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*Image Aberration**

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INTRODUCTION

ABERRATION in a photogrammetric system is the displacement of an image equal to the distance the system travels in the time it takes the image forming light to pass from the lens to the light sensitive surface, or the displacement of an object equal to the distance the object travels in the time it takes the image forming light to pass from the object to the lens. Aberration is the sum of the displacements if both the object and photogrammetric system are in motion. The total displacement, therefore, varies with the image distance, object distance, image velocity vector, and object velocity vector.

The angular deviation S subtended by the total image displacement arising from the sum of the object and photogrammetric system motion, on the other hand, depends only on the ratio of the velocities times the sine of the angle enclosed between the direction of motion and the direction of the image-object line. This is expressed with the equation

$$S = \frac{v}{V} \sin \theta$$

where

v = the velocity of the system or object
V = the velocity of light.

Assuming the direction of motion is perpendicular to the direction of the object and the velocity of the system is one mile per second,

$$s'' = 1.''0$$

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when the direction of the object is collinear with the direction of motion

$$s'' = 0\rlap{.}''0$$

In general, the angular deviation of an image is one second of arc with each mile-per-second of velocity. With the advent of artificial satellites, velocities of 5 to 7 miles per second are reached yielding a possible angular deviation of 5 to 7 seconds. Star places are corrected for the orbital velocity of the earth which reaches a maximum value of 20″. Photogrammetrists, like astronomers, must be prepared to correct images for the velocity of a satellite whether the satellite is photographed from the ground or the ground is photographed from a satellite.

Aberration differs from image-motion inasmuch as image-motion depends only on the time the shutter is open and the resultant velocity of the object and camera. Aberration is independent of shutter speed. Image-motion creates an image smear. Image aberration involves no loss in image resolution.

General equations are developed for the correction of image coordinates for any velocity and any orientation of the camera that requires only approximate ephemeris data.

Data Reduction Equations for Orbital Velocity Aberration

A. Definition

The apparent direction of an object with respect to a camera coordinate system is defined by the film coordinates of the image referred to the principal-point and the rear nodal-point of the camera, if the camera has no motion with respect to the object, or conversely. Such is the case when an earthbound camera photographs a fixed earthbound object. If either the object or camera are moving in a direction other than toward or away from each other, the apparent direction of the object defined by the image coordinates is deviated an angle S from the true direction as a consequence of the relative velocity of light and the object or camera. The resultant velocity of the camera and object may be treated as the velocity of the camera with respect to a fixed object.

The deviation of a direction arising from the relative velocity of light and the image collecting system or object is called aberration. Aberration S of a photographic image is illustrated in Figures 1 and 2. Figure 1 illustrates camera-motion and Figure 2, object-motion. Consider camera-motion, first.

In the time it takes light energy to travel from L to i the camera has moved from L to L', therefore,

$$Li' = V(T_1 - T_0)$$

$$LL' = i'i'' = v_c(T_1 - T_0)$$

where

V = velocity of light
v = velocity of camera
$(T_1 - T_0)$ = time lapse

The velocity of the camera is deduced from ephemeris data

$$v_c = \sqrt{\frac{GM}{a(1 - e^2)}[1 + e^2 + 2e\cos(\mu - \omega)]}$$

$$\sin\mu = \frac{\sin\delta_L}{\sin i}$$

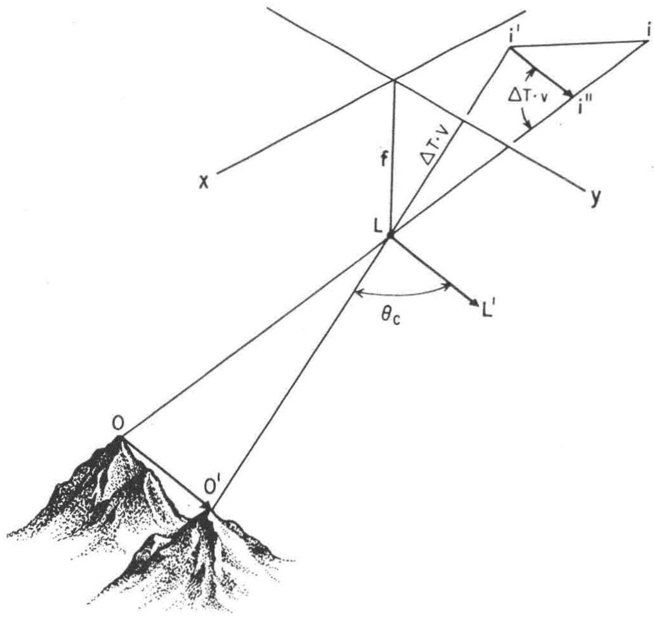


FIG. 1. Camera Motion.

δ_L = the declination of the camera station.
From the previous equalities

$$i'i'' = Li' \frac{v_c}{V}$$

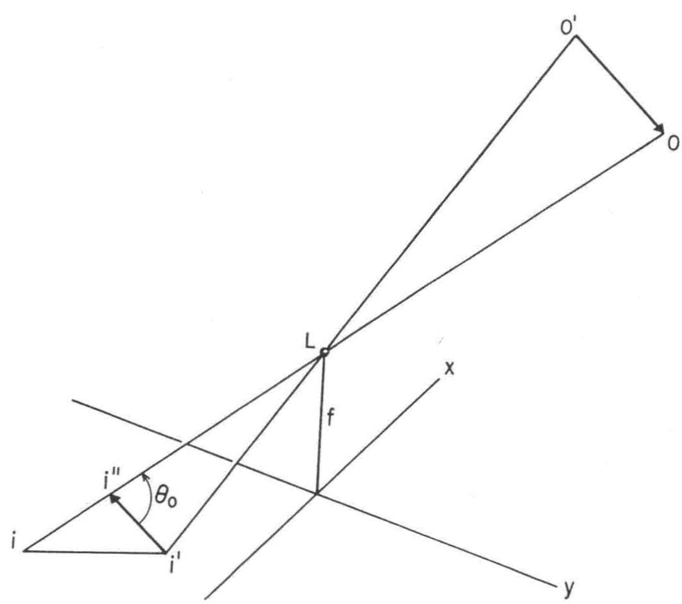


FIG. 2. Object Motion.

By the law of sines

$$S_c = \frac{i'i''}{Li'} \sin \theta_c = \frac{v_c}{V} \sin \theta_c$$

where θ is the angle between the direction of motion and the direction of the image-object line.

Now consider object motion illustrated in Figure 2.

In the time it takes light energy to travel from O' to L the object O' has moved from O' to O. Whence,

$$LO' = V(T_1 - T_0)$$

$$O'O = v_0(T_1 - T_0)$$

But

$$\frac{i'i''}{Li} = \frac{O'O}{LO'}$$

Therefore

$$i'i'' = Li' \frac{v_0}{V}$$

for object-motion as well, and similarly

$$S_0 = \frac{v_0}{V} \sin \theta_0$$

Thus object-motion may be treated exactly like camera-motion.

In both instances $i'i''$ is the spatial image-motion referred to the actual photographic image. The corrections for aberration projected to the film plane $i'i_c$ and $i'i_0$ are required.

The aberration of images measured on ordinary aerial negatives may be completely neglected owing to the low velocity of conventional photographic aircraft relative to the velocity of light. The velocity of a 90 minute satellite is nominally 5 miles-per-second.

Putting

$$\theta = 90°$$

$$S'' = \frac{5}{186{,}000 \sin 1''} = 5'' \text{ approximately}$$

whether the satellite is photographed from the ground or the ground is photographed from a satellite.

B. Aberration Correction Equations for Camera and Object Velocity

The aberration correction applied to image-coordinates is dependent on the orientation of the exposure camera. Assume the camera, either in orbit or on the ground, is star oriented. The orientation of the exposure camera with respect to the celestial sphere is defined:

	x	y	f
X	α_x	α_y	α_z
Y	β_x	β_y	β_z
Z	γ_x	γ_y	γ_z

These values are obtained by some proven data-reduction procedure. The orientation of the orbital axes with respect to the celestial sphere is similarly defined:

	X'	Y'	Z'
X	$\alpha_X{}'$	$\alpha_Y{}'$	$\alpha_Z{}'$
Y	$\beta_X{}'$	$\beta_Y{}'$	$\beta_Z{}'$
Z	$\gamma_X{}'$	$\gamma_Y{}'$	$\gamma_Z{}'$

Let the direction of motion be designated Z'' which is a line in the plane of the orbit tangential to the orbital path at the satellite station. The direction angles of Z'' are expressed as follows:

$$\left.\begin{aligned} \cos \alpha_Z{}'' &= \cos RA_Z{}'' \cos \delta_Z{}'' \\ \cos \beta_Z{}'' &= \sin RA_Z{}'' \cos \delta_Z{}'' \\ \cos \gamma_Z{}'' &= \sin \delta_Z{}'' \end{aligned}\right\} F = (e, a, i, \Omega, \omega, \delta)$$

or

$$\begin{aligned} \cos \alpha_Z{}'' &= \cos \Omega \sin (\phi - \omega) - \sin \Omega \cos (\phi - \omega) \cos i \\ \cos \beta_Z{}'' &= \sin \Omega \sin (\phi - \omega) + \cos \Omega \cos (\phi - \omega) \cos i \\ \cos \gamma_Z{}'' &= \cos (\phi - \omega) \sin i \end{aligned}$$

where

$$\tan \phi = \frac{\sin \nu}{(\cos \nu + e)}$$

$$\nu = (\mu - \omega)$$

The three coordinate systems defined are illustrated in Figure 3. Since

$$\begin{aligned} \cos \alpha_X{}' &= \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ \cos \beta_X{}' &= \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\ \cos \gamma_X{}' &= \sin \omega \sin i \\ \cos \alpha_Y{}' &= - \cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i \\ \cos \beta_Y{}' &= - \sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i \\ \cos \gamma_Y{}' &= \cos \omega \sin i \\ \cos \alpha_Z{}' &= - \sin \Omega \sin i \\ \cos \beta_Z{}' &= \cos \Omega \sin i \\ \cos \gamma_Z{}' &= \cos i \end{aligned}$$

The direction-cosines of the motion vector may be obtained by rotation of the direction-cosines of the orbital X' and Y' axes through the slope angle ϕ at the position of the satellite:

$$\begin{aligned} \cos \alpha_Z{}'' &= \sin \phi \cos \alpha_X{}' + \cos \phi \cos \alpha_Y{}' \\ \cos \beta_Z{}'' &= \sin \phi \cos \beta_X{}' + \cos \phi \cos \beta_Y{}' \\ \cos \gamma_Z{}'' &= \sin \phi \cos \gamma_X{}' + \cos \phi \cos \gamma_Y{}' \end{aligned}$$

ϕ, like v (velocity), may be determined with approximate orbital ephemeris data.

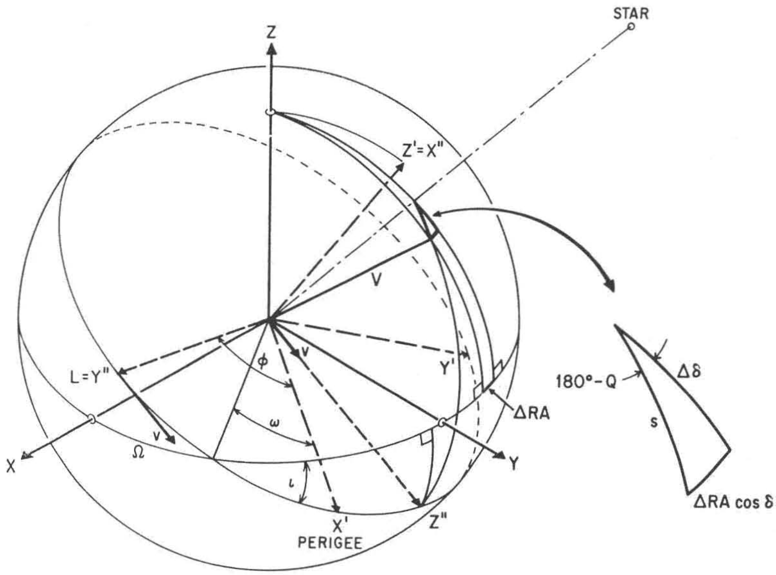


FIG. 3. Relation of X'', Y'', Z'' to X', Y', Z' and X, Y, Z

The direction-cosines of the direction of motion Z'' combined with the direction-cosines of the camera axes give the direction-cosines of the velocity vector with respect to the camera axes:

$$\cos\alpha_x \cos\alpha_{Z''} + \cos\beta_x \cos\beta_{Z''} + \cos\gamma_x \cos\gamma_{Z''} = \cos Z_x''$$

$$\cos\alpha_y \cos\alpha_{Z''} + \cos\beta_y \cos\beta_{Z''} + \cos\gamma_y \cos\gamma_{Z''} = \cos Z_y''$$

$$\cos\alpha_z \cos\alpha_{Z''} + \cos\beta_z \cos\beta_{Z''} + \cos\gamma_z \cos\gamma_{Z''} = \cos Z_z''$$

Assume now the camera coordinates are transformed to be collinear with the direction of motion:

$$x \cos X_x'' + y \cos X_y'' + f \cos X_z'' = x''$$

$$x \cos Y_x'' + y \cos Y_y'' + f \cos Y_z'' = y''$$

$$x \cos Z_x'' + y \cos Z_y'' + f \cos Z_z'' = z''$$

The motion perpendicular to Z'' is zero whence we obtain by taking the first order differentials of the variables:

$$dx \cos X_x'' + dy \cos X_y'' + df \cos X_z'' = 0 \tag{1}$$

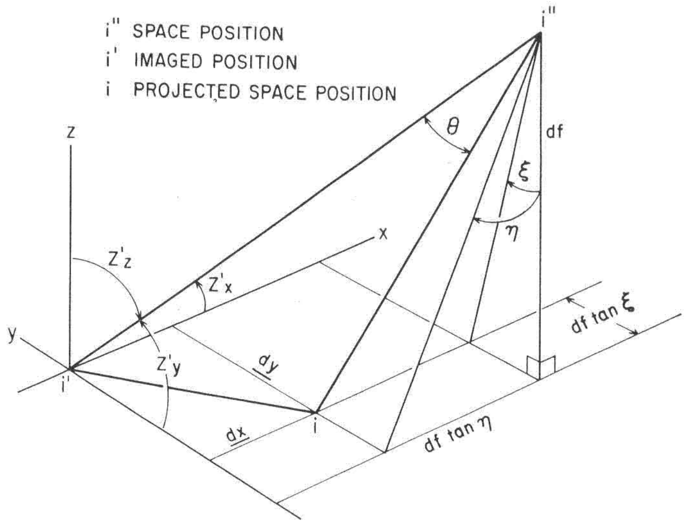
$$dx \cos Y_x'' + dy \cos Y_y'' + df \cos Y_z'' = 0 \tag{2}$$

$$dx \cos Z_x'' + dy \cos Z_z'' + df \cos Z_z'' = dZ''. \tag{3}$$

Now

$$dz'' = i'i'' = L\frac{v}{V}$$

Therefore, performing the indicated multiplication produces the following



$$\underline{dx} = \frac{v}{V} L \left(\cos Z'_x - \frac{x}{f} \cos Z'_z\right) \qquad \underline{dy} = \frac{v}{V} L \left(\cos Z'_y - \frac{y}{f} \cos Z'_z\right)$$

FIG. 4. Geometry of Spatial Image Aberration.

expressions for dx, dy, and df:

$$dx = (1) \cos X_x'' + (2) \cos Y_x'' + (3) \cos Z_x'' = L \cdot \frac{v}{V} \cos Z_x''$$

$$dy = (1) \cos X_y'' + (2) \cos Y_y'' + (3) \cos Z_y'' = L \cdot \frac{v}{V} \cos Z_y''$$

$$df = (1) \cos X_z'' + (2) \cos Y_z'' + (3) \cos Z_z'' = L \cdot \frac{v}{V} \cos Z_z''$$

These are spatial corrections to image i' for aberration. An equivalent film plane correction defined by the position of i with zero aberration is desired:

$$dx = dx - \frac{x}{f} \cdot df$$

$$dy = dy - \frac{y}{f} \cdot df$$

which reduces to

$$dx = L \cdot \frac{v}{V} \left(\cos Z_x'' - \frac{x}{f} \cos Z_z''\right) \tag{4}$$

$$dy = L \cdot \frac{v}{V} \left(\cos Z_y'' - \frac{y}{f} \cos Z_z''\right) \tag{5}$$

These are the general equations for correcting the image-coordinates of a satellite image photographed from the ground or a ground-point image photographed from a satellite for any orientation of the camera axes. The corrections are illustrated in Figure 4.

Any number of motions may be expressed by a summation. Suppose for instance both camera and object are moving, whence

$$dx = \frac{L}{V}\left[v_c\left(\cos Z_{x_c}'' - \frac{x}{f}\cos Z_{z_c}''\right) + v_0\left(\cos Z_{x_0}'' - \frac{x}{f}\cos Z_{z_0}''\right)\right]$$

$$dy = \frac{L}{V}\left[v_c\left(\cos Z_{y_c}'' - \frac{y}{f}\cos Z_{z_c}''\right) + v_0\left(\cos Z_{y_0}'' - \frac{y}{f}\cos Z_{z_0}''\right)\right]$$

where *c* and *o* denote camera and object velocities. The latter more complex situation may occur with satellite photography of large rapidly rotating planets or when exposures from one orbit are made of a satellite in a different orbit.

The illustrations and slides which are an important part of this presentation were made by Andy Schnapp of the Autometric Corporation.

New Significance of Errors of Inner Orientation

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ABSTRACT: *The errors of inner orientation have assumed a new significance due to recent and promising possibilities offered by the improved methods of providing accurate auxiliary data for determining outer orientation, and by the development of an analytical plotter which is able to accept orientation data in numerical form. The accuracy of inner orientation can be greatly improved by using projected fiducial marks of the right shape and size. An experimental fiducial mark projector has been built and tested, and the results indicate that pointing to such fiducial marks can be made with a standard deviation of ±1 micron. The ensuing reduction in the influence of the errors of inner orientation on the elements of outer orientation improves not only the possibility of studying their absolute accuracy, but also of making use of auxiliary data, and of benefitting from the many advantages offered by the analytical plotter to advance the automation of photogrammetric mapping procedures.*

IT HAS been said that photogrammetry is primarily the science of orientations. Certainly, inner, relative and absolute orientations do occupy a very prominent position indeed in photogrammetric practice and theory. The inner orientation is the most fundamental of the three since it is an essential element when defining accurately the geometric properties of bundles of projecting rays used in subsequent relative and absolute orientations. Errors of inner orientation are equally fundamental, and their effects are always present in the outcome of subsequent operations, and thus affect all accuracy studies as well as all results.

The effects of errors in inner orientation on the final outcome of photogrammetric evaluation have been carefully analyzed.[1] The results of studies of this problem show that:

—*if* approximately vertical photographs are considered,
—*if* the terrain is relatively flat, and
—*if* the outer orientation of the photographs is obtained by performing relative and absolute orientations,

then the effect of even considerable errors of inner orientation are negligible. The results further indicate that this outcome is due to the fact that the errors caused by erroneous inner orientation are to a great extent compensated by relative and absolute orientations. This happens as a matter of course—undoubtedly a very fortunate circumstance.

From the development of photogrammetry during the past few years new possibilities have emerged which have interesting and