layouts, property surveys may not require new photography or waiting for photographic weather or season. If changes have been slight, photography which is one, two, or more years old may be used with recent changes added readily to the photogrammetric manuscripts in the field. Similarly, control once established and photo identified may be used for more than one project, even if the ground identifying markers have been destroyed.

What then should we as photogrammetrists do to aid the surveyor to use photogrammetry in his work? Once the land surveyor has been convinced that photogrammetry can be a useful tool for his operations, his first problem is to select the proper equipment. There are many types of equipment which can be used for this job. All of these equipments will operate from essentially the same photography. So, therefore, photography is not a factor in the choice of equipment. The problem of trained operator availability, or the time required to train an operator for the equipment, is of a serious consideration. Beyond this, the surveyor must consider both start-up and operational cost. Start-up costs include the cost of the plotter and any auxiliary equipment required. Some of the equipment from which the surveyor may choose his set up are the Bausch & Lomb Balplex Plotter, the Kelsh Plotter, the Zeiss Stereotope, the Wild A-6 or A-8, the Santoni Stereo-simplex and other similar equipment. We at Bausch & Lomb, of course, believe that the most feasible, the most economical and best possible equipment for this use is the Balplex. However, there may be others of different opinions! The land surveyor would be well advised to review the equipments which are available with competent technical personnel before selecting his final instrument. Particular attention should be paid to continuing operating costs. After the initial cost of the equipment, these continuing costs will determine the efficiency and the economy of utilizing photogrammetry in small land surveys.

In review then, we may say that the small land surveyor is an engineer or surveyor who is working either on his own or with a small engineering firm. He works in many areas which are close to each of us in engineering and we believe, as photogrammetrists, that the science of photogrammetry can help and support him in this work. We, as the Society of Photogrammetry, have an obligation to bring the capabilities of our science to the attention of the small land surveyors and to aid them in the development of techniques and procedures whereby they may accomplish this task. We, at Bausch & Lomb, and other manufacturers, have an obligation as instrument designers to aid these new photogrammetrists in the selection of their equipment, in the training of their personnel, and, where practice dictates, to develop new and specialized equipment for their use. As I stated in the beginning of this talk, at the present, we do not have answers to all of these problems. However, I hope that today we have at least raised some questions and opened the minds of some of our people to a new, open and rewarding field in which photogrammetry may be utilized.

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*Capabilities and Limitations of Spatial Aerotriangulation**†

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(*Abstract is on the next page*)

AEROTRIANGULATION, or bridging as it is sometimes termed, could be defined as the systematic process for providing control over an extended area by measurements on aerial photographs. Like all techniques of measurement, aerotriangulation has its limitations. Its use is governed on one hand by technical difficulties affecting accuracy, and on the other hand by economic considerations. Accuracy at any price is seldom demanded.

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† Paper presented at the 1962 St. Louis ACSM-ASP Convention.

ABSTRACT: *The potentialities and limitations of spatial aerotriangulation in reducing the amount of ground-control necessary for photogrammetric mapping are discussed in this paper. The importance of advance planning in projects involving bridging is pointed out. Definite design criteria are given for aerotriangulation projects, sharply reducing the necessity of relying heavily on luck or extensive experience.*

With few exceptions (e.g. basic research and military projects), the desired accuracy is a function of the requirements of economy and the purpose of the measurement.

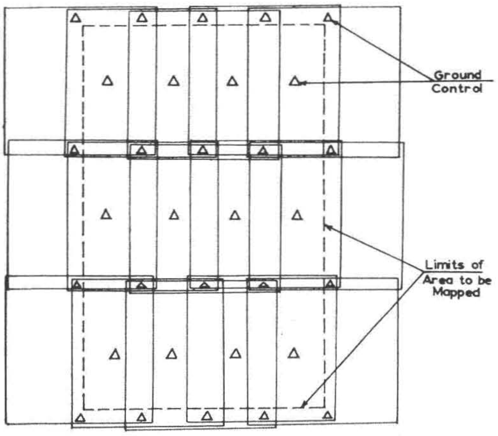

FIG. 1

In photogrammetric mapping, ground-control is of major importance since it is the basis on which photogrammetry provides the geometric net-work necessary for the construction of the map. It is obvious that the accuracy of a map can be no better than that of the ground control used. Ground control is, in general, one of the most expensive phases of photogrammetric mapping. This is the reason why ways and means have been devised to reduce the amount of such control necessary for mapping.

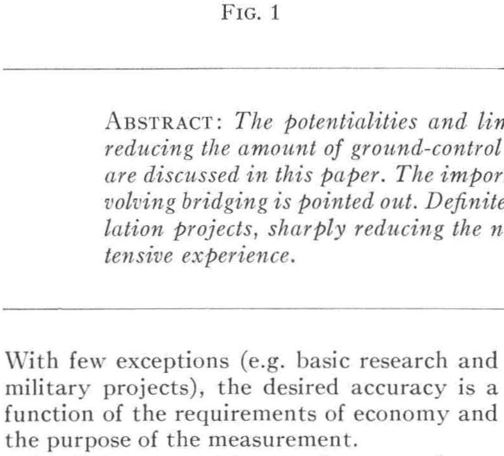

FIG. 2

Figure 1 shows a photographic coverage of an area to be mapped photogrammetrically. Some years ago, one had to insist on having ground control for each and every stereomodel. Five control points (one in each corner of the stereomodel and one in the center) were considered the standard requirement. With some planning, arrangements could be made for some of the ground control points to be of service to more than one model (as shown in Figure 1). Such an arrangement was considered as still too expensive, and the idea of bridging was introduced in practice.

As will be pointed out later in this paper, the required accuracy of the map plays a major role in deciding whether or not bridging could be used in a certain project. In large-scale photogrammetric mapping with very high accuracy requirements, one might need to have every stereomodel terrestrially controlled. For the sake of economy, one has to investigate the possibility of using aerotriangulation techniques before deciding to fully control the area to be mapped. The steps of such an investigation are given in details in this paper.

Through aerotriangulation, the control points necessary to control each model are deduced photogrammetrically. Figure 2 shows one possible pattern of photographic coverage and ground control. In this case, every strip is provided with sets of ground-control points. The distance between any two consecutive sets of ground-control points is governed by the "Maximum Bridging Distance." This will be discussed later in this paper.

Figure 3 schematically shows another popular pattern of flights and of ground

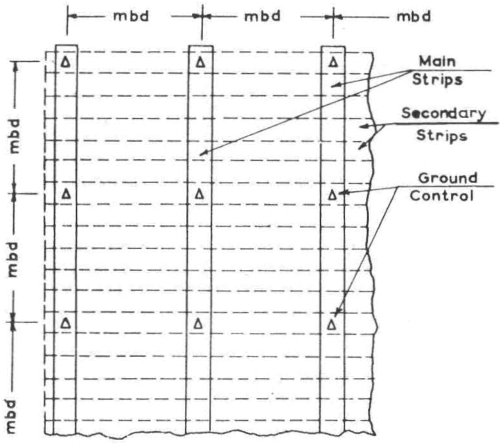

Fig. 3

control. The area to be mapped is flown in two perpendicular directions. One direction establishes the secondary strips which provide the full photographic coverage of the area to be mapped, while the other direction yields the main strips which are used to control the secondary strips. As shown in Figure 4, only the main strips are terrestrially controlled (ground control marked with △ in Figure 4). After being adjusted on the basis of this ground control, the main strips provide what is generally termed "Photogrammetric-Control" (marked with □ in Figure 4) which is used to control and adjust the secondary strips. After being adjusted on the basis of this photogrammetric-control, the secondary strips provide, photogrammetrically, the control necessary for mapping individual stereomodels. Such photogrammetric control is marked with o in Figure 4. Here again the maximum bridging distance has to be respected to guarantee satisfactory residual errors after the adjustment.

The time and effort spent to determine the photogrammetric-control necessary for the secondary strips and stereomodels are far less than in the case of terrestrially determined ground control. In addition, it is important to note that photogrammetric control points can be chosen in ideal positions. Artificial points (carefully pricked in the emulsion) play an important role in this respect.

A comparison of Figures 1, 2 and 3 shows how effective aerotriangulation is in cutting to a minimum the amount of ground control necessary for mapping. The results are savings of much time, effort and money in this expensive phase of photogrammetric mapping.

Aerotriangulation, or bridging, is not a novel idea as it may seem. Back in 1935, Otto von Gruber, one of the most distinguished pioneers of Photogrammetry, published an article in the German Journal *Bildmessung Und Luftbildwesen* entitled "Beitrag zur Theorie und Praxis von Aeropolygonierung und Aeronivellement." This essay on the theory and practice of aerotriangulation is considered *The Classic of Aerotriangulation* as it outlines the principles which are the basis for many of the modern efforts in this field.

Aerotriangulation has had a long and eventful childhood and has survived many infancy diseases. It has been on the laps of scientists for almost three decades, and for several years has been in practical use. It is interesting to note that while some practitioners use aerotriangulation successfully, even in the case of large-scale mapping, others are bitterly opposed to the use of this technique, even in cases involving small-scale mapping. The latter position is generally attributed to unsatisfactory results obtained in a number of trials with bridging.

The author is inclined to believe that the outcome of any aerotriangulation project depends mainly on *advance planning*. In such planning, one has to be familiar with the obtainable quality of the different methods to be able to predict the expected accuracy and to make sure that the tolerance demands will be fulfilled. In other words, one has to thoroughly study the propagation of errors in aerotriangulation, before and after the adjustment. In the absence of such a study, the matter of good or bad results is, of course, just a matter of good or tough *luck*. I am sure you agree with me that *luck* should have no place in photogrammetric work.

As one can expect, the accuracy of photogrammetric-control is somewhat lower than that of terrestrially determined ground-control. To ensure a specified accuracy in photogrammetric-control, the bridged distance has to be confined within certain limits. In case of a long strip, it is not sufficient to have ground-control only at the beginning, and end of the strip as indicated in almost all textbooks of photogrammetry. Referring to Figure 5, some sort of ground-control has to be established at intervals throughout the long strip. The Maximum Bridging Distance (*mbd*) to be allowed in a certain project depends, among other factors, on the standard of accuracy required in photogrammetric-control, the photo-scale, the forward overlap and the quality of photogrammetric measure-

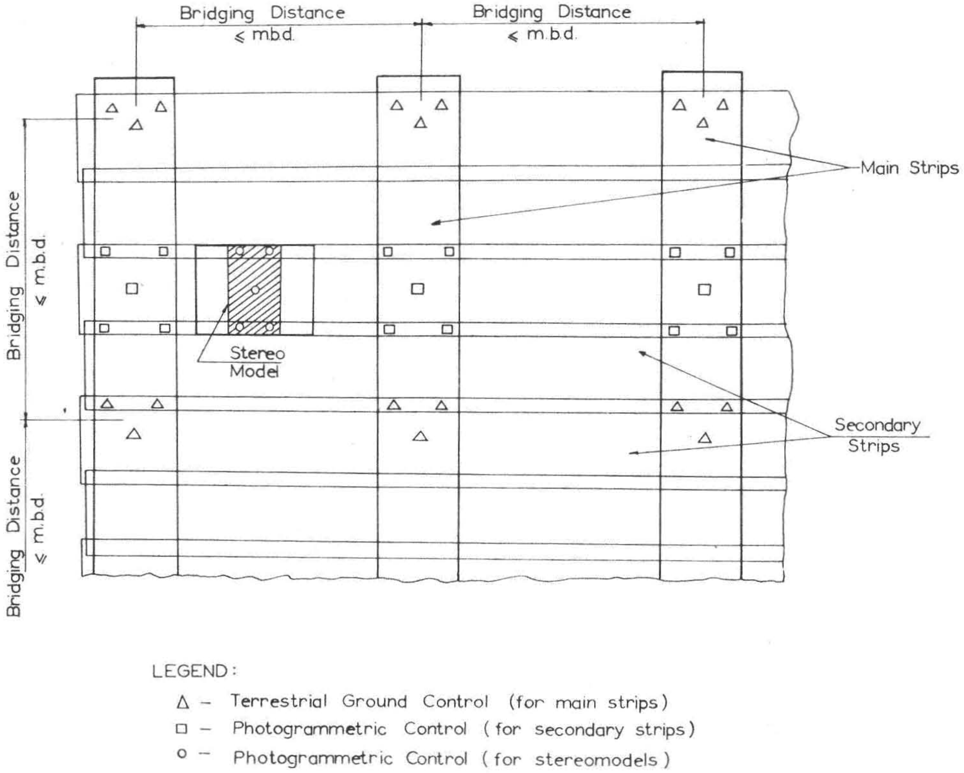

FIG. 4.

ments (a function of the quality of the photography, the stereoplotter or stereocomparator used, and the operator), as well as on the methods used for executing and adjusting aerotriangulations.

The basic formulae for determining the maximum bridging distance were developed at the University of Illinois and reported by the author in 1961 as:[2]

$$\left.\begin{aligned} \text{mbd (feet)} &= 0.43\, B\sqrt{\frac{\mu f S}{\mu_0 Z}} \\ \text{and} \qquad & \\ \text{mbd (meters)} &= 0.047\, B\sqrt{\frac{\mu f S}{\mu_0 Z}} \end{aligned}\right\} \quad (1)$$

where

S is the modulus of the map-scale (map-scale number)

f is the principal-distance of the camera (in inches of millimeters)

B is the average length of aerial base (in feet or meters)

$Z=H$ is the average flight-height above ground (in feet or meters)

μ is the tolerated mean square error in planimetry (in inches or millimeters) measured on the publication scale of the map

μ_0 accuracy (mean square error) of the measurement of parallax in the image-plane (in inches or millimeters).

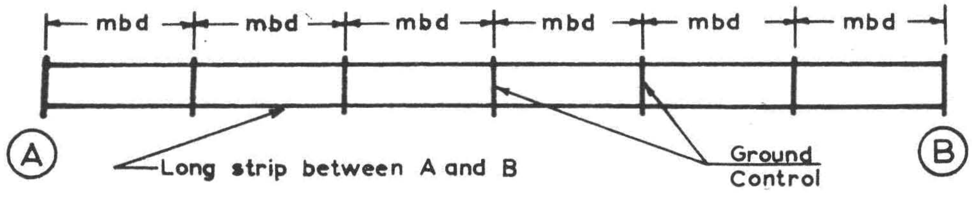

FIG. 5.

The value of μ_0 should be determined experimentally. In modern photogrammetric systems of average precision μ_0 is in the order of 0.01 mm. (0.005 mm in highly precise systems).

The expected accuracy (mean square error) in the derived elevations of aerotriangulated points is expressed by the following formula:[2]

$$\mu_H = 2\mu_0 \frac{Z^2}{Bf}(4.35 - 1.25\,N + 0.375\,N^2 - 0.0625\,N^3 + 0.015625\,N^4)^{1/2} \qquad (2)$$

where:

N is the number of models bridged. In this equation μ_0 and f should be in inches or millimeters, while Z and B should be in feet or meters, to get μ_H in feet or meters.

In Equations 1 and 2, μ and μ_H represent the mean square error in planimetry and elevations due to triangulation and adjustment of a single strip. If block adjustment according to Figures 3 & 4 is used, then the mean square error in planimetry and elevations of spot points would be approximately equal to $\mu\sqrt{2}$ and $\mu_H\sqrt{2}$ respectively (since the triangulation and adjustment are undertaken on the main strips first, then on the secondary strips).

In addition to the errors due to aerotriangulation, one should, of course, take all other sources of error into consideration. Such errors are those due to identification, ground-control, plotting and drafting.

If $\mu_{p_{tr}}$ is the mean square error in the position of a point p due to aerotriangulation and its adjustment (equal to μ in case of single strips, and $\mu\sqrt{2}$ in case of blocks of strips according to the pattern shown in Figures 3 & 4),

$\mu_{H_{tr}}$ is the mean square error in the elevation of a point due to aerotriangulation and its adjustment (equal to μ_H in case of single strips, and $\mu_H\sqrt{2}$ in case of block strips according to the pattern shown in Figures 3 & 4),

μ_{P_T} is the total mean square error in the position of a point p,

μ_{H_T} is the total mean square error in elevation of a point p,

μ_{cds} is the mean square error in the position of a point due to transformation of coordinates,

μ_i is the mean square error due to identification of the point p in the terrain, as well as on the photography during plotting,

μ_{pp} is the mean square error due to the deformation of the bundle of rays and due to the operator,

μ_d is the mean square error due to drafting work,

m_p is the mean square value of the error in planimetry to the combined effect of all sources with the exception of Aerotriangulation and its adjustment ($\mu_{p_{tr}}$),

m_H is the mean square value of the error in elevation due to the effect of all sources with the exception of Aerotriangulation and its adjustment ($\mu_{H_{tr}}$),

and

m_{y_p} is the mean square error in measuring y parallax (in image plane),

then, the following relation could be given:

$$\mu_{p_T}{}^2 = \mu^2{}_{p_{tr}} + \mu^2{}_{cds} + \mu_i{}^2 + \mu^2{}_{pp} + \mu_d{}^2 \qquad (3)$$

or

$$\mu^2{}_{p_T} = \mu^2{}_{p_{tr}} + m_p{}^2 \qquad (4)$$

The values of such factors depend, of course, on the photogrammetric system used. In case of a reasonably reliable operator, and a stereoscopic instrument of precision comparable to that of the Wild A8 Stereoplotter, a photogrammetric system involving a reliable camera with a precision normal-angle lens (such as the Wild Aviotar—$f=165$ mm, picture-format 120/120 mm) and $B:H$ ratio equal to 2:7, the following empirical value could be adopted:

$$m_p = 0.20\,^0/_{00}Z$$

If, instead of the normal-angle lens, the above outlined system involves a wide-angle lens (such as the Wild Aviogon—$f=115$ mm, picture-format 180/180 mm) and $B:H=3:5$, the following empirical value could be adopted:

$$m_p = 0.15\,^0/_{00}Z$$

As far as the accuracy in elevations is concerned, the following parallax formula applies:

$$m_H = \frac{Z^2}{Bf}\cdot m_{y_P} \qquad (5)$$

For all practical purposes, however, the following general figures could be used, with care:

For the previously mentioned system involving normal-angle cameras:

$$m_H = 0.25\,^0/_{00}Z$$

and for the previously mentioned system involving wide-angle cameras:

$$m_H = 0.15{}^0/_{00}Z$$

Following the same line of thinking which led to Equations 3 and 4, one could say that:

$$\mu^2{}_{H_T} = \mu^2{}_{H_{tr}} + m_H{}^2 \tag{6}$$

Equations 1 through 6 are to be carefully considered in the course of designing mapping projects involving bridging, to make sure that the expected accuracy of the method will satisfy the specifications.

Sometimes Equations 4 and 6 will indicate that the specified accuracy cannot be met by aerotriangulation. This is particularly true in cases involving large-scale mapping with very high standards of accuracy. In such cases, each and every model has to be terrestrially controlled (see Figure 1).

In order to give the practitioner an idea about the attainable accuracy in actual work with bridging techniques, two examples will be cited; one for large-scale mapping, and the other for small-scale mapping.

Concerning large-scale mapping, the following project was reported by Kasper and Scholl in 1956:[3]

"Aerotriangulation in Voralberg, Austria, carried out by the Federal Office of Weights and Measures, Vienna, Wild RC7a Plate Camera, Aviogon $f=10$ cm, wide-angle lens, flight-height above ground $h=1200$ meters, Picture-scale 1:12000, longitudinal-overlap 60%, plotting in Wild A7 autograph, length of strip 4 km, 6 stereomodels, containing groups of ground-control points of 4 to 5 points each at both ends of the strips, an analytical compensation by means of IBM punch card computers—according to the method of Roelofs and van der Weele—the comparison of the coordinates of 72 control-points determined by terrestrial means with the transformed and compensated autograph coordinates gave the following mean square errors:

$$m_x = \pm 4.8 \text{ cm } (\pm 1.9 \text{ inches})$$
$$m_y = \pm 6.0 \text{ cm } (\pm 2.4 \text{ inches})."$$

In other words, the mean square error in position would be in the order of 7.7 cm (3.0 inches).

An example for aerotriangulation with small-scale photography was given by Schlund in 1961:[4] Photographic Bloc "Vercors" in France, designed by and photographed for the International Society for Photogrammetry (Commission III), Wild RC7a plate-camera, Aviogon $f=100$ mm wide-angle lens, flight-height above ground averaging 8100 meters, picture-scale 1:81000, longitudinal overlap 70%, lateral overlap about 20%, plotting in Wild A7 Stereoplotter, size of block 50 km/55 km, three main strips and six secondary strips, three sets of ground-control points for each main strip only, adjustment according to Schlund, the comparison of the coordinates of 47 control-points determined by terrestrial means with the transformed and compensated autograph coordinates gave the following mean square errors:

$$m_x = \pm 2.0 \text{ meters}$$
$$m_y = \pm 2.1 \text{ meters}$$

In other words, the mean square error in position m_L was in the order of 2.9 meters. Thr mean square error in elevation was determined by comparing the terrestrially determined elevations of 66 points to the photogrammetrically obtained values and was found to be $m_H = \pm 1.5$ meters.

CONCLUSIONS

Aerotriangulation is a double-edged technique that has to be used with utmost care if decent results are to be expected. It is very effective not only in reducing the amount of ground-control necessary for mapping to a minimum, but also in providing ideal photogrammetric-control in each stereomodel. *Planning* is of utmost importance in projects involving bridging. The density of ground-control should be considered very carefully to make sure that the expected accuracy of the method adopted will satisfy the specifications. There should be no reliance on *luck* in photogrammetric work; thorough planning is necessary to ensure acceptable results.

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