# *Geometric Calibration of Antennae by Photogrammetry**

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## Introduction

The United States has now completed another segment of the Distant Early Warning (DEW) Line as part of a defensive warning system against hostile aircraft approaching the North American continent over the polar route. At the same time it has been planning and constructing a Ballistic Missile Early Warning System (BMEWS) in an attempt to build a defensive warning system which will serve to combat the fast-changing field of hostile offensive weapons.

One of the primary functions of any warning system is the relay of communications from the location at which trouble has been detected, to the center of the systems operation from which retaliatory measures may be initiated. In the recently completed DEW line eastern extension across Greenland to Iceland, large parabolic dish antennae were built to receive and transmit vital communications. This line of communication was tied in directly to the existing line, spanning Canada to the United States, and finally to Colorado Springs, Colorado, the systems center.

Due to the large distances involved, such as the more than 200 miles between the eastern coast of Greenland and the western coast of Iceland, a special radio frequency must be used which makes possible transmitting messages far beyond the horizon limit of conventional high frequency radio. It is extremely important that the antennae used in this communication system be positioned precisely, and that the reflector surface of each antenna be erected so the antennae may receive and transmit at the highest efficiency possible. Losses due to improper positioning are extremely costly and are cumulative with time.

This paper is intended to propose a photogrammetric method for checking the finished surface of an antenna reflector of such a communication system, after construction and erection have been completed.

## Discussion of the Problem

There are several problems involved with the checking of the surface of a reflector. First of all, the structure itself is large. The largest communication antenna on the DEW line eastern extension is a parabolic dish type with a diameter of 120 feet. At the BMEWS station at Thule, Greenland, four surveillance radar antennae have been erected; each of these is 163 feet high and 400 feet long. As can be seen, manual methods of measuring points on the antenna surface, which will be discussed later, become quite tedious and require many man hours of labor. The antenna is also a rather difficult piece of construction to work on, especially for anyone with unsure footing or a fear of height.

A second problem involved is at what time the check of the surface is to be made. Since the final reflector surface sheeting is smooth and unclimbable, if checking various points of the surface requires that a man visit at each point holding a tape or a marker, it is quite obvious that the check must be made before the final surface sheeting is positioned. The alternate place to check then would be at the individual girts on which the surface sheeting is finally placed. The horizontal girt members are connected to each antenna truss by means of angle plates, and are slotted so as to be adjustable. Then if the girt positions are checked and not the final surface, it is quite possible that an irregularity in the final surface could be undetected through movement of the girts before or during positioning of the final surface.

A third problem is the length of time allotted the engineer to check the girt positioning, assuming that this is the method used. Since the girts should not be checked until all steel has been placed and since the contractor cannot lay off his men from the job for a day or two, especially in remote areas, the engineer finds that the contractor is ready to place the surface sheeting before he starts his surface check. Only the most expedient

* In 1961, this paper was submitted in competition for Bausch and Lomb Photogrammetric Award of the American Society of Photogrammetry and was the winner in the Undergraduate Division.
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survey party can keep ahead of the contractor's steelmen.

A fourth problem is that of allowable tolerances. Depending on the interpretation of the erection specifications the tolerance of positioning any one point on the reflector surface is to be between $\pm\frac{1}{8}$ and $\pm\frac{1}{4}$ of an inch. This tolerance is not as severe in the manual methods as it is in the proposed photogrammetric method.

## Present Methods

During the summer of 1960, the author was employed as a member of a geodetic survey party at a DEW line eastern extension location on the eastern coast of Greenland. The methods described in this paper were those that were either used or under consideration for use at that site.

### ANGLE INTERSECTION:

The original method under consideration consisted of checking the angles of intersection of various points on the antenna from two theodolite stations in front of the antenna against computed theoretically correct angles. Angles computed were those to the end and middle of each girt. Allowable tolerances were determined in terms of allowable angular error. The theodolite stations were placed 60 feet in front of the antenna, and 60 feet either side of the feed-horn tower. Due to the very steep vertical angles encountered, angle eyepieces were to be used. However, the method was not used because of the anticipated tediousness of operation along with the late arrival of the angle eyepieces.

### TAPING:

A second method, which was adopted consisted of checking the measured distances of the various points from the focal-center of the antenna, atop the feed-horn tower, against computed distances. Distances were checked only to each end of each girt, theorizing that if each end of the girt were in position, the middle of the girt must also be in position. For taping, a piece of piano wire was used of length slightly less than the minimum distance to be measured and attached at one end to a hook, which fitted over the girt, and at the other end to an ordinary steel tape. The entire system was standardized as a unit and proved to be satisfactory. (Note Figures 1 and 2.)

Each of these methods appear quite lengthy in nature, especially considering the size of antenna reflector surface, and the number of

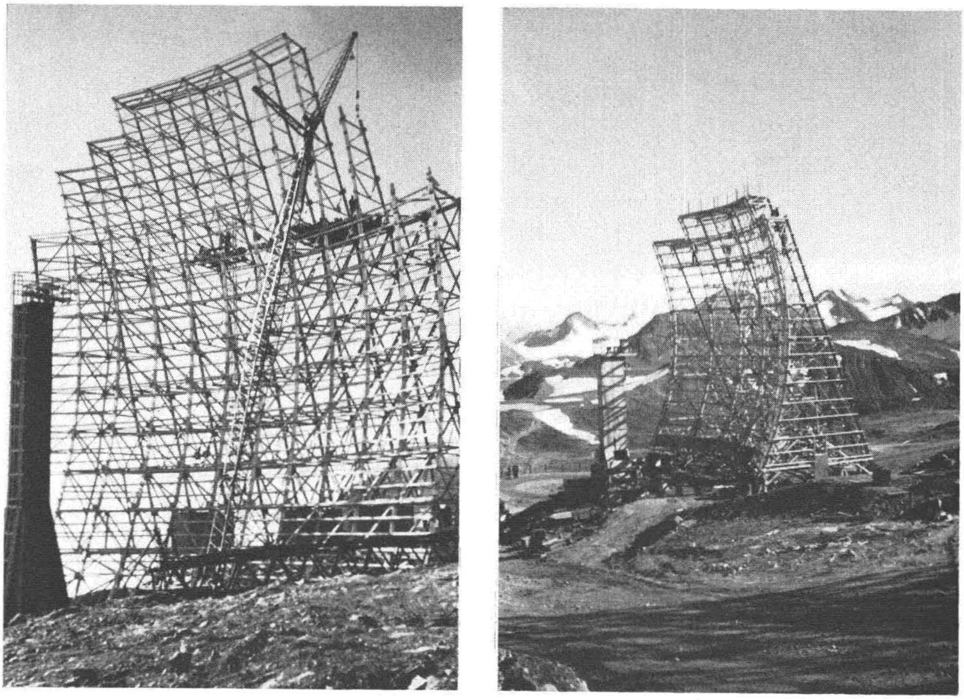

FIG. 1. *Left*—120 foot diameter antenna under construction. Feed-horn tower in the left foreground. *Right*—60 foot diameter antenna. Note the final reflector surface sheeting being positioned at the base of the antenna.

points to be checked. On the 120 foot diameter antenna, there were 881 points to be checked. Furthermore, the methods appear to be less desirable than a method which could check the final surface after the reflector sheeting has been placed.

## Proposed Photogrammetric Method

The author feels that a more accurate and economic solution than that which the ground survey provides in the geometric calibration of an antenna, may be achieved in applying photogrammetric methods. For purposes of demonstration of the proposed method the author will refer to a 120 foot diameter parabolic dish-type antenna. However, it is felt that the method in general will be applicable to any similar type of structure.

### EQUIPMENT:

Due to the small allowable tolerances of construction of $\pm\frac{1}{8}$ to $\pm\frac{1}{4}$ of an inch, it will be necessary that the camera to object distance be rather small. Conventional aerial photography methods therefore cannot be used. Because of the size of the antenna and the range of vertical angle coverage of a terrestrial phototheodolite, conventional terrestrial photography methods appear unusable. It is suggested that a standard aerial camera be used with an adapted lens for short object distances, a six-inch focal-length and nine-inch-by-nine-inch format. The photographs could be taken from a rigid platform supported by a crane boom, at the elevation of the focal-center and at various intervals across the front of the antenna. The crane is suggested because it is readily accessible and extremely economical, compared to the use of a helicopter for example. (Note Figure 3.)

For purposes of compilation and surface determination from the photographs, a first-order universal stereoplotter such as the Wild

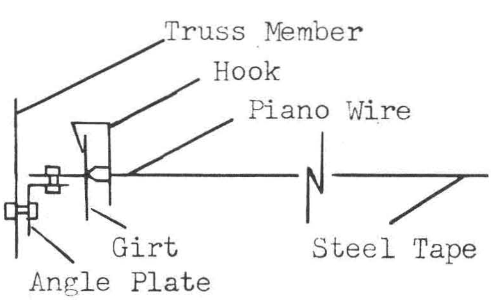


FIG. 2. Taping Mechanism.

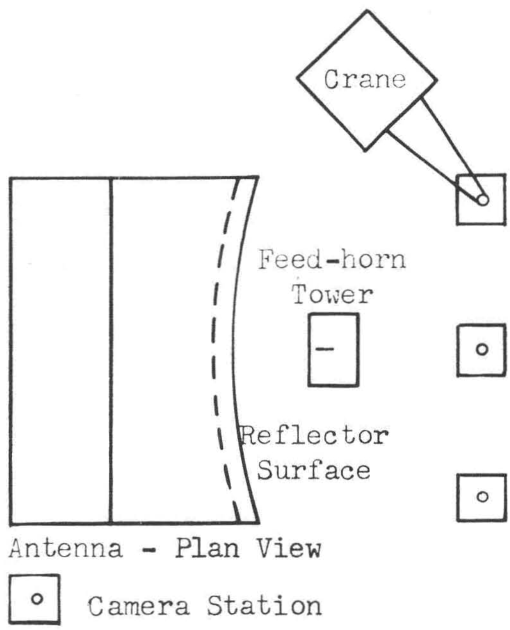


FIG. 3.

Autograph is suggested. The precision of such an instrument will allow compilation of data within the required tolerance.

### CAMERA STATION DESIGN:

The positioning of the camera station will be limited by the type of camera used, the degree of accuracy required, the size of the antenna and the presence of any obstructions which would prevent stereo vision of the reflector surface. Figure 4, a projection on a horizontal plane, gives the geometry.

For an allowable tolerance, $h=\frac{1}{8}$ inch $=3.18$ mm. Assuming that .01 mm. can be measured on a photograph by the Wild Autograph, $aa'=.01$ mm. For a nine-inch format and 60 per-cent overlap, $oa'=(.4)\ (9)=91.4$ mm. For an aerial camera with a focal-length of six inches, $f=152.4$ mm. From geometric considerations: (Note Figure 4.)

$$\frac{oa'}{AA'}=\frac{f}{h} \qquad \frac{aa'}{AA'}=\frac{f}{H}$$

$$= h\frac{oa'}{AA'} \qquad f = H\frac{aa'}{AA'}$$

$$h\frac{oa'}{AA'}=H\frac{aa'}{AA'} \quad \text{or} \quad H=h\frac{oa'}{aa'}$$

$$H=3.18\frac{91.4}{.01}=29{,}065.2 \text{ mm.} = 95.4 \text{ feet}$$

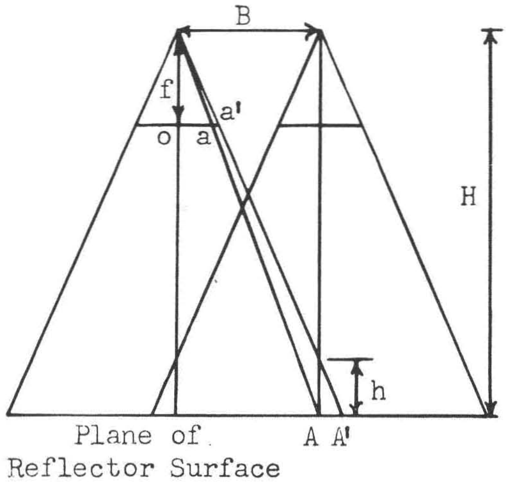


FIG. 4.

From other geometric considerations it can be shown that;

$$H - h = \frac{B}{oa'} f \quad \text{or} \quad B = H - h \frac{oa}{f}$$

$$B = (29065.2 - 3.2)\left(\frac{91.4}{152.4}\right)$$

$$= (29062.0)(.60) = 17,437.2 \text{ mm.}$$

$$= 57.2 \text{ feet}$$

In checking the number of photographs required to make a complete stereo coverage of the reflector surface, assume that a camera with a six-inch focal-length utilizing a nine-inch-by-nine-inch format will cover an angle of 74°. For a camera-object distance then of 95.4 feet, the coverage of the object width and height can be found as follows. (See Figure 5.)

$$L = 2H \tan \alpha/2$$

$$L = 2H \tan 37°$$

$$L = (2)(95.4)(.75355)$$

$$L = 143.8 \text{ feet}$$

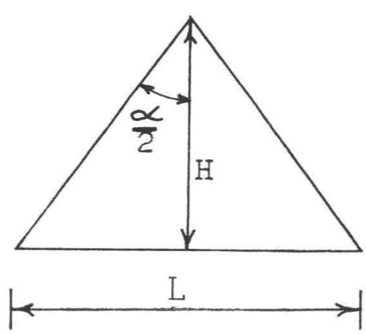


FIG. 5.

Next, consider the interference by the feedhorn tower of the stereo coverage. Since the position of the camera stations is farther away from the antenna than the feedhorn tower, the feedhorn tower will appear in the photographs. Therefore it becomes necessary to locate the camera stations such that complete stereo may be viewed at all points. It is suggested then that three camera stations be used, spaced as shown in Figure 6. This will provide ample coverage of the antenna for use in the stereoplotter.

RESULTS TO BE OBTAINED:

The photographs next must be analyzed in a first-order stereoplotter. Ground control for the model could consist of six points on the antenna, three along each side edge. Two methods of checking the surface are suggested. One would be to determine the $x$, $y$, and $z$ coordinates of each point and check these against computed values. It is believed that this method will be the more accurate, and if an electronic printout recorder for model coordinates is available, the method will also be fast and simple. A second method would be to draw contours of equal distances from the camera station as they appear on the antenna. These contours for the surface in question should appear as concentric circles. Any irregularities in the surface would then be quickly detectable. It is believed that in the absence of an electronic printout recorder, this latter method would be the easiest to accomplish, however a few points could be spot checked with use of the former method to insure that the surface as a whole is positioned properly.

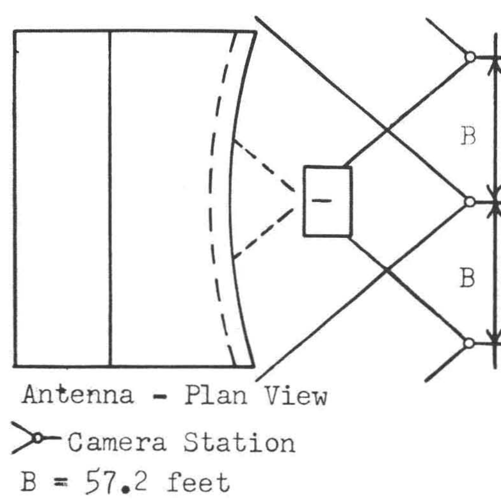


FIG. 6.

The points to be checked could be the corners of the surface sheeting sections. These points are desirable for two reasons: (1) they are easily identified and may be precisely located in the stereoplotter; and (2) the theoretically correct position of these points may be easily calculated for purposes of comparison.

The amount of relief on the antenna is large compared to the object distance. Because of the physical limitations of first-order stereoplotters with regard to the $z$ direction of movement, after compiling the outer portions of the antenna it will be necessary to increase the $x$ base so that the inner portions of the antenna may be compiled.

## Conclusions

When several different methods are being considered for a project, the method used is that which is the most economical. To check the reflector surface of an antenna, as described previously by ground survey methods, requires at least five men for two to four working days depending on weather conditions. The result is a check on the position of the girts and not on the position of the final reflector surface. By photogrammetric methods, it is estimated that the manpower needed would be a crane operator, photographer and three man survey party for one hour to obtain the photographs and ground control. Also required would be one man one day for the processing of the photographs and one man two days for the preparation of the model and the compilation of the results. More important however is the fact that the surface checked is the final surface and any irregularities found, which might not have been detected in the ground survey method, when corrected in the antenna, will greatly affect the efficiency of operation of the antenna. It is realized that any changes to be made in the reflector surface, after the final surface sheeting has been placed would be more difficult than those which could be made before. However, the author does not believe that the difference in difficulty is so great as to substantially decrease the relative efficiency of the method. It is also realized that if the location of the antenna is in a remote area, such as the eastern coast of Greenland where planes go through once a week, time for transportation of the photographic negatives will have to be taken into consideration. However, since the photographs are taken after the final surface sheeting has been placed, construction is not being delayed. Therefore the time delay for photography and measurement becomes of little significance

Although throughout the majority of this paper reference has been made to a 120 foot diameter parabolic dish antenna, the author anticipates that the principles of the method described could be applied to the reflector surface of any mathematical shape and of any size. Such a photogrammetric method not only reduces checking time in the field but provides a representation of the final surface. The solution therefore becomes more complete and hence is better.

## Bibliography

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# THE COVER

"Hole in the Ground" is the geographical name applied to a collapsed volcanic crater or depression in central Oregon. The hot magma has come in contact with a high water table or surface water at the time of ejection and resulting explosions and collapse have left the crater as it exists today. The term generally applied to this type of depression which has been created in a hydrous environment is maar. The "Hole in the Ground" is a so-called simple maar, not complicated by repeated explosions super-imposing layers and is in the northern Lake county northeast of Crater Lake—Mr. Delano