*The Supragraph—A New Maximum Precision Plotter with Mechanical Analog Computers**†

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TO GIVE an idea of the very interesting solution embodied in this new instrument, the Supragraph, I will begin with a few general considerations.

The characteristical features of the precision plotters known and in use for decades are the following:

1) The (optical or mechanical) projection.
2) The reproduction of the relatively identical situation of the bundles of rays (as regards angles and distances) which they had at the instant of exposure.
3) The convenient observation of the model (free from y-parallaxes) made possible by 1) and 2).
4) The determination of model coordinates with the aid of measuring spindles.

However, the oldest photogrammetric precision instrument is not the photogrammetric plotter but the stereocomparator whose resurrection is being witnessed in this age of electronic computers. Contrary to precision plotters, it is distinguished by the following characteristics:

1) there is no reproduction of a model by projection
2) the observation of the entire model is not free from disturbances (freedom from parallaxes is possible only point-by-point)
3) the coordinates determined are image coordinates instead of model coordinates.

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The Supragraph is an instrument which combines both the aforementioned types of equipment. With the aid of a simple switch, the very precise plotting instrument which supplies model coordinates with an extremely high accuracy, can be converted into a stereocomparator permitting measurement of image coordinates.

As will be easily realized, there is a mathematical relationship between image-coordinates and model-coordinates. The photos placed in a comparator contain the same image points as the photos positioned and oriented in a plotting instrument. Consequently, if it were possible in a stereocomparator-type plotting instrument to superimpose on both photos such an additional motion in x and y-direction that the image-point under observation appears free from parallaxes in the same position in which an identical point would appear in a projector-type instrument, then the most important characteristics would be applicable also to this instrument.

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To be able to incorporate the aforementioned possibility in a design, one must first investigate the mathematical relationships between model and image. Before going into these details, there will be enumerated the following conditions and prerequisites which in part, will simplify these mathematical relationships:

1) The photograms placed in the instrument can be centered, rotated and displaced in accordance with the base components. But they will always remain in a horizontal plane for comparator observation.
2) On the image side, the coordinate zero-point coincides with the principal-point, and on the model-side with the nadir. The direction of principal axes on the image side is the intersecting line between the xz plane of the instrument and the image plane.
3) The measurement of coordinate values is made in the model space (i.e. in the terrain). In other words: When the instrument is used as a plotter, the values set on the handwheels and the pedal disk are already model coordinates. This conditions a motion of the individual photos which, although it is dependent on the handwheels and the pedal disk, is varied according to a mathematical law.

Description of the Supragraph as a Plotting Instrument

Figure 1 shows that when the ground point P is set with the aid of the handwheels and the pedal disk, the respective amount of displacement in the horizontal image

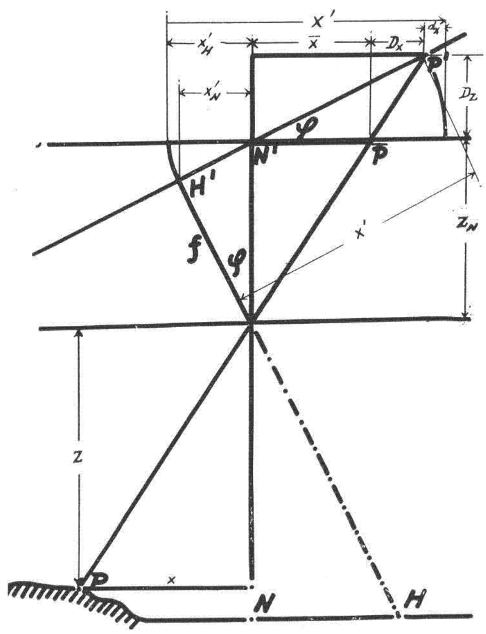

FIG. 1. Tilt of the image-plane around the nadir-point

plane will be x'; this corresponds to the distance $H'-P'$. The mathematical relationship between these two magnitudes is dependent on the elements of interior and relative orientation of each image as well as on the coordinates and the altitude of the model-point. The following formula results for the x-component:

$$x' = \bar{x} + D_x + d_x + x_H' \tag{1}$$

After the respective transformation and with the unknown magnitudes expressed by the known values of the elements of relative and interior orientation, this formula will adopt the following form:

$$x' = \underbrace{\frac{x}{z}\left[f\cdot\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}+\frac{(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)\cdot f\cdot\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}}{z-(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)}\right]}_{\bar{x}+Dx}$$

$$+\underbrace{\frac{(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)\cdot f\cdot\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}\cdot\mathrm{tg}\,\varphi}{[z-(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)]\cdot[\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}+1}}_{dx}+\underbrace{f\cdot\mathrm{tg}\,\varphi}_{x_H'} \tag{2}$$

After another transformation it will look as follows:

$$x' = \frac{f}{z+\dfrac{[z-(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)]\cdot[1-\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}}{\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}}-x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega} \tag{3}$$

$$\cdot\left[x+\mathrm{tg}\,\varphi\cdot\frac{2z+\dfrac{[z-(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)]\cdot[1-\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}}{\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}}-(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)}{\sqrt{1+\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}}\right]$$

This formula already makes provision for the fact that the distance x' does not coincide with the principal-axis, but that it is divided into its components in accordance with the tip-and-tilt actually existing. Taking a closer look at the formula, one will notice that identical terms are represented several times. Consequently these terms need not be determined in the instrument several times, but only once. For better clearness, the following abbreviations are introduced for these repeated expressions:

$$\begin{aligned} m &= x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega \\ a &= \sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1} \\ k &= \frac{[z-(x\cdot\mathrm{tg}\,\varphi+y\cdot\mathrm{tg}\,\omega)]\cdot[1-\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}]}{\sqrt{\mathrm{tg}^2\varphi+\mathrm{tg}^2\omega+1}} \end{aligned} \tag{4}$$

The formula is thus given the following form:

$$x' = \frac{f}{z+k-m}\cdot x+\mathrm{tg}\,\varphi\cdot\frac{2z+k-m}{1+a} \tag{5}$$

The respective equation for y' reads:

$$y' = \frac{f}{z+k-m}\cdot y+\mathrm{tg}\,\omega\cdot\frac{2z+k-m}{1+a} \tag{6}$$

The aforementioned equations can be solved by numerical computation which, however, requires at first an analog-to-digital conversion and—once the result of the computation is available—an additional digital-to-analog conversion. In the Supragraph, these equations are solved with sufficient accuracy with the aid of mechanical analog computers. The data of relative orientation, the components of the equivalent focal-length of the cameras and the model coordinates are introduced in the analog computers as known magnitudes. In accordance with the type of equations, the various analog computers of the Supragraph are divided into

one M computer
one K computer
one A computer and
one Δ computer

each for the x and y-component of each instrument side. These computers are the so-called "auxiliary computers."

Each photocarrier is part of a principal computer which converts the results received from the auxiliary computers into the respective x' and y'-motions, taking into consideration the calibrated focal-length and the flying-height.

For a better understanding of the diagrammatic sketches one should point out that in the Supragraph the majority of the settings and the partial results of the various auxiliary computers are transformed into revolutions of spindles or shafts. In this case there is a possibility of having some auxiliary computations carried out at a considerably enlarged scale. The required amounts of displacement are frequently retransformed into revolutions of spindles with the aid of enlarging angle levers and micro-switches. Precision differential gears are used for summing up or subtracting certain partial results. A new design of gimbal connecting shafts applied for the first time in this instrument makes possible connecting the various computers free from backlash. The computers are thus entirely independent with regard to position, because they are only connected by rotating shafts. The diagrammatic sketches show the various computers strongly generalized and simplified.

The Basic Design of the Supragraph

Figure 2 shows a front view of the instrument, giving an idea of its compact design. This is a satisfactory protection against dust. The base cabinets on each side

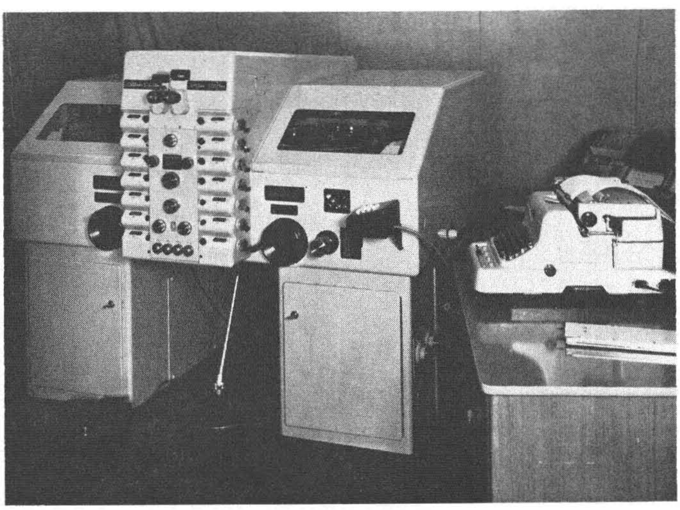

FIG. 2. Front view of the Supragraph

house the auxiliary computers; above them there is a drive assembly not visible in the photograph. The upper part of the instrument contains the two photo-carriers with the principal computers. In the front center one can distinguish the two eyepieces, the setting knobs and the counters for the orientation elements as well as the handwheels. Above the eyepieces there are the seven-digit coordinate counters whose last read-out unit is one micron, and a numbering counter. All setting knobs for the orientation elements are protected against accidental disadjustment by magnetic locks. The swing, bx and f-settings are provided with motor drives permitting rapid changes of adjustment. Also the x, y and z-motions can be driven by motors. The resulting warming-up of the precision measuring spindles is counteracted by built-in cooling fans which are switched on automatically together with the respective drive motors. Behind the door of the lower left part of the instrument, there is an electrical switchboard from which the operator may control all important functions of the instrument. From this switchboard the instrument can be switched from plotter to stereocomparator and vice versa. In addition, the sense of rotation of the x, y and z-direction can be inverted or even locked. Furthermore, the switchboard serves for controlling the sense of rotation of the counters. And finally, the switchboard is used for setting the elevation counter from meters to feet, as well as for controlling certain functions of a coordinatograph which might be connected to the instrument.

The solution of the aforementioned equations, will be explained with the aid of diagrammatic sketches which show how the various terms of the formula are computed.

1) *M* COMPUTER (Figure 3)

According to the above mentioned formula, the letter m stands for $x\cdot tg\varphi + y\cdot tg\omega$. By operating the knurled knobs for φ (1) and ω (2), also the two cam computers (3) and (4) are set to a certain value.

The two cam disks permit the value $(tg\varphi - \varphi)$ or $(tg\omega - \omega)$ to be sensed, which is immediately retransformed into a rotation. With the aid of the differential gears (5 and 6), the amounts φ and ω are automatically added to these values. Consequently, the values supplied by the differential gears are the following:

$$tg\,\varphi - \varphi + \varphi = tg\,\varphi$$

$$tg\,\omega - \omega + \omega = tg\,\omega$$

Both these values are transmitted to one auxiliary computer. Each of these in the sketch has the shape of a two-armed lever whose pivot is stationary. Each right-hand arm of the lever is lowered or lifted from its horizontal zero position by the amount $tg\varphi$ or $tg\omega$ at the distance 1 from the point of rotation. The starting point of the left arm of the lever is separated from the point of rotation by the horizontal distance x, i.e. the x-distance from the nadir of the point under observation. As will be easily realized, the amount by which the platform (7) will be lifted will be $x\ tg\varphi$ or $y\ tg\omega$. As a result, the point of rotation (8) of the lever will change its position by the amount

$$\frac{x\cdot tg\,\varphi + y\cdot tg\,\omega}{2}.$$

By an appropriate selection of threads, the amount of rotation can be brought to $x\cdot tg\varphi + y\cdot tg\omega$.

The amount m has thus been computed. As will be noted from the diagram, m changes continuously at the time the handwheels are operated, while the modifications caused by the setting of the orientation values are of a unique nature, since $tg\varphi$ and $tg\omega$ will not be changed any more during the plotting operation.

2) THE *A* COMPUTER (Figure 4)

This computer is a three dimensional cam computer (9). This permits the entire square root value $\sqrt{tg^2\varphi + tg^2\omega + 1}$ to be obtained 1,000 times enlarged. The cam body which is shown in the diagram in a rather abstract form, is driven by φ and ω. φ produces a rotation and ω a displacement of the cam body.

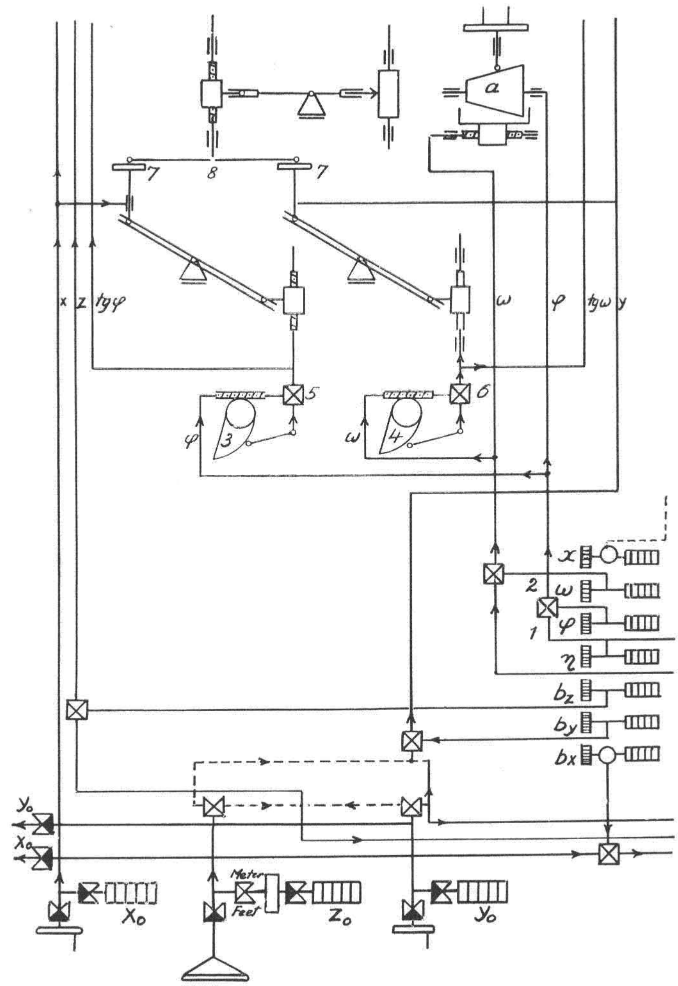

FIG. 3. *M*-Computer and drives.

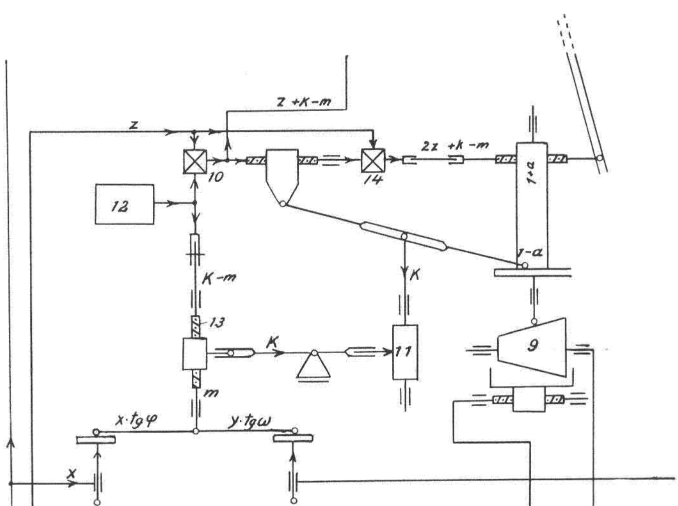

FIG. 4. *A*-Computer and *K*-Computer.

Contrary to the schematic diagram, the take-off is accomplished with the aid of a two-armed lever which forms directly the values

$$1 + a \quad \text{and}$$
$$1 - a.$$

The value $1+a$ is used both in the principal x' and y' formula, while the value $1-a$ is required for forming the K-value, as shall be seen later.

3) K COMPUTER (Figure 4)

Before studying the K computer, one must consider an additional factor. The following value for K has ben quoted:

$$K = \frac{[z - (x \cdot \text{tg}\,\varphi + y \cdot \text{tg}\,\omega)] \cdot [1 - \sqrt{\text{tg}^2\,\varphi + \text{tg}^2\,\omega + 1}]}{\sqrt{\text{tg}^2\,\varphi + \text{tg}^2\,\omega + 1}};$$

consequently, K can also be expressed in the following form:

$$K = \frac{(z - m) \cdot (1 - a)}{a}$$

By means of a transformation, the following relation can be derived, which is more convenient for the computing operation:

$$\frac{1 - a}{1} = \frac{K}{z + K - m}$$

To the K computer leads, first of all, that shaft which carries the m formed in the M computer, which in the differential gear (10) is negatively added to the z-value coming direct from the pedal disk. Thus, the value z-m is formed. The K computer will simultaneously form the K-value, displacing the micro-switch (11) by the value K. This connects the motor (12) which makes the spindle (13) rotate until the micro-switch has been disconnected again. It will be easily noted that the spindle carrying the m-value from the M computer, does in reality contain already the value K-m. The value $z+K-m$ is, in the differential gear No. 14, again added to the value of z, so that the value $2z+K-m$ is produced, which is transmitted to the Δ-computer.

The *Δ-computer* (Figure 5) controlled through a microswitch (15) is a multiplier to which the value $1+a$ is transmitted from the A computer, the value $2z+K-m$ from the K computer and the values tgφ for the δx-value and tgω for the δy-value from the cam computers.

The final result of the Δ-computer is transmitted to the principal computer. The *principal computer* (Figure 6) receives the following results from the various auxiliary computers or setting mechanisms:

1) x from the x-handwheel
2) $\delta x = tg\varphi[2z+k-m]/1+a$ from the Δ-computer
3) $fx-\Delta fx$ from the f and Δfx setting mechanism
4) $fy-\Delta fy$ from the f and Δfy setting mechanism
5) $z+\Delta z = z+k-m$ from the K computer.

The principal computer will now perform a process which—from a mathematical point of view—might still be considered as some kind of projection. Since this mathematical process of projection is divided into the two components x' and y', the instrument offers the advantage that for the two components f-values (c.f.l.) can be introduced which are differentially different. In other words, errors of affinity in the

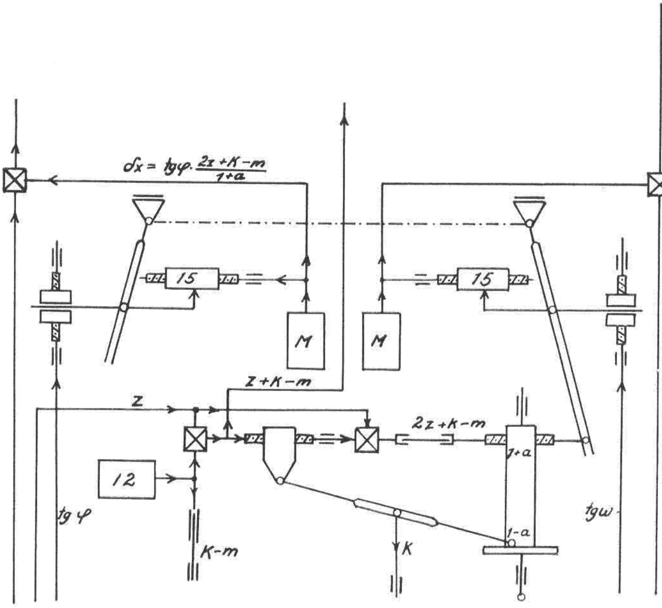

FIG. 5. Δ-Computer.

photogram, which may be produced by irregular film shrinkage, can be compensated for by an appropriate choice of the plotting focal lengths fx and fy.

In addition, maximum stability is guaranteed by locating the swivel arms in the xy-plane, because the arms are under stress in one plane only, and because they have a statically favorable cross-section which makes deformations practically impossible.

The ball bearings used are magnetically held on the arms, so that there are no

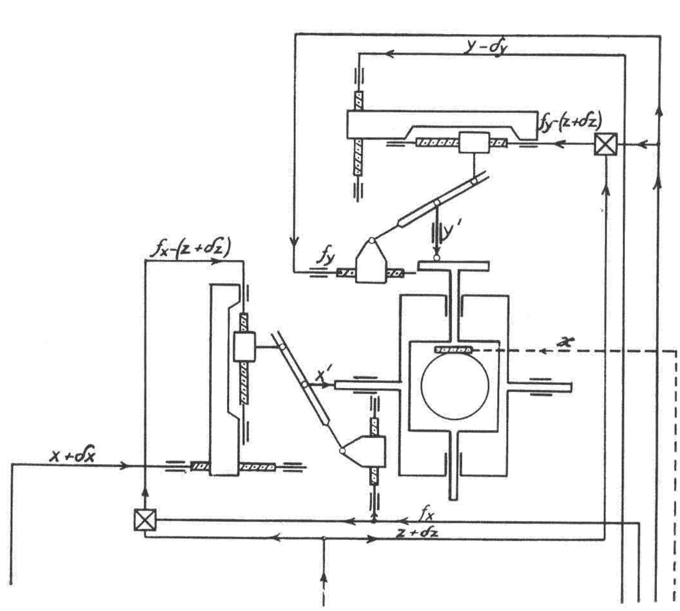

FIG. 6. Main Computer.

additional influences due to springs of some sort or other. In order to eliminate the possibility of damaging the arms while inserting the photograms which are centered on the correction plates outside the instrument, the ball bearings are withdrawn from the swivel arms when the upper part of the instrument is opened. This is achieved with the aid of small built-in electric motors which also take care of restoring the ball bearings to their original position when the instrument is closed.

The functioning of the principal computer will be easily understood with the aid of the diagram.

From the handwheels come the values x_0 and y_0 which—as was mentioned above—are model coordinates. These values are added to the δx and δy-values formed in the Δ-computers. Consequently, the value

$$x + \operatorname{tg} \varphi \cdot \frac{2z + k - m}{1 + a}$$

is formed, which—according to the aforementioned formula—must still be multiplied with the value

$$\frac{f}{z + k - m}.$$

The x' or y'-value obtained by this computing operation is the actual amount of displacement of the photo. It should be pointed out in this connection that an increase of the z-value does not influence the visual stereoscopic image which will keep the same scale; only the x' and y' motions will decrease with increasing z-value. However, the scale of the visual stereoscopic model can be varied by means of interchangeable eyepieces ranging from 6× to 12× enlargement.

As a plotting instrument, the Supragraph will behave like any other precision plotter known to us. All setting motions for the elements of relative and absolute orientation produce the same apparent motions as in other instruments based on the principle of projection. A refinement of orientation methods is, however, indispensable, because residual parallaxes of quite a few microns will already be noted and because an attempt to eliminate them according to conventional trial and error methods would necessarily fail.

The gimbal errors inherent in many projector-type instruments were avoided in the Supragraph. A relatively simple optical system and the ingenious way of projecting the measuring mark—variable in color and in shape—into the path of rays have led to an extremely high resolving power. The smallest measuring mark has a diameter of some 20 microns. With the aid of aspherical correction plates, any type of presently known photography can be plotted in the instrument, provided that the calibrated focal-lengths fall within the range from 48 mm. to 215 mm. or that the photos have been transformed to a value within this range.

When the instrument is used as a *stereocomparator*, all functions of the auxiliary computers are disconnected with the aid of a switch. This means that no variable δx or δy-value is added to the x_0 and y_0-motions coming from the handwheels. The function of the principal computers is thus left untouched. Still, it is recommended to set the value $f = z$ in the principal computer, because the figures appearing in the counters and the Ecomat unit will then correspond to the actual dimensions within the photogram.

The coordinate values determined are image coordinates instead of model coordinates. For more convenient operation, the drives of the comparator are somewhat different from those of the plotting instrument. The Δx-motion, for instance, is controlled by the pedal disk, while the left handwheel takes care of the common x-motion of both photos. In a similar manner, the y-motions are divided between y-handwheel and by.

Coordinates are recorded by a magnetic counter incorporated in the Supragraph. This magnetic counter permits a reading of 1 micron in the last digit. When the instrument is used as a plotter, the unit of the last digit may, of course, differ from the round value of 1 micron. A card punch reading out the results in the form of punched cards can be controlled through an electrical typewriter.

Test Results

Nine-point measurements, stereoscopic-model measurements and an aerotriangulation with the aid of precision grid plates were made when the instrument was turned over to the customer.

The nine-point measurements were transformed to the nominal figure of the measuring grid by means of a Helmert transformation with all nine points used. The differences encountered in this operation were used in order to compute the mean point errors according to the following formula:

$$m_p = \sqrt{\frac{[v_x v_x] + [v_y v_y]}{n - 2}}.$$

$f = z = 48$ mm. $m_p = \pm 3.1$ microns

$f = z = 88$ mm. $m_p = \pm 3.4$ microns

$f = z = 150$ mm. $m_p = \pm 2.5$ microns

$f = z = 210$ mm. $m_p = \pm 2.6$ microns

$f = 88$ mm. $z = 132$ mm. $m_p = \pm 3.2$ microns

$f = 150$ mm. $z = 225$ mm. $m_p = \pm 2.75$ microns

$f = 150$ mm. $z = 75$ mm. $m_p = \pm 2.4$ microns.

The stereoscopic measurements were carried out with the aid of 45 grid points of intersection. The deviations from an ideal plane were used for computing the mean heighting error:

	Base in	Base out
$f = z = 88$ mm.	± 2.2 microns	± 3.3 microns
$f = z = 150$ mm.	± 3.5 microns	± 3.2 microns
$f = z = 210$ mm.	± 4.18 microns	± 6.9 microns

Another test consisted in a grid triangulation. After six models, the longitudinal deviation amounted to only 35 microns. This can be easily explained by the differential height gaps of the transfer points.

The coming weeks will bring comparable results for comparator measurements carried out in the Supragraph.

It will have been noted from these outlines that the Supragraph is an instrument which combines a large number of outstanding features. These features are summarized as follows:

1) Owing to its basic design as Stereocomparator with auxiliary computers, the Supragraph combines a *Precision Stereocomparator and a maximum-precision first-order Plotter* in one instrument. In order to use the Supragraph as a Stereocomparator, there is only need to disconnect the auxiliary computers.
2) The design principle of the guide rods working only in one plane, in connection with the subdivision into x and y-components, makes possible achieving *accuracies* which were heretofore impossible.

   The focal-length setting according to x and y-component makes possible eliminating affine film shrinkage and thus obtaining an additional, methodical increase in accuracy.

3) With values from 48 to 210 mm., the *focal-length range* is unequalled. 136° ultra-wide-angle photography can therefore be plotted just as well as long-focus normal-angle photography.
4) The instrument is contained in a completely closed housing, so that it presents maximum *resistance against ambient influences*, including fluctuations of temperature.
5) The Supragraph offers a maximum of *convenience*, because all operations including the setting of focal-lengths, the exchange of measuring marks, etc., as well as all commutations, can be easily made from the operator's seat.

Precision counters make it possible to *exactly set orientation values*, thus facilitating the orientation process.

Limit switches with indicators avoid overrunning at the limits of the working ranges and indicate the final position achieved.

It is believed that the new Zeiss Supragraph will be another important step forward on the way to photogrammetric progress.

*Automatic Map Compilation**

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ABSTRACT: *A discussion of research and development resulting in a successful prototype model of an optical-electronic system that can be added to existing conventional plotters to produce both altitude information and orthophotos automatically from pairs of aerial photographs.*

EARLY in 1960, Ramo Wooldridge was awarded a contract to produce a prototype system for the automatic production of altitude data and orthophotos from a stereomodel as projected on a Kelsh plotter. Some months later RW was awarded a second contract to develop a breadboard model of an advance compilation system intended to be faster, more versatile, and more precise than might be expected by simply adding automatic features to an existing plotting instrument. Both systems are being developed for the Army Geodesy Intelligence Mapping Research and Development Agency. The feasibility of both systems has been demonstrated, and they are now in the final stage of checkout and test. This paper describes the first system in some detail and touches on some of the salient features of the second. It is expected that the second system will be described in more detail in a later paper.

Both systems are the outgrowth of a number of programs carried on over the past ten years. The Automatic Stereomapping System attached to a Kelsh plotter will automatically produce altitude information by following the stereo-image along profile lines while simultaneously printing out an orthophoto. The Map Compilation System produces the same type of information from stereo-pairs without an optical model.

The basic principle of automatic height sensing from stereo-pairs is diagrammed in Figure 1. Two projectors, $P1$ and $P2$, are shown with rays from a high-light point in the imagery converging on the model surface at P. If an aperture A is moved horizontally in a direction parallel to the flight-line (left to right) at a position H above the surface of the model, the ray from $P1$ through A will reach photomultiplier $PM1$ before the corresponding ray from $P2$ through A reaches photo-

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