where

$\Delta U_i = U_i$ (Given)—U_i (Computed)

$\Delta V_i = V_i$ (Given—V_i (Computed)

$\Delta W_i = W_i$ (Given) W_i (Computed)

n = number of points used in the transformation (i.e., the number of points common to the two coordinate systems).

Remarks on the IBM 650 Program:

The above described problem has been programmed for the IBM 650. Due to the limited storage capacity of this electronic computer, no attempt was made to deal with the case of unequal weights of observations. The case of unequal weights will be considered in a future program for analytical aerotriangulation. This, however, will be programmed for the IBM 7090 to be acquired in the immediate future by the University of Illinois.

The solution of the analytical relative orientation generally requires three iterations and can handle up to 21 points. The approximate time required for relative orientation using 9 points is about four minutes per model.

The absolute orientation also requires about three iterations and takes around four minutes for the determination of the transformation constants.

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Ortho-Contour Photography

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The image produced by the usual method of photography is inevitably perspective. Its aspect is so similar to that which we observe visually that it can be said to be the simplest for grasping the concept of the object. In photogrammetry, however, great efforts are expended in order to get an ortho-projective diagram of contours of the object from the perspective images. This is the function of most items of "mapping equipment." It would be quite convenient if the ortho-projective image of the object itself, or even better, the ortho-projective image of its contours could be photographed directly. A method which enables us to accomplish the former process was invented by Cooke, and reported by Prickett and Morris in 1950.[1] This method necessitates the use of a on-spherical lens large enough to cover the object. Although they have ameliorated this defect by the attempt to widen the area to be

[1] R. Prickett and M. Morris: The Orthocamera: Orthogonal Photographic Scanning Camera. PHOTOGRAMMETRIC ENGINEERING, XVI (1950), pp. 823–830.

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photographed through lateral scanning, inconvenience still remains in that we must prepare a sufficiently large scanning frame to anticipate cases in which we must treat large objects.

In the ortho-projective image photographed in this way, the contour of every optical depth is photographed with a uniform rate of contraction (or magnification), but the contours are not explicit because they are photographed in consecutive positions to give the continuous whole image. Conversely, if all the contours of an object are photographed on a plate at a uniform rate of contraction, would they give a projective image of the object? It turns out that they will, provided there are no lateral shifts among the images of contours. The scheme later explained realizes this principle, enabling one to take photograph of contours, discrete or continuous, at a fixed contraction rate on a single photographic plate with no shifts from the optical axis of the camera lens, and consequently with no shifts among them.

The diagram (Figure 1) shows the principle of this method as a ground plan, L and P represent the camera lens and the photographic plate respectively. In this I is an imaginary plane perpendicular to the lens axis, and conjugate with the plate P. From the light source S which is set beside the lens L at a considerable distance, a vertical-plane light beam B is projected on the plane I, making a vertical illuminated line C on the plane. We provide a slit $S1$ just in front of the photographic plate P, in order that C and Sl will also conjugate. In other words, the illuminating plane B and the photographing plane $S1$-C intersect each other solely on the imaginary plane I, which is conjugate with the plate P. Keeping the conjugate relation between C and Sl, we then make the direction of B rotate around S and also move Sl correspondingly in the horizontal direction.

Let us now put an actual object O at the place including I. A point on the surface of O is illuminated just once in the course of such a whole scanning, and if the point is also on the imaginary plane I, making an image on the plate P by the slit Sl is allowable. However, if the point is not on I, it is also not on the photographing plane Sl-C, and its image is blocked from the plate P by the slit diaphragm. Accordingly, after one whole scanning, a curve which is determined by the intersection of O and I; i.e., one contour is recorded on the plate P. We then shift the object O in the direction parallel to the lens axis, keeping the other parts of the equipment fixed. As the imaginary plane I intersects the surface of O at another place now, the second contour curve will be exposed after another scanning on the same photographic plate. Since the shift of the object is parallel to the lens axis and the relation between I–L and L–P is kept constant, it is obvious that the second contour gives a sharp image at the fixed contraction rate and without shift from the lens axis. By repeating this operation, a group of such contours of the object can be obtained.

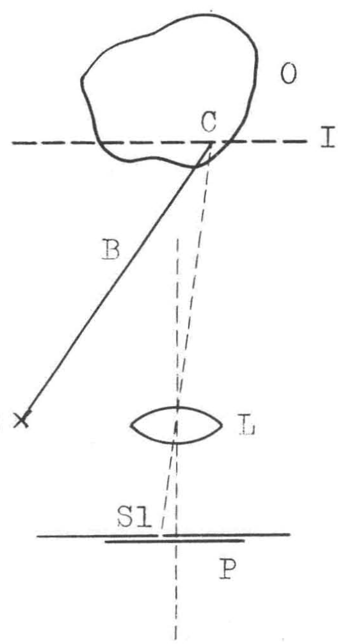

FIG. 1. Scanner schematic.

Following this principle, a working model was actually constructed as shown in Figure 2 and explained below.

A vertical rod R is fixed on to the horizontal base bench B, and the camera lens L is so set at the top of the rod that its optical axis is parallel to the base bench. Also a lateral arm P is fixed to the bench, and near its both ends vertical axes A_1 and A_2 are fixed. Another horizontal bench E is attached perpendicularly to B, and a slide bearing a vertical stud A_0 on the top is free to move along its top edge. Three arms M_0, M_1, and M_2 connect R–A_0, A_0–A_1, and A_0–A_2, respectively. Here, the axes R, A_1, and A_2 allow M_0, M_1, and M_2 to turn around them only, whereas the stud A_0 fits into the slots cut along the central lines of the three arms, and permits them not only to turn around but to slide freely by it. In other words, the central lines of the arms

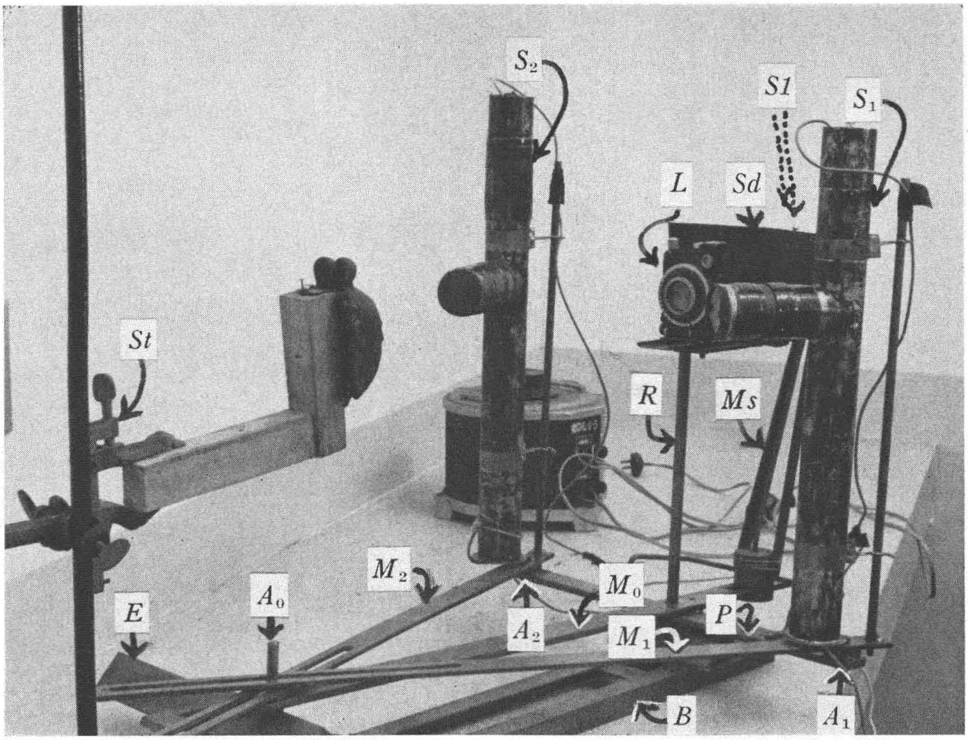

FIG. 2. Working model.

M_0, M_1, and M_2 always aim at the axis of the stud A_0. On the arms M_1 and M_2, near to A_1 and A_2, light sources S_1 and S_2 are attached. As the light sources, we first used tube lamps of single-wire tungsten filament held vertically but because not stable these have been replaced with slits illuminated by fluorescent lamps. The light emitted from each of these light sources is thrown on to the prolongation of the center line of the axis A_0 by a lens having a narrow vertical window, as a practically plane vertical beam, while the other superfluous light emitted from the lamp is completely cut off by a metal cover.

The cassette *C* holding a photographic plate is set perpendicularly to the optical axis behind the lens *L* in the position which is conjugate with the vertical plane through the bench *E*. The silt-diaphragm *Sd* slides just in front of *C*. The slit opening *Sl* in this diaphragm is made to move by the arm *Ms* fixed to the arm M_0 so that it is always exactly above the central line of M_0. In the earlier experiments, we made *Sd* slide between guides attached to the frame which holds the cassette, but the undesirable friction which this produced caused rejection of this attempt. As a substitute we fixed the slit-diaphragm directly to the arm *Ms;* consequently in its present arrangement, the slit *Sl* makes a circular motion. Thus geometry of image formation is not as good at the ends as at the central part of the plate because the slit is separated a little from the plate at the ends. The object to be photographed is set on the stand *St*, which can be slid on the base bench *B;* the scanning is accomplished by smoothly moving the slide bearing the axis A_0 over its full way on the bench *E*. Adopting two illuminating beams is not contrary to the principle in the preceding explanation, provided they intersect each other precisely on the central line of the axis A_0. The use of two illuminating beams prevents the loss of illumination on the sides of the object.

The photograph of a snail shell shown in Figure 4 was taken in this way. Because the equipment constructed was not very precise, the contour curves might be said to be not sharply defined. In this example, the contour planes were spaced every 10 mm. If spaced more closely the whole image of the shell will become continuous, of which every part will give a sharp image at the fixed contraction rate; i.e., a perfect ortho-projective image of the shell will be given.

As an example of a rather simple configuration an ortho-contour photograph of a cylin-

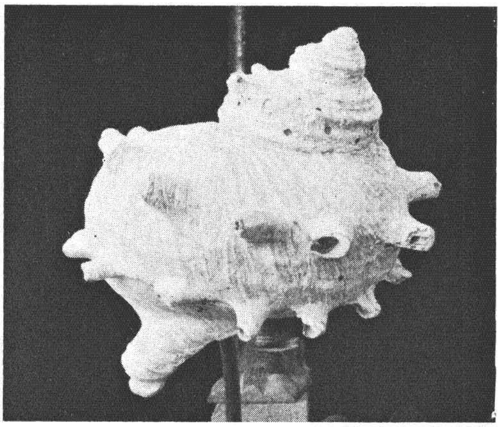

FIG. 3. Usual photograph.

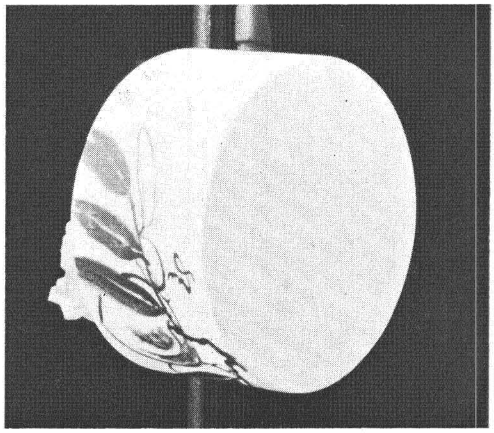

FIG. 5. Usual photograph.

der taken by this equipment is shown in Figure 6. The photograph is not one of the most successfully photographed images. We are attempting to show by this simple example the characteristics of this photography and what must be improved in future experiments. These improvements are as follows:

(a) The scanning speed must be more uniform. In every contour curve we note variations in photographic intensity. Two causes are suspected. The fine periodical variation comes from the imperfect finishing of gears used in connection with a motor to drive the slider. The larger irregular variation seems to have been caused by the imperfect precision of the whole equipment which is almost hand-made.

(b) The contours cannot be said to be very sharp, because the slits were kept rather wide. These curves can be made far finer by reducing the widths of the illuminating beams and also that of the photograph slit, thus keeping the geometrical relations rigorously correct.

(c) With finer contour curves, we can photograph them at closer intervals. In this example, the contour interval on the object was approximately every 10 mm. In this photograph there will be noted eight contour lines on the plane surface of the cylinder; further that the curves defined by the ends of these straight lines do not connect to each other smoothly enough to give an ellipse. However, with narrower contours taken at a closer interval, this discontinuity is expected to be less noticeable.

(d) In this hastily built equipment, the illuminating beams and the photographing slit are connected in their

FIG. 4. Ortho-contour photograph.

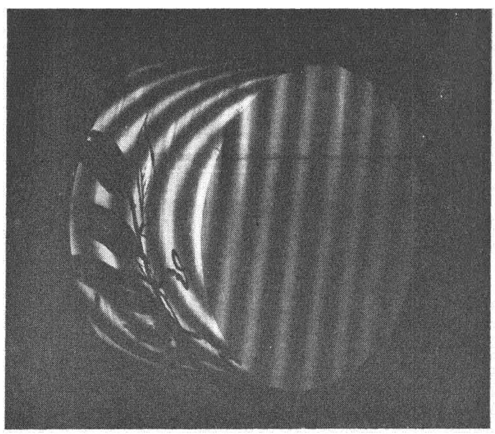

FIG. 6. Ortho-contour photograph.

movement but not in their widths. More rigorous conjugate relation between the widths will give a clearer image.

In contrast to the defects listed above, the following two are intrinsic to the principle of this method, and cannot be avoided by improvements.

a. As made evident by the article (c) an edge of an object, for instance a building which is not parallel to the photographic plate, is not recorded as a sharp line. This is because each contour is photographed as a perspective image in itself, regardless of how finely it may be taken.
b. There are cases of some dead surfaces on the object; in the example shown, the end parts of the contours on the cylindrical surface are missing. Although it is certain that the contours run through the missing area, their parts cannot be seen through the small camera lens, being hidden by other parts of the object. Generally speaking, a possibility exists of missing the largest contour in photographing a round object.

This method of photographing ortho-contours involves no great technical difficulties in constructing the equipment. A small camera lens, provided it is free of distortion, will permit photographing the ortho-image of a large object such as a building. In such a case, the whole illuminating-photographing equipment must be shifted in the direction of the lens axis instead of the object, and the operation may be carried out at night. Furthermore, a universal camera which can be applied to objects of any dimension requires lens of variable focal-length and a correspondingly variable plate distance, as well as suitable regulation of the illuminating unit. The necessary linkage mechanisms may be very complicated, but the basic principle is always the simple one above described.