Any number of motions may be expressed by a summation. Suppose for instance both camera and object are moving, whence

$$dx = \frac{L}{V}\left[v_c\left(\cos Z_{x_c}'' - \frac{x}{f}\cos Z_{z_c}''\right) + v_0\left(\cos Z_{x_0}'' - \frac{x}{f}\cos Z_{z_0}''\right)\right]$$

$$dy = \frac{L}{V}\left[v_c\left(\cos Z_{y_c}'' - \frac{y}{f}\cos Z_{z_c}''\right) + v_0\left(\cos Z_{y_0}'' - \frac{y}{f}\cos Z_{z_0}''\right)\right]$$

where *c* and *o* denote camera and object velocities. The latter more complex situation may occur with satellite photography of large rapidly rotating planets or when exposures from one orbit are made of a satellite in a different orbit.

The illustrations and slides which are an important part of this presentation were made by Andy Schnapp of the Autometric Corporation.

New Significance of Errors of Inner Orientation

U. V. HELAVA,
Division of Applied Physics
National Research Council
Ottawa, Can.

ABSTRACT: *The errors of inner orientation have assumed a new significance due to recent and promising possibilities offered by the improved methods of providing accurate auxiliary data for determining outer orientation, and by the development of an analytical plotter which is able to accept orientation data in numerical form. The accuracy of inner orientation can be greatly improved by using projected fiducial marks of the right shape and size. An experimental fiducial mark projector has been built and tested, and the results indicate that pointing to such fiducial marks can be made with a standard deviation of ±1 micron. The ensuing reduction in the influence of the errors of inner orientation on the elements of outer orientation improves not only the possibility of studying their absolute accuracy, but also of making use of auxiliary data, and of benefitting from the many advantages offered by the analytical plotter to advance the automation of photogrammetric mapping procedures.*

IT HAS been said that photogrammetry is primarily the science of orientations. Certainly, inner, relative and absolute orientations do occupy a very prominent position indeed in photogrammetric practice and theory. The inner orientation is the most fundamental of the three since it is an essential element when defining accurately the geometric properties of bundles of projecting rays used in subsequent relative and absolute orientations. Errors of inner orientation are equally fundamental, and their effects are always present in the outcome of subsequent operations, and thus affect all accuracy studies as well as all results.

The effects of errors in inner orientation on the final outcome of photogrammetric evaluation have been carefully analyzed.[1] The results of studies of this problem show that:

- *if* approximately vertical photographs are considered,
- *if* the terrain is relatively flat, and
- *if* the outer orientation of the photographs is obtained by performing relative and absolute orientations,

then the effect of even considerable errors of inner orientation are negligible. The results further indicate that this outcome is due to the fact that the errors caused by erroneous inner orientation are to a great extent compensated by relative and absolute orientations. This happens as a matter of course—undoubtedly a very fortunate circumstance.

From the development of photogrammetry during the past few years new possibilities have emerged which have interesting and

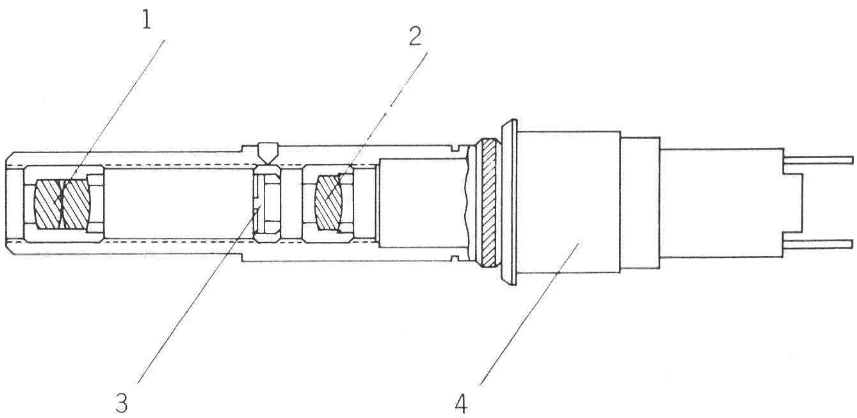

FIG. 1. Diagram showing the most important parts of experimental fiducial mark projector: (1) projecting lens, (2) collimator lens, (3) fiducial mark, (4) lamp housing.

promising applications, but for which the favorable conditions mentioned in the previous paragraph do not apply. One such possibility is that in the analytical plotter the elements of outer orientation can be introduced, both accurately and fast, in numerical form. This offers great advantages for automation in photogrammetric mapping procedure,[2] and in the use of auxiliary data. Both automation and the use of precise auxiliary data, such as that obtained from the orbital information of a camera carrying satellite, are topical aspects of photogrammetry, and likely to be more so in the future. However, in both cases the accuracy to which the inner orientation of the photogrammetric camera is known, or can be re-established in a plotting instrument, is of crucial importance when results of the highest accuracy are required.

A study of the consequences of errors of inner orientation of a photogrammetric camera, and of the errors committed when this inner orientation is re-established for plotting purposes, leads to the following conclusions:

1) The greatest immediate benefits will result from an improved determination and identification of the principal-point.
2) The successful use of auxiliary data for the direct determination of outer orientation requires that the position of the principal-point be accurately determined and that it can be accurately re-established.
3) The automatic introduction of orientation elements requires that the position of the principal-point can be accurately re-established, but no special requirements for the initial determination of that position are necessary.

These conclusions focus our attention on the fiducial-marks of the photogrammetric camera and on the methods used to re-establish the correct position of the photogrammetric image for the evaluation process. Centering the photograph on the plate carrier of a stereoplotter constitutes the most commonly used method of re-establishing the correct position of the image. The centering is accomplished by a trial-and-error procedure;

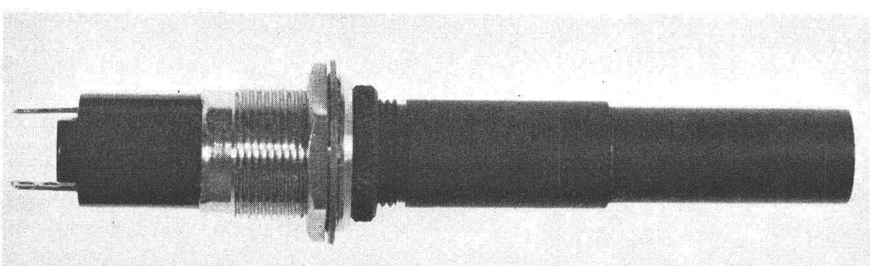

FIG. 2. Photograph of experimental fiducial mark projector.

TABLE I

Average of nine observers	Design n:o							
	1	2	3	4	5	6	7	8
m_x microns	3.4	2.0	2.2	2.2	1.3	1.0	2.1	1.7
m_y microns	3.0	2.1	2.8	2.5	1.5	1.2	2.2	2.2

consequently, its accuracy is rather low. Analytical methods, including the analytical plotter, utilize a mathematical centering based on the measurement of the coordinates of the fiducial marks. This represents a considerable improvement over the trial and error procedure, in terms of both the accuracy obtained and the reliability of the results. The method of least squares may be employed when the mathematical centering method is used. This makes possible utilizing redundant observations and to express the results in well defined terms. However, in practice, the accuracy obtainable is limited by the ability of the operator to perform pointing at the fiducial marks. This conclusion directs our attention with increased sharpness to the fiducial marks of the photogrammetric camera.

The fiducial marks now commonly used in photogrammetric cameras are unsatisfactory for high precision pointing. This is so for mainly two reasons: the fiducial marks are too crude and are so constructed that the measuring mark is partly or fully obstructed at the most important moment of measurement. Consequently, it is not possible to perform pointing to a fiducial mark with an accuracy that is comparable to the very high measuring accuracy of modern photogrammetric instruments, particularly those designed for analytical methods. The use of optical projection of fiducial marks to the image-plane has been studied at the National Research Council as a promising possibility to improve this situation.

Figure 1 shows the construction of an experimental fiducial mark projector that was used to produce images of fiducial marks of various shapes for testing purposes. The figure is self explanatory. Also see Figure 2. The shapes used in the experiment are shown in Figure 3. The scale of the projected image was such that the diameter of test figure n:o 6 (small circular dot) was 50 microns.

Pointing on the test fiducial marks was performed by nine different operators on a Jena 1818 Stereocomparator. Only monocular observations were made using the parallax movements of the instrument, and the results were recorded by an independent observer so as to avoid unconscious bias. Ten series were made, each series consisting of one pointing at each individual design. The results are collected in Table I.

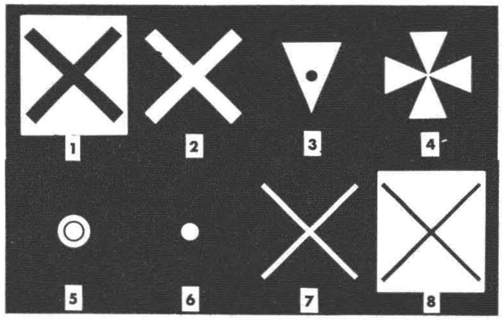

FIG. 3. Fiducial mark designs used in the experiment.

All the results are excellent. Even in the worst case, an improvement was obtained which was approximately three times that normally achieved when using conventional fiducial marks. In the best case (n:o 6) the improvement was about tenfold. Even though all the results are excellent, those obtained when using a circular fiducial mark are by far the best in the group.

An ideal situation for precise pointing when using a black circular measuring mark calls for a fiducial mark consisting of a circular translucent dot on a dark background, the diameter of the dot being slightly larger than that of the measuring mark. Design number six fulfils these conditions completely, while in design number five, concentric rings have been added to provide for measuring marks of different diameters. The additional rings seem to impair the results, although the inner dot is identical with that of design six. Therefore, it appears advisable to strive towards standard size fiducial and measuring marks.

The use of optically projected fiducial marks of suitable design provides a means of improving the accuracy of inner orientation so as to bring it in line with the accuracy of modern photogrammetric instruments and methods. The accuracy to which the principal point can be located is thus improved accordingly, further improvement being possible by increasing the number of fiducial marks. However, a large number of fiducial marks is practical only when the pointing can be accomplished automatically. The ensuing reduction in the influence of the errors of inner orientation on the elements of outer orientation improves the possibility of studying their absolute accuracy, of making use of auxiliary data and of benefiting from all the advantages offered by analytical methods and the analytical plotter to advance the automation and streamlining of photogrammetric mapping procedures.

ACKNOWLEDGMENT

The author wishes to acknowledge the contributions of Mr. G. Lempereur, who designed the experimental fiducial mark projector and targets and helped in obtaining the results summarized in Table I.

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1. Brandenberger, A., "Fehlertheorie der Inneren Orientierung von Steilaufnahmen," Zurich, 1948.
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PHOTOGRAMMETRY AS A SCIENCE AND AS A TOOL (AN INDEX)

COMPILED BY ABRAHAM ANSON

AN EXPLANATION BY THE COMPILER

Each succeeding issue of PHOTOGRAMMETRIC ENGINEERING from its first publication in 1934 evidences that scientists and engineers in many fields of physics, medicine, law, forestry, geology, cartography and engineering have discovered and demonstrated new uses for photogrammetry.

This compilation contains a selection of papers published in PHOTOGRAMMETRIC ENGINEERING which deal with specific problems, beginning with the first volume in 1934 and continuing to 1962. The papers are grouped according to an arbitrary classification for field of interest. In an attempt to avoid duplication, a paper listed in one field could be used to apply to others. Roman numerals indicate the volume; where two numbers are given, the first is the volume and the second is the issue; the author's name is in italics; the number following the volume number is the beginning page. Where one number is included in the bracket, the volume has been successively paginated. Additional references to other publications, where available, have also been listed.

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