accuracy determined as standard error of unit weight from the adjustment of discrepancies in suitable conditions, which should be as reliable or "tight" as possible. In photogrammetry there are many similar relations, sometimes more complicated than the simple example discussed. The replicated measurements of parallaxes can give good information about the *precision* of individual measurements and of the average of replicated measurements, but the *accuracy* of the measured parallaxes must be determined from conditions, as for instance those of the relative orientation that all pairs of rays shall intersect, or from elevation differences, determined by geodetic measurements. For determining the geometrical quality of a measuring operation or for the study of the error propagation through a measuring procedure—for instance photogrammetry—the most reliable information is obtained as accuracy, expressed as standard errors in terms of functions of standard errors

[3] The corrections to measured data are denoted v according to established practice. v is the first letter in the German word Verbesserung. Note the difference between v' and v.

[4] Because there is only one redundant quantity the determination of the accuracy is weak.

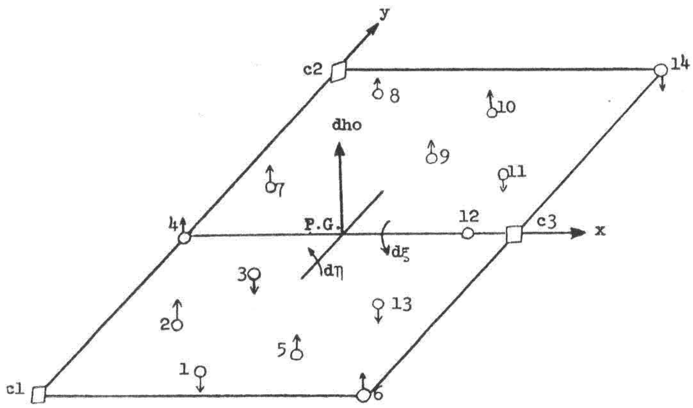

FIG. 2. Determination of the basic accuracy of elevation measurements from least squares adjustment of indirect observations.

Least square adjustment with one parameter dh_0 regarded as a *constant error.*

$$v_1 + e_1 = dh_0;\ v_1 = dh_0 - e_1$$
$$\cdots\cdots\cdots\cdots$$
$$v_n + e_n = dh_0;\ v_n = dh_0 - e_n$$

Normal equation:

$$ndh_0 = \sum e$$
$$dh_0 = \frac{\sum e}{n}$$
$$\sum v^2 = \sum e^2 - \frac{(\sum e)^2}{n}$$
$$s_0 = \sqrt{\frac{\sum v^2}{n-1}}$$
$$s_{s_0} = \frac{s_0}{\sqrt{2(n-1)}}$$

s_0 = Standard error of unit weight from one-parameter adjustment.

Least squares adjustment with three parameters dh_0, $d\eta$, and $d\xi$.

$$v_1(+e_1) = dh_0 + X_1 d\eta + Y_1 d\xi - e_1$$
$$\cdots\cdots\cdots\cdots$$
$$v_n(+e_n) = dh_0 + X_n d\eta + Y_n d\xi - e_n$$

Normal equations:

$$ndh_0 = \sum e$$
$$d\eta \sum X^2 + d\xi \sum XY = \sum Xe$$
$$d\eta \sum XY + d\xi \sum Y^2 = \sum Ye$$
$$\sum v^2 = \sum e^2 - dh_0 \sum e - d\eta \sum Xe - d\xi \sum Ye$$
$$s_0 = \sqrt{\frac{\sum v^2}{n-3}}$$
$$s_{s_0} = \frac{s_0}{\sqrt{2(n-3)}}$$

s_0 = Standard error of unit weight from 3-parameter adjustment.

of unit weight of the basic observations.[5] For such purposes the method of least squares is of great importance, in particular because a well defined, unique procedure is given which usually leads to very simple computations, see Figure 2. It is a special case of the famous Maximum likelihood method from statistics.

It must, however, be emphasized that the method of least squares is no magic procedure which can convert poor observations into results of high geometrical quality. But for investigating the geometrical quality of the basic observations and in particular for the distinguishing between regular and irregular errors of such observations—primarily in connection with calibration procedures—the method is the most powerful tool that we have and we should use it a great deal more.

In applying the method of least squares to such problems the need for a clear terminology becomes evident. In particular it is desirable that the terms *accuracy* and *precision be used in qualitative sense only, and not in connection with quantitative information about the geometrical quality.*

Theory of errors and adjustment procedures become more and more important in all measuring sciences with the increased demand for higher geometrical quality; also in photogrammetry. It is therefore necessary that the basic concepts and terms be well defined and clearly expressed. Otherwise the entire development may become a giant on feet of clay.

[5] In order to increase the reliability of the determination of the standard error of unit weight, the number of redundant measurements should be high.