Principles of Stereoscopic Instrumentation for PPI Photography

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ABSTRACT: *Stereo instrumentation for overlapping radar photographs corrects any relief displacement and furnishes target elevation. The instrumentation and associated computational processes are described for a projection plotter, where the simple optical system permits drawing ray diagrams that are readily interpreted. In outward appearance, the projection equipment is similar to conventional plotters of the same type, with the exception of an additional prismatic element in each optical path, that is inserted to remove the* y *component of parallax from the general radar parallax vector. The prismatic correction is entered automatically, so that the actual operation of the stereoplotter is the same as for aerial photographs. The method of data reduction, however, is substantially more complicated. The rigorous solution is derived for the case that both radar frames are taken at the same altitude, and the flow diagram for an on-line computational program is presented.*

THE experimental investigations of radar stereoscopy conducted by Kober and his associates at Wright Field succeeded in showing that elevations and depressions were observable even with the crude displays then available. Esten's subsequent analysis of the parallax[1] for slant-range sweeps introduced the basic concept of the parallax vector. The extension of the theory to the ground-range display by Levine established the potential utility of parallax methods in determining terrain elevation.[2]

Kober's initial work on radar stereo techniques was essentially qualitative, and the ensuing treatments of parallax were not concerned with the method of its measurement. It is timely, therefore, to describe the instrumentation suitable for large-scale data reduction. In this paper we shall develop the precise equations and essential features of a projection plotter designed specifically for this application. The initial section reviews the relation of radar parallax to relief displacement.

RELIEF DISPLACEMENT AND RADAR PARALLAX

On a ground-range sweep, a target at slant range S appears on the display at a distance

$$\rho = m_R(S^2 - H^2)^{1/2} \tag{1}$$

where H is the aircraft altitude above the plane earth, and m_R is the scale of the radar

[1] R. D. Esten, "Radar Relief Displacement and Radar Parallax," U. S. Army Engineer Research and Development Laboratories Report 1294, Ft. Belvoir, Va., 12 May 1953. Alternative approximations for targets that lie on the flight line (and therefore applicable to data from side-looking systems) are developed in C. Colbert, "Parallax Equations for Determining Topographic Elevations from Radar Scope Photography," Westgate Laboratory, Inc., Yellow Springs, Ohio, November, 1954. A practical difficulty in applying either of these solutions arises from their being based on the slant-range display, since targets on the datum surface are not superposed when two frames are displaced according to the scaled separation of the nadir points. This limitation is described further in D. Levine, Radar Parallax for Slant-Range Displays, Sec. 2.7 of "Mapping from Airborne Radar Scope Presentations: Second Interim Technical Report," Northrop Aircraft, Inc. Report NAI-58-72, Hawthorne, Calif., 27 December 1957.

[2] D. Levine, "Radargrammetry," McGraw-Hill Book Company, Inc., New York, 1960; pp. 152–169.

photograph.[3] For a target located at ground range R and having an elevation h, the slant range is obtained from

$$S^2 = R^2 + (H - h)^2 \quad (2)$$

The distance indicated on the display, as illustrated in Figure 1, is

$$\rho = m_R[R^2 + (H - h)^2 - H^2]^{1/2} \quad (3)$$

When the same target appears in different radar frames, the geometry of the relief displacements is represented in Figure 2, which is drawn at the scale of the radar photographs in order to illustrate the (x_R, y_R) coordinate system of the overlapping frames. The target appears on the display at (x_{R1}, y_{R1}) when the aircraft is above N_1, and at (x_{R2}, y_{R2}) when above N_2. The x_R axis is oriented along the line N_1N_2 with positive x_R in the direction of flight, and positive y_R 90 degrees counterclockwise, with the origin midway between N_1 and N_2. For the specific target location of Figure 2, x_{R2} is positive, and x_{R1}, y_{R1}, and y_{R2} are negative numbers.

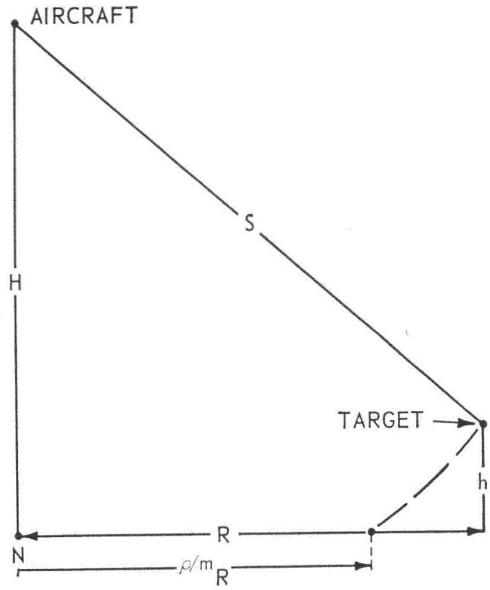

FIG. 1. Displacement of an elevated target.

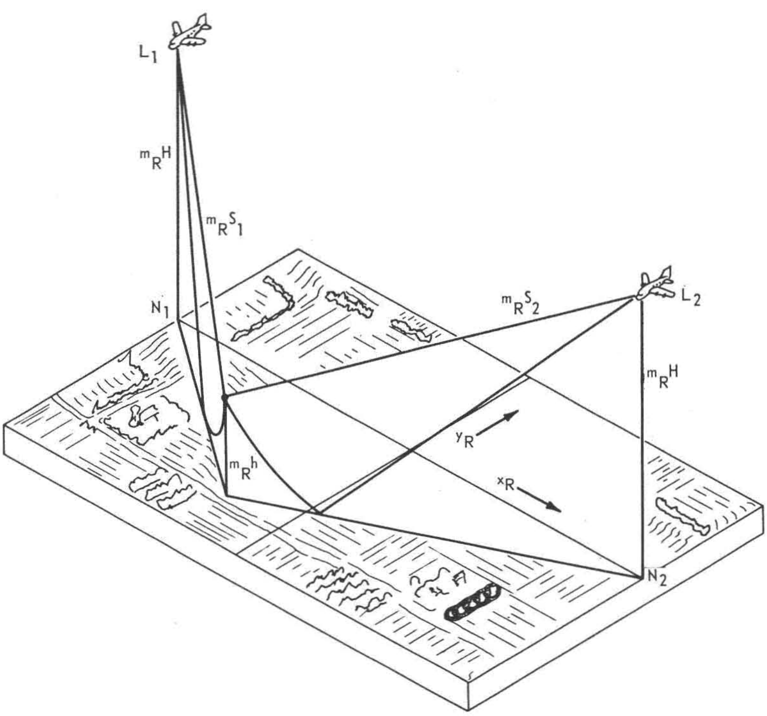

FIG. 2. Relief displacement of an elevated target in frames centered at N_1 and N_2.

[3] All symbols are listed at the end of the paper for convenient reference.

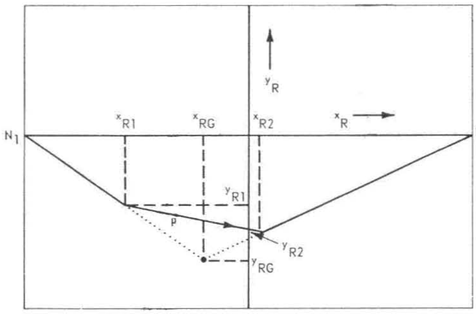

FIG. 3. Superposed radar frames.

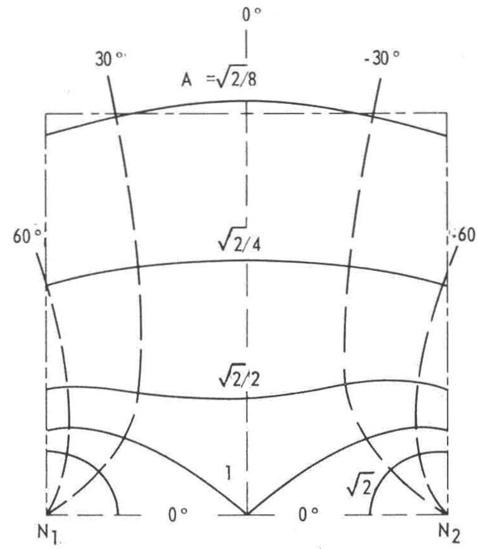

FIG. 4. Curves of parallax magnitude and angle. There is symmetry about the line $\overline{N_1N_2}$, so that only the upper portion of the overlap region is shown.

When the two radar frames are superposed with their coordinate axes aligned, then the nadir points N_1 and N_2 are separated by the scaled value of the airbase, and the target of Figure 2 appears as in Figure 3. Because of the relief displacement, the two images of this target are separated by the radar parallax. Using complex notation, the target location in the first frame is at

$$z_{R1} = x_{R1} + iy_{R1} \tag{4}$$

and in the second frame it is at

$$z_{R2} = x_{R2} + iy_{R2} \tag{5}$$

The separation between these two points defines the radar parallax, this complex quantity being identified in Figure 3. For positive x_{R2} and negative values of x_{R1}, y_{R1}, y_{R2} as in this example, the parallax is equal to

$$p = x_{R2} - x_{R1} + i(y_{R2} - y_{R1}) = z_{R2} - z_{R1} \tag{6}$$

An approximate analysis of radar parallax leads to the curves[4] of parallax magnitude and angle in Figure 4. The parallax magnitude is expressed in terms of the parameter A by means of the equation

$$|p| = \frac{2|h|(2H - h)A}{b_R} \tag{7}$$

where b_R is the airbase. The curves of this drawing are interpreted in Figure 5 in terms of the parallax vector for a target of fixed elevation. In this sketch when the target is located at the mid-point of the airbase, it has a reference parallax magnitude of unity. Then its parallax vector has a minimum magnitude of 0.176 within the region outlined. The parallax angle goes to 45° at the corners of the area in question, so that the minimum x component of parallax is 0.125. Consequently, the illustration shows an 8 to 1 variation of the x component of parallax. However, the range of values is actually even greater, since the parallax magnitude exceeds unity near the nadir points N_1 and N_2. A precise statement of the pertinent properties of the parallax vector follows:

[4] D. Levine, *op. cit.*, pp. 152–169. Figure 4 is taken from Fig. 4-25 of this reference.

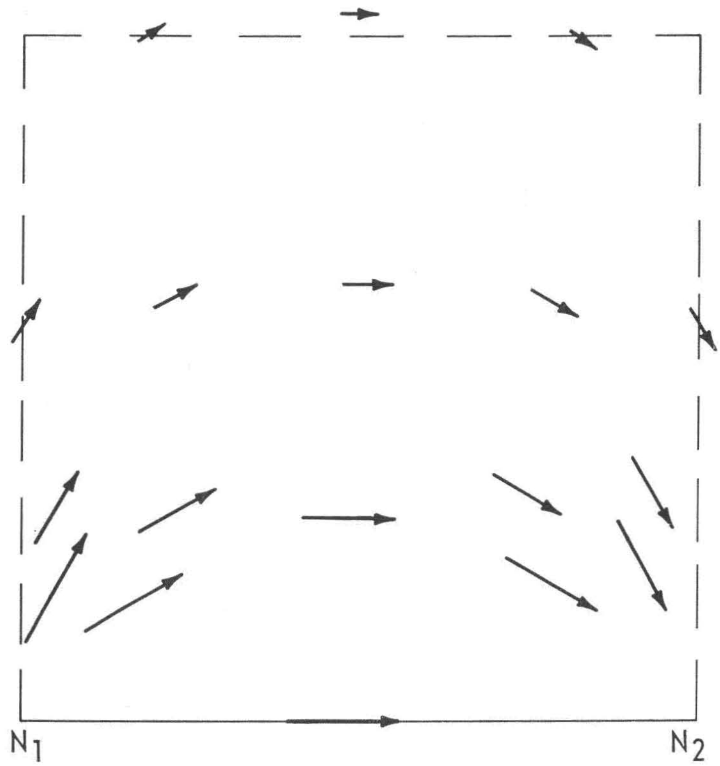

FIG. 5. Parallax vectors within the overlap region of Fig. 4 for a target above the datum surface.

(1) For a target of height h the minimum x component of parallax within the region of Figure 5 is one-eighth of the parallax magnitude for the same target located at the mid-point of the airbase.

(2) For a target of height h the y component of parallax is zero along the airbase and its perpendicular bisector.

When the radar parallax is known, the height and location of a target can be determined. Appropriate instrumentation for rapid and precise data reduction is described in the next section, after which the mathematical analysis is presented.

THE PROJECTION PLOTTER

Projection plotters are widely used in photogrammetry to determine the height from relief displacement in aerial photographs. They employ two projectors, one for each of two overlapping frames, with the intersection of rays from a single target being observed on a viewing screen located on a tracing table. The height of the viewing screen is adjusted so that rays from an elevated (depressed) target intersect on its surface, and the location of the tracing table then gives the position of the target.[5]

We shall now determine how to employ similar equipment with radar PPI photo-

[5] Typical projection systems for aerial photography are described in MANUAL OF PHOTOGRAMMETRY American Society of Photogrammetry, 1952, pp. 633–639 and 663–721; J. P. Burns, "A Comparison of the Kelsh and Balplex Plotters for Large-Scale Mapping," PHOTOGRAMMETRIC ENGINEERING, Vol. XXIV, 1958, pp. 74–76; "Photogrammetric Equipment," Bausch and Lomb Optical Co. Catalog F-301; "Balplex Plotters," Bausch and Lomb Optical Co. Catalog F-310; "Multiplex Mapping," *Dept. of the Army Tech. Manual* TM 5-244, June, 1954; D. Lyon, "Basic Metrical Photography," revised edition, Box 492 Arsenal Station, St. Louis, 1960, pp. 6–81 through 6–89B. Such common features as use of complimentary color filters in the projectors, and viewing with colored lenses are not explained here, since the principles involved are given in these references.

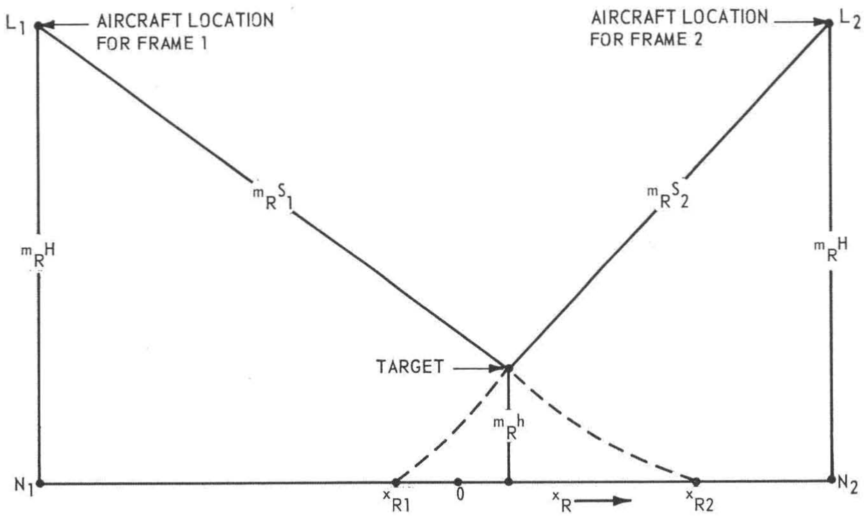

FIG. 6. Elevation displacement for equal flight altitudes at L_1 and L_2.

graphs having ground-range sweeps. For this purpose, we shall initially consider a target located on the airbase, because of the simplification in the geometrical representation. The relief displacements for such a target on two overlapping frames are illustrated in Figure 6, which is drawn at the scale of the radar photographs. The image appears at x_{R1} in frame 1 and at x_{R2} in frame 2. In the projection plotter of Figure 7, the ground nadir of each frame is on the optical axis. Positive x_R is toward the left in each frame, so that the corresponding axis in the datum surface is directed toward the right. With this arrangement, the intersection of the rays corresponding to the elevated target of Figure 6 occurs below the datum surface, and constitutes a depression in the stereo model.

The alternative arrangement of Figure 8 interchanges the two frames, and places the homologous images of the nadir points[6] on the optical axes, with positive x_R again toward the left. Now the rays of an elevated object intersect above the datum surface, so that the stereo model is erect. It may be noted that the first model appears to have greater sensitivity as measured by $|dh_p/dh|$, where h_p is the distance between the viewing screen and the datum surface. Even with such an advantage, the inverted stereo model might cause excessive operator confusion, thereby degrading the overall accuracy. Since the utility of this model can be established only by laboratory tests, and there is no corresponding problem associated with the normal model, the latter has been selected for further analysis. Essentially the same methods are applicable to the reversed model, if it should eventually prove to be of value.

When the radar frames are oriented as in Figure 8, an elevated target anywhere within the overlap area appears to lie above the datum surface. For example, the target of Figure 2 is drawn in accurate perspective projection in Figure 9. However, unlike the example of Figure 8, the rays originating from a specific elevated target in frame 1 do not actually intersect the rays from the corresponding image in frame 2. The reason for this condition may be noted in Figure 5, where the parallax vector has a y component unless the target is on either the airbase or its perpendicular bisector. Before the rays can be made to intersect by adjusting the height of the viewing screen, this y component of parallax must be eliminated. Otherwise the accuracy of the measurement is likely to be degraded. Removal of the y parallax

[6] The homologous image of the nadir point N_2 is located at N_2' in frame 1; that is, N_2' and N_2 are both images of the same point on the ground.

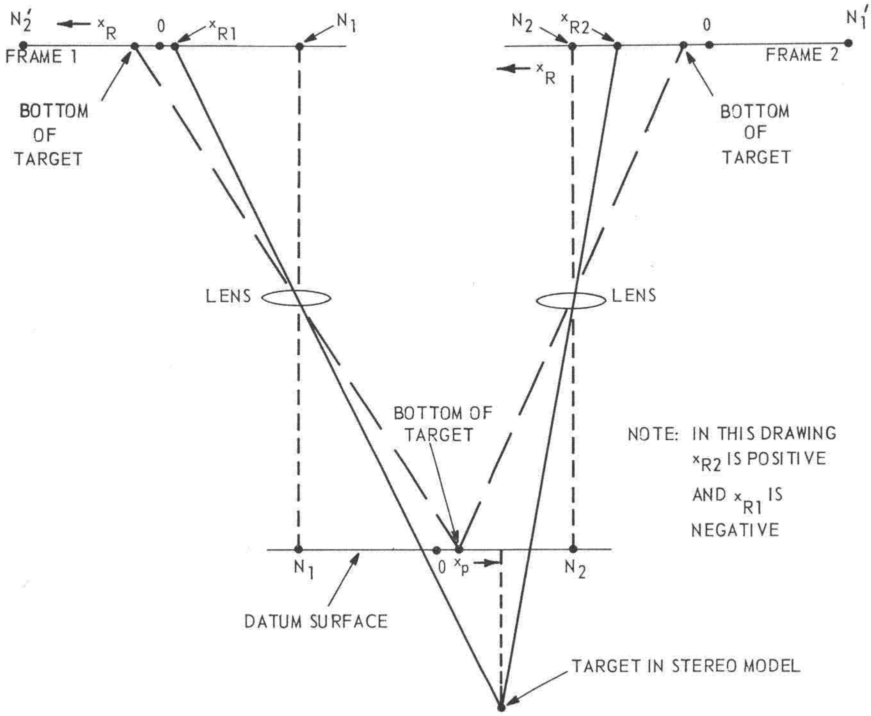

FIG. 7. Optical projection of the radar displays for the elevated target of Fig. 6.

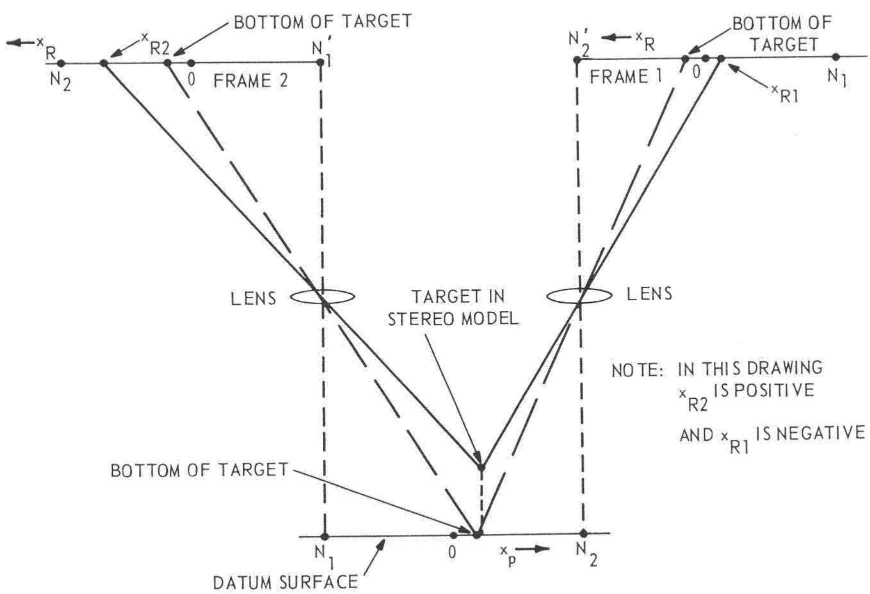

FIG. 8. Optical projection of the radar displays for a normal stereo model.

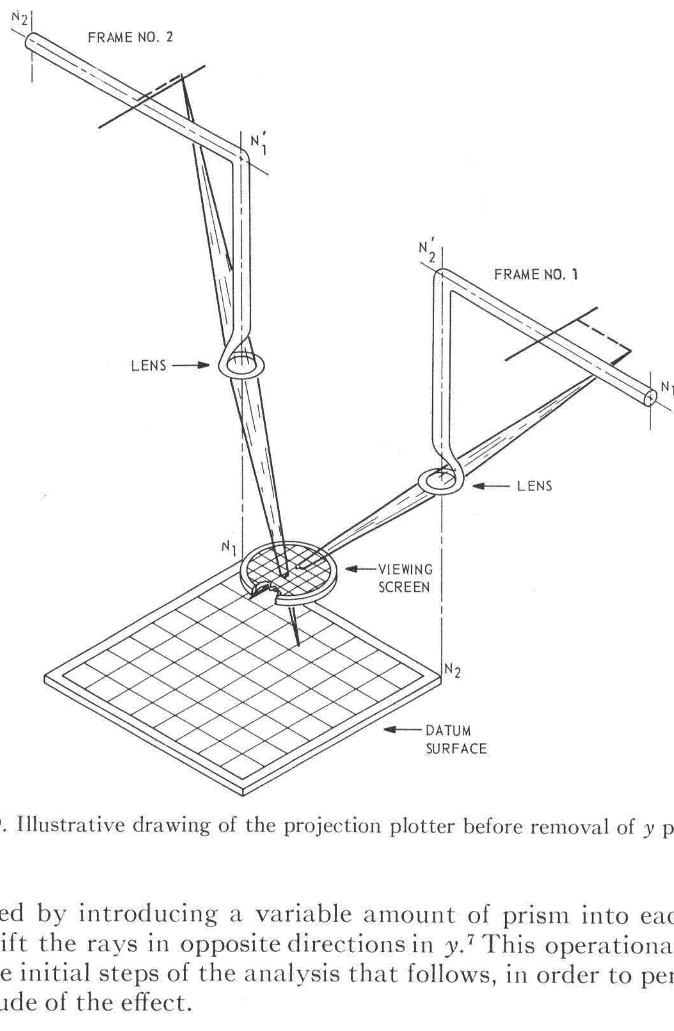

FIG. 9. Illustrative drawing of the projection plotter before removal of y parallax.

is accomplished by introducing a variable amount of prism into each optical path in order to shift the rays in opposite directions in y.[7] This operational feature is not included in the initial steps of the analysis that follows, in order to permit evaluation of the magnitude of the effect.

POSITION OF THE VIEWING SCREEN

An elevated target appears at (x_{R1}, y_{R1}) in frame 1 and at (x_{R2}, y_{R2}) in frame 2. When both of these sets of coordinates are known, it is possible to determine the precise location of the viewing screen. The derivation is instructive in its own right, since it demonstrates that the viewing screen has a unique location for the operating conditions that are imposed.

The artist's sketch of Figure 9 is repeated in Figure 10 with a right-handed (x_p, y_p, h_p) set of coordinates located within the stereo model. The origin is at the middle of the airbase in the datum surface, with positive x_p in the direction of flight, and h_p representing height above the datum surface. The image of the elevated target in frame 2 has coordinates $(-b_M/2-b_R/2-x_{R2}, \; -y_{R2}, \; L_R+L_M)$. The ray originating at this image passes through the center of the lens at $(-b_M/2, 0, L_M)$ so that the equation of the ray is

[7] Alternatively, the x parallax can be eliminated by optical means so that the rays intersect, and the viewing screen then adjusted to the point of intersection. However, the y component of parallax is zero along both the airbase and its perpendicular bisector; consequently, determination of elevation everywhere within the region of overlap requires that the x component of parallax be utilized.

$$\frac{x_p + b_M/2}{b_R/2 + x_{R2}} = \frac{y_p}{y_{R2}} = \frac{h_p - L_M}{-L_R} \tag{8}$$

Similarly, the image in frame 1 is at $(b_M/2+b_R/2-x_{R1},\ -y_{R1},\ L_R+L_M)$, and the ray originating there passes through the other lens at $(b_M/2,\ 0,\ L_M)$. Consequently, the equation of this ray is

$$\frac{x_p - b_M/2}{b_R/2 - x_{R1}} = \frac{y_p}{-y_{R1}} = \frac{h_p - L_M}{L_R} \tag{9}$$

At the surface of the viewing screen both rays have the same height coordinate, h_p. Furthermore, the operator adjusts the height of the viewing screen until both rays have the same x_p coordinate. But in general the y_p coordinates then differ, as in Figures 9 and 10. Consequently, for the ray originating in frame 1, we let

$$y_p = y_{p1} \tag{10}$$

at the surface of the viewing screen, and for the intersection of the ray from frame 2 we set

$$y_p = y_{p2} \tag{11}$$

With the substitution of Equation (11), Equation (8) becomes

$$-L_R\left(x_p + \frac{b_M}{2}\right) = \left(\frac{b_R}{2} + x_{R2}\right)(h_p - L_M) \tag{12}$$

$$y_{p2} = \frac{y_{R2}}{L_R}(L_M - h_p) \tag{13}$$

and the substitution of Equation (10) reduces Equation (9) to

$$L_R\left(x_p - \frac{b_M}{2}\right) = \left(\frac{b_R}{2} - x_{R1}\right)(h_p - L_M) \tag{14}$$

$$y_{p1} = \frac{y_{R1}}{L_R}(L_M - h_p) \tag{15}$$

Addition of Equations (12) and (14) gives

$$-L_R b_M = (b_R + x_{R2} - x_{R1})(h_p - L_M)$$

or

$$(b_R + x_{R2} - x_{R1})h_p = b_R L_M - L_R b_M + L_M(x_{R2} - x_{R1})$$

It is convenient to define the magnification of the projector m, where

$$m = \frac{b_M}{b_R} = \frac{L_M}{L_R} \tag{16}$$

Substitution into the preceding equation then expresses the height of the viewing screen as

$$h_p = \frac{L_M(x_{R2} - x_{R1})}{b_R + x_{R2} - x_{R1}} \tag{17}$$

When this result is introduced into Equation (14) we obtain

$$L_R x_p = L_R \frac{b_M}{2} + \left(\frac{b_R}{2} - x_{R1}\right) L_M \left(\frac{x_{R2} - x_{R1}}{b_R + x_{R2} - x_{R1}} - 1\right)$$

whence the x_p coordinate is

$$x_p = \frac{b_M}{2} - \frac{(b_R/2 - x_{R1})mb_R}{b_R + x_{R2} - x_{R1}} \equiv \frac{b_M}{2}\left(1 - \frac{b_R - 2x_{R1}}{b_R + x_{R2} - x_{R1}}\right) \equiv \frac{b_M(x_{R1} + x_{R2})}{2(b_R + x_{R2} - x_{R1})} \quad (18)$$

The y_{p2} coordinate is obtained from Equations (13) and (17) as

$$y_{p2} = \frac{y_{R2}}{L_R} L_M \left(1 - \frac{x_{R2} - x_{R1}}{b_R + x_{R2} - x_{R1}}\right) = \frac{my_{R2}b_R}{b_R + x_{R2} - x_{R1}} = \frac{b_M y_{R2}}{b_R + x_{R2} - x_{R1}} \quad (19)$$

and y_{p1} follows similarly from Equation (15), so that

$$y_{p1} = \frac{my_{R1}b_R}{b_R + x_{R2} - x_{R1}} = \frac{b_M y_{R1}}{b_R + x_{R2} - x_{R1}} \quad (20)$$

Example: Given that at the scale of the photograph

$x_{R1} =$	$-$ 19.2 mm.	$x_{R2} =$	1.3 mm.
$y_{R1} =$	$-$ 10.8 mm.	$y_{R2} =$	$-$ 15.1 mm.
$b_R =$	69.5 mm.	$b_M =$	69.5 mm.
$m =$	1.00	$L_M =$	60.0 mm.

Find h_p, x_p, y_{p1}, and y_{p2}.

The height of the viewing screen is

$$h_p = \frac{60.0(20.5)}{90.0} = 13.7 \text{ mm.}$$

and the coordinates are

$$x_p = \frac{34.75(-17.9)}{90.0} = -6.9 \text{ mm.}$$

$$y_{p2} = \frac{-10.8(69.5)}{90.0} = -8.3 \text{ mm.}$$

$$y_{p1} = \frac{-15.1(69.5)}{90.0} = -11.7 \text{ mm.}$$

The computations of this example were employed in drafting Figure 10.

Location of the Images

A problem closer to the immediate topic is to find the image coordinates in each frame when the viewing screen is adjusted to remove the x component of parallax in the stereo model. Now we are given x_p, h_p, y_{p1}, and y_{p2}, and want to obtain x_{R1}, y_{R1}, x_{R2} and y_{R2}. From Equations (12) through (15) we can immediately write the solutions:

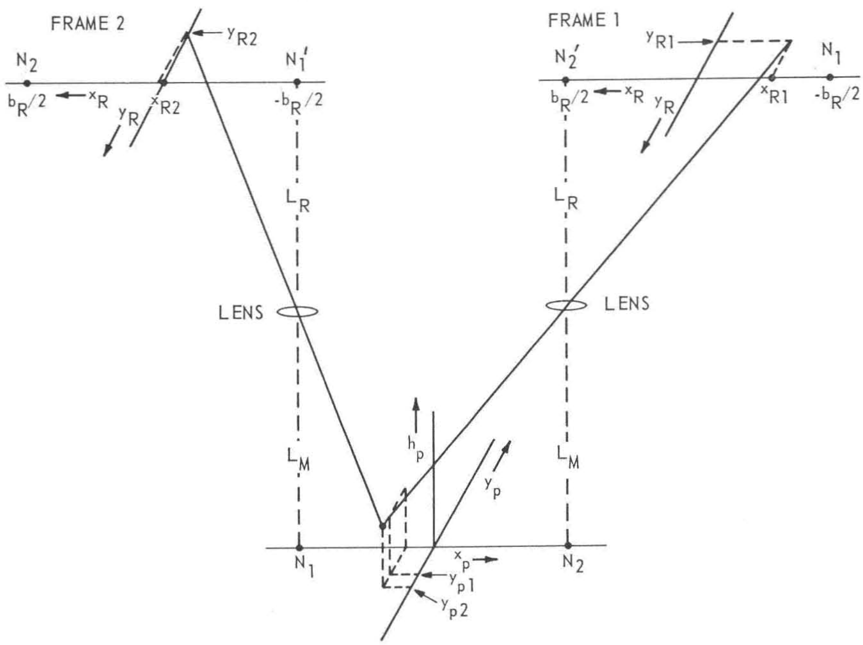

FIG. 10. Line drawing of the projection plotter before removal of the y parallax.

$$x_{R1} = \frac{b_R(h_p - L_M)/2 - L_R(x_p - b_M/2)}{h_p - L_M} = \frac{L_R x_p - b_R h_p/2}{L_M - h_p} \tag{21}$$

$$y_{R1} = \frac{L_R y_{p1}}{L_M - h_p} \tag{22}$$

$$x_{R2} = \frac{-b_R(h_p - L_M)/2 - L_R(x_p + b_M/2)}{h_p - L_M} = \frac{L_R x_p + b_R h_p/2}{L_M - h_p} \tag{23}$$

$$y_{R2} = \frac{L_R y_{p2}}{L_M - h_p} \tag{24}$$

These equations are useful in the subsequent solution for the position of the target.

TARGET COORDINATES

The target coordinates in the image plane are obtained without difficulty from Fig. 3. The target lies somewhere on the line passing through the nadir point of frame 1, N_1, and the point (x_{R1}, y_{R1}). The equation of this line is

$$y_R = \frac{y_{R1}}{x_{R1} + b_R/2}\left(x_R + \frac{b_R}{2}\right) \tag{25}$$

The target also lies on the line passing through the nadir point of frame 2, N_2, and the point (x_{R2}, y_{R2}), that satisfies the relation

$$y_R = \frac{y_{R2}}{x_{R2} - b_R/2}\left(x_R - \frac{b_R}{2}\right) \tag{26}$$

Equations (21) through (24) can be substituted at this point, to express the lines as

$$\begin{aligned} y_R &= \frac{L_R y_{p1}}{L_R x_p - b_R h_p/2 + b_R(L_M - h_p)/2}\left(x_R + \frac{b_R}{2}\right) \\ &\equiv \frac{L_R y_{p1}}{L_R x_p + b_R(L_M/2 - h_p)}\left(x_R + \frac{b_R}{2}\right) \end{aligned} \tag{27}$$

and

$$\begin{aligned} y_R &= \frac{L_R y_{p2}}{L_R x_p + b_R h_p/2 - b_R(L_M - h_p)/2}\left(x_R - \frac{b_R}{2}\right) \\ &\equiv \frac{L_R y_{p2}}{L_R x_p - b_R(L_M/2 - h_p)}\left(x_R - \frac{b_R}{2}\right) \end{aligned} \tag{28}$$

Before proceeding further it is convenient to normalize the height of the viewing screen h_p by means of division by the distance between lens and datum surface:

$$H_p \equiv \frac{h_p}{L_M} \tag{29}$$

With this definition and Equation (16), Equations (27) and (28) become

$$y_R = \frac{2y_{p1}}{2x_p + b_M(1 - 2H_p)}\left(x_R + \frac{b_R}{2}\right) \tag{30}$$

$$y_R = \frac{2y_{p2}}{2x_p - b_M(1 - 2H_p)}\left(x_R - \frac{b_R}{2}\right) \tag{31}$$

The x_R coordinate of the point of intersection denoted by x_{RG}, is obtained by equating the two relations for y_R:

$$\frac{2y_{p1}}{2x_p + b_M(1 - 2H_p)}\left(x_{RG} + \frac{b_R}{2}\right) = \frac{2y_{p2}}{2x_p - b_M(1 - 2H_p)}\left(x_{RG} - \frac{b_R}{2}\right)$$

whence

$$y_{p1}[2x_p - b_M(1 - 2H_p)]\left(x_{RG} + \frac{b_R}{2}\right) = y_{p2}[2x_p + b_M(1 - 2H_p)]\left(x_{RG} - \frac{b_R}{2}\right)$$

so that

$$\begin{aligned} &[2x_p(y_{p1} - y_{p2}) - b_M(1 - 2H_p)(y_{p1} + y_{p2})]x_{RG} \\ &\quad = \frac{b_R}{2}[-2x_p(y_{p1} + y_{p2}) + b_M(1 - 2H_p)(y_{p1} - y_{p2})] \end{aligned}$$

and

$$x_{RG} = \frac{b_R[-2x_p(y_{p1} + y_{p2}) + b_M(1 - 2H_p)(y_{p1} - y_{p2})]}{2[2x_p(y_{p1} - y_{p2}) - b_M(1 - 2H_p)(y_{p1} + y_{p2})]} \tag{32}$$

Since the two rays strike the viewing screen at y_{p1} and y_{p2}, we may say that the y coordinate of the viewing screen is midway between them, defining its position as

$$y_p \equiv \frac{y_{p1} + y_{p2}}{2} \tag{33}$$

Each ray is an equal distance Δy_p from this mid point, so that

$$\Delta y_p \equiv \frac{y_{p2} - y_{p1}}{2} \tag{34}$$

When a prismatic element is inserted to remove the y component of parallax, Δy_p is readily related to the prism power and the height of the viewing screen. Consequently, y_p and Δy_p are obtained from the projection plotter more directly than y_{p1} and y_{p2}. The latter quantities are actually obtained from the relations

$$y_{p1} = y_p - \Delta y_p \tag{35}$$

$$y_{p2} = y_p + \Delta y_p \tag{36}$$

With these definitions, the x coordinate of the target in the image-plane is given by

$$x_{RG} = \frac{b_R[2x_p y_p + b_M(1 - 2H_p)\Delta y_p]}{2[2x_p\Delta y_p + b_M(1 - 2H_p)y_p]} \tag{37}$$

The y_R coordinate is obtained by substituting Equation (37) into Equation (30):

$$\begin{aligned} y_{RG} &= \frac{b_R y_{p1}}{2x_p + b_M(1 - 2H_p)}\left(\frac{2x_p y_p + b_M(1 - 2H_p)\Delta y_p}{2x_p\Delta y_p + b_M(1 - 2H_p)y_p} + 1\right) \\ &= \frac{b_R y_{p1} y_{p2}}{2x_p\Delta y_p + b_M(1 - 2H_p)y_p} = \frac{b_R[y_p^2 - (\Delta y_p)^2]}{2x_p\Delta y_p + b_M(1 - 2H_p)y_p} \end{aligned} \tag{38}$$

The target coordinates in the plane of the datum surface (x_M, y_M) are exactly m times larger, so that

$$x_M = m x_{RG} = \frac{b_M[2x_p y_p + b_M(1 - 2H_p)\Delta y_p]}{2[2x_p\Delta y_p + b_M(1 - 2H_p)y_p]} \tag{39}$$

$$y_M = m y_{RG} = \frac{b_M[y_p^2 - (\Delta y_p)^2]}{2x_p\Delta y_p + b_M(1 - 2H_p)y_p} \tag{40}$$

Alternative Relations for Target Coordinates

A second set of equations for the target coordinates is obtained upon starting with Equation (3), which we rewrite as

$$\rho^2 = m_R^2[R^2 - h(2H - h)] \tag{41}$$

When the aircraft is above N_1 in Figure 3, this relation can be expressed as

$$\left(-\frac{b_R}{2} - x_{R1}\right)^2 + y_{R1}^2 = \left(-\frac{b_R}{2} - x_{RG}\right)^2 + y_{RG}^2 - h_R(2H_R - h_R)$$

where

$$\begin{cases} h_R = m_R h \\ H_R = m_R H \end{cases} \tag{41a}$$

The equation then simplifies to

$$b_R x_{R1} + x_{R1}^2 + y_{R1}^2 = b_R x_{RG} + x_{RG}^2 + y_{RG}^2 - h_R(2H_R - h_R) \tag{42}$$

Similarly, when the aircraft is above N_2, the equation becomes

$$\left(\frac{b_R}{2} - x_{R2}\right)^2 + y_{R2}^2 = \left(\frac{b_R}{2} - x_{RG}\right)^2 + y_{RG}^2 - h_R(2H_R - h_R)$$

or

$$-b_R x_{R2} + x_{R2}^2 + y_{R2}^2 = -b_R x_{RG} + x_{RG}^2 + y_{RG}^2 - h_R(2H_R - h_R) \tag{43}$$

When Equation (43) is subtracted from Equation (42), there remains

$$x_{R1}^2 - x_{R2}^2 + b_R(x_{R1} + x_{R2}) + y_{R1}^2 - y_{R2}^2 = 2b_R x_{RG}$$

or

$$2b_R x_{RG} = (x_{R1} - x_{R2} + b_R)(x_{R1} + x_{R2}) + y_{R1}^2 - y_{R2}^2$$

Substitution of Equations (21) through (24) gives

$$2b_R x_{RG} = \left(\frac{-b_R h_p}{L_M - h_p} + b_R\right)\frac{2L_R x_p}{L_M - h_p} + \frac{L_R^2(y_{p1}^2 - y_{p2}^2)}{(L_M - h_p)^2}$$

or, upon introducing Equations (35) and (36),

$$2b_R x_{RG} = \frac{L_R}{(L_M - h_p)^2}\left[2b_R(L_M - 2h_p)x_p - 4L_R y_p \Delta y_p\right]$$

Both sides of this equation are multiplied by $m^2/2$ to obtain

$$m b_M x_{RG} = \frac{L_M}{(L_M - h_p)^2}\left[b_M(L_M - 2h_p)x_p - 2L_M y_p \Delta y_p\right]$$

and substitution of Equation (29) finally yields

$$x_{RG} = \frac{b_M(1 - 2H_p)x_p - 2y_p\Delta y_p}{m b_M(1 - H_p)^2} \tag{44}$$

The y coordinate is obtained upon substituting into Equation (30):

$$\begin{aligned} y_{RG} &= \frac{2(y_p - \Delta y_p)}{2x_p + b_M(1 - 2H_p)}\left(x_{RG} + \frac{b_R}{2}\right) \\ &= \frac{2(y_p - \Delta y_p)}{m[2x_p + b_M(1 - 2H_p)]}\left(\frac{b_M(1 - 2H_p)x_p - 2y_p\Delta y_p}{b_M(1 - H_p)^2} + \frac{b_M}{2}\right) \end{aligned} \tag{45}$$

The target coordinates in the plane of the datum surface, being m times larger, are

$$x_M = \frac{b_M(1 - 2H_p)x_p - 2y_p\Delta y_p}{b_M(1 - H_p)^2} \tag{46}$$

$$y_M = \frac{2(y_p - \Delta y_p)}{2x_p + b_M(1 - 2H_p)}\left(\frac{b_M(1 - 2H_p)x_p - 2y_p\Delta y_p}{b_M(1 - H_p)^2} + \frac{b_M}{2}\right) \tag{47}$$

Before undertaking further discussion of these results, we shall work out a numerical example.

Illustrative Example

The application of the preceding equations will be indicated by a series of problems. In the first of these, the initial position of an elevated target is given, and we determine its displacement in different radar frames. In succeeding stages we compute the position of the viewing screen, and then work backward using the latter information to ascertain the location of the target, this position having been given in the very first problem.

PROBLEM 1

An elevated target appears on two frames separated by 20,000.0 m., the aircraft altitude being 5,000.0 m. The scale of the radar photographs is 1:200,000. Consequently the airbase at this scale is 100 mm., so that the origin of the (x_R, y_R) coordinate is 50 mm. from each nadir point. In terms of an image of this coordinate system projected onto the ground, a reflective body is located 2,000 m. above the ground surface with coordinates

$$x_{RG}' = -\ 6{,}000 \text{ m.}$$
$$y_{RG}' = -\ 5{,}000 \text{ m.}$$

where the primes identify the ground-coordinate system. Find the coordinates in each radar frame.

The distance between the target and the nadir-point of frame 1, N_1, is

$$R_1 = 1{,}000(4^2 + 5^2)^{1/2} = 6{,}403.1 \text{ m.}$$

and the angle at N_1 is

$$\tan\theta_1 = \frac{-5{,}000}{+4.000} = \frac{-1.25}{+1}$$

whence the angle is in the fourth quadrant, and

$$\theta_1 = -\ 51°20'24.69''$$

In frame 1 the distance on the display from N_1 to the target is obtained by substituting into Equation (41):

$$\rho_1 = \frac{1}{200{,}000}\{(6{,}403.1)^2 - 2{,}000[(2)(5{,}000) - 2{,}000]\}^{1/2}$$
$$= \frac{1}{200}(41 - 16)^{1/2} = 0.025 \text{ m.} \tag{41b}$$

which has components of 15.618 mm. along x_R and 19.522 mm. along y_R so that its coordinates are

$$x_{R1} = -\ 34.382 \text{ mm.}$$
$$y_{R1} = -\ 19.522 \text{ mm.}$$

The distance between the target and the nadir point of frame 2, N_2, is

$$R_2 = 1{,}000(16^2 + 5^2)^{1/2} = 16{,}763.0 \text{ m.}$$

and the angle at N_2 is

$$\tan\theta_2 = \frac{-5{,}000}{-16{,}000} = \frac{-0.31250}{-1}$$

whence the angle is in the third quadrant, and

$$\theta_2 = 197°21'14.48''$$

In frame 2 the distance on the display from N_2 to the target is obtained from

Equation (41) as

$$\rho_2 = \frac{1}{200.000}\left\{(16{,}763.0)^2 - 2{,}000[(2)(5{,}000) - 2{,}000]\right\}^{1/2}$$

$$= \frac{1}{200}(281 - 16)^{1/2} = 81.394 \text{ mm.}$$

which has components of 77.689 mm. along x_R and 24.278 mm. along y_R, so that its coordinates are

$$x_{R2} = -27.689 \text{ mm.}$$

$$y_{R2} = -24.278 \text{ mm.}$$

PROBLEM 2

The radar frames of Problem 1 are placed in a projection plotter having

$$\begin{cases} m = 25 \\ L_M = 600.000 \text{ mm.} \end{cases}$$

Find the height and position of the viewing screen.

The airbase in the mapping plane is given by Equation (16) as

$$b_M = mb_R = 2{,}500.000 \text{ mm.}$$

and the height of the platen follows from Equation (17):

$$h_p = \frac{600(6.693)}{106.693} = 37.639 \text{ mm.} \tag{17a}$$

The x_p coordinate of the viewing screen is

$$x_p = \frac{2{,}500(-62.071)}{2(106.693)} = -727.215 \text{ mm.} \tag{18a}$$

and the y_p coordinates are

$$y_{p1} = \frac{2{,}500(-19.522)}{106.693} = -457.434 \text{ mm.} \tag{20a}$$

$$y_{p2} = \frac{2{,}500(-24.278)}{106.693} = -568.975 \text{ mm.} \tag{19a}$$

so that

$$y_p = \frac{-457.434 - 568.975}{2} = -513.204 \text{ mm.} \tag{33a}$$

$$\Delta y_p = \frac{-568.975 + 457.434}{2} = -55.770 \text{ mm.} \tag{34a}$$

The target position at (−30, −25) in the film-plane becomes (−750, −625) in the mapping plane because of the optical magnification of the projectors. These coordinates may be compared with those of the viewing-screen, which is centered at (−727.215, −513.204). The positional discrepancy between the location of the viewing-screen and the correct site is 22.785 mm. in x and 111.796 mm. in y. Since the scale of the manuscript is 1:8,000, these displacements correspond to 182.28 m.

in x and 894.37 m. in y. This simple example illustrates the necessity of computing the exact map coordinates from the measurable quantities in the projection plotter.

PROBLEM 3

Given the values of h_p, x_p, y_p, and Δy_p in the preceding example, find the target location in the image plane by means of Equations (37) and (38).

The normalized height of the viewing screen is

$$H_p = \frac{h_p}{L_M} = \frac{37.639}{600.000} = 0.062\ 732 \tag{29a}$$

$$x_{RG} = \frac{100}{2} \frac{[2(-727.215)(-513.204) + 2{,}500(1 - 0.125\ 464)(-55.770)]}{[2(-727.215)(-55.770) + 2{,}500(1 - 0.125\ 464)(-513.204)]}$$

$$= \frac{50(746{,}419 - 121{,}932)}{81{,}114 - 1{,}122{,}040} = \frac{50(624{,}487)}{-1{,}040{,}930} = -\ 29.997 \text{ mm.} \tag{37a}$$

$$y_{RG} = \frac{100[(-513.204)^2 - (-55.770)^2]}{-1{,}040{,}930}$$

$$= \frac{100(263{,}378 - 3{,}110)}{-1{,}040{,}930} = \frac{26{,}026{,}800}{-1{,}040{,}930} = -\ 25.003 \text{ mm.} \tag{38a}$$

The target location in x_R and y_R is within 0.003 mm. of its known position, the error arising in rounding the last figure in the course of the computation.

PROBLEM 4

Given h_p, x_p, y_p, and Δy_p as in Problem 3, find the target location in the image plane, using Equations (44) and (45).

$$x_{RG} = \frac{2{,}500(1 - 0.125\ 464)(-727.215) - 2(-513.204)(-55.770)}{25(2{,}500)(1 - 0.062\ 732)^2}$$

$$= \frac{-1{,}589{,}980 - 57{,}243}{54{,}904} = \frac{1{,}647{,}220}{54{,}904} = -\ 30.002 \text{ mm.} \tag{44a}$$

$$y_{RG} = \frac{2(-513.204 + 55.770)}{2(-727.215) + 2{,}500(1 - 0.125\ 464)} \left(-30.002 + \frac{100}{2}\right)$$

$$= \frac{2(-457.434)(19.998)}{-1{,}454.43 + 2{,}186.34} = \frac{(-457.434)(39.996)}{731.91} = -\ 24.997 \text{ mm.} \tag{45a}$$

The coordinates are within 0.003 mm. of the known location of the target, the error having the same source as in the preceding example.

AUTOMATIC REDUCTION OF y PARALLAX

The four physically measurable quantities in a projection plotter are the position and height of the viewing screen, and the separation of the two rays in y. These elements, denoted by x_p, y_p, h_p, and Δy_p, are employed to ascertain the height and location of an elevated target, symbolized by h_M, x_M, and y_M. There are, then, four known elements that determine three unknowns, and as a consequence there are two independent expressions for x_M, Equations (39) and (46). If we eliminate x_M between these two equations, then any one of the four measurable elements can be expressed in terms of the other three. In particular, the y parallax, which is proportional to Δy_p, is determined by x_p, y_p, and h_p, so that it can be removed automatically within the equipment as the viewing screen is translated in x_p and y_p, and its elevation h_p is altered. The operation of the equipment then becomes completely similar to that of a standard projection plotter for aerial photographs.

Upon equating Equations (39) and (46), we obtain

$$\frac{b_M[2x_py_p + b_M(1 - 2H_p)\Delta y_p]}{2[2x_p\Delta y_p + b_M(1 - 2H_p)y_p]} = \frac{b_M(1 - 2H_p)x_p - 2y_p\Delta y_p}{b_M(1 - H_p)^2}$$

whence

$$\begin{aligned} &2b_M^2(1 - H_p)^2x_py_p + b_M^3(1 - 2H_p)(1 - H_p)^2\Delta y_p \\ &= 4b_M(1 - 2H_p)x_p^2\Delta y_p - 8x_py_p(\Delta y_p)^2 + 2b_M^2(1 - 2H_p)^2x_py_p - 4b_M(1 - 2H_p)y_p^2\Delta y_p \end{aligned}$$

or

$$\begin{aligned} 8x_py_p(\Delta y_p)^2 + b_M(1 - 2H_p)[b_M^2(1 - H_p)^2 + 4(y_p^2 - x_p^2)]\Delta y_p \\ + 2b_M^2H_p(2 - 3H_p)x_py_p = 0 \end{aligned} \quad (48)$$

If either $x_p=0$ or $y_p=0$, or both, then this equation reduces to $\Delta y_p=0$. This conclusion constitutes a proof of the earlier assertion that the y parallax is zero along the airbase ($y_p=0$) and along its perpendicular bisector ($x_p=0$). Before manipulating the general expression further, we shall consider an example based on the results of Problem 2.

PROBLEM 5

Given $h_p=37.639$ mm., $x_p=-727.215$ mm. and $y_p=-513.204$ mm., find Δy_p when $L_M=600$ mm. and $b_M=2{,}500$ mm.

For this problem, each term in Equation (48) is first divided by $8b_M^2x_py_p$, and then evaluated, with the result being

$$\left(\frac{\Delta y_p}{b_M}\right)^2 + 1.29\ 718\left(\frac{\Delta y_p}{b_M}\right) + 0.028\ 4145 = 0 \quad (48a)$$

which has two solutions given by

$$\frac{\Delta y_p}{b_M} = \frac{-1.29\ 718 \pm 1.25\ 260}{2} \quad (48b)$$

The plus sign is taken for the second term, the other root being extraneous; consequently,

$$\frac{\Delta y_p}{b_M} \doteq -0.02\ 229 \quad (48c)$$

and

$$\Delta y_p = -55.73 \text{ mm.} \quad (48d)$$

which differs by 4 in the last significant figure from the correct answer of -55.77 mm.

Returning to the general relation of Equation (48), we have already described the solution for $x_p=0$ or $y_p=0$. When neither of these coordinates is zero, we may express the equation as

$$\left(\frac{\Delta y_p}{b_M}\right)^2 + \frac{(1 - 2H_p)}{8x_py_p}[b_M^2(1 - H_p)^2 + 4(y_p^2 - x_p^2)]\left(\frac{\Delta y_p}{b_M}\right) + \frac{H_p(2 - 3H_p)}{4} = 0 \quad (49)$$

which is of the form

$$t^2 + Bt + C = 0 \tag{49a}$$

with roots

$$t_1 = \frac{-B + \sqrt{B^2 - 4C}}{2} \tag{49b}$$

$$t_2 = \frac{-B - \sqrt{B^2 - 4C}}{2} \tag{49c}$$

As B becomes very large, $t_1 \rightarrow 0$ and $t_2 \rightarrow \infty$. In Equation (49) the coefficient of the linear term increases as x_p or y_p approaches zero, in which case the root should also approach zero, as discussed previously. Consequently, t_1 is the only root having physical significance for our application, so that the solution of Equation (49) may be written as

$$\begin{aligned}\frac{\Delta y_p}{b_M} &= \frac{B}{2}\left[-1 + \left(1 - \frac{4C}{B^2}\right)^{1/2}\right] = -\frac{B}{2}\left(\frac{2C}{B^2} + \frac{2C^2}{B^4} + \frac{4C^3}{B^6} + \cdots\right) \\ &= -\frac{C}{B}\left(1 + \frac{C}{B^2} + \frac{2C^2}{B^4} + \cdots\right)\end{aligned} \tag{50}$$

Substitution for B and C from Equation (49) gives a one-term approximation of

$$\Delta y_p \doteq -\frac{2b_M H_p(2 - 3H_p)x_p y_p}{(1 - 2H_p)[b_M{}^2(1 - H_p)^2 + 4(y_p{}^2 - x_p{}^2)]} \tag{51}$$

PROBLEM 6

Find Δy_p for the values of Problem 5, using the one-term approximation of Equation (51).

$$\Delta y_p = \frac{-2(2{,}500)(0.062\ 732)(1.811\ 804)(-727.215)(-513.204)}{(0.874\ 536)(4{,}428{,}589)} = -\ 54.762 \text{ mm.} \tag{51a}$$

so that the approximation is correct within 2 per cent for this problem. Additional terms in the series expansion can be retained, but it is probably simpler to solve Equation (49) directly when greater accuracy is required. Equation (51) is useful, however, in indicating the general dependence of the y parallax on the other physical parameters.

Target Elevation

The target elevation enters into Equations (42) and (43), which are repeated below for convenience:

$$-h_R(2H_R - h_R) = x_{R1}{}^2 + b_R x_{R1} + y_{R1}{}^2 - b_R x_{RG} - x_{RG}{}^2 - y_{RG}{}^2 \tag{52}$$

and

$$-h_R(2H_R - h_R) = x_{R2}{}^2 - b_R x_{R2} + y_{R2}{}^2 + b_R x_{RG} - x_{RG}{}^2 - y_{RG}{}^2 \tag{53}$$

The elevation at the scale of the map manuscript, h_M, is of more direct interest than its value at the scale of the radar film, so that we define

$$\begin{aligned} h_M &= m h_R \\ H_M &= m H_R \end{aligned} \tag{54}$$

and multiply Equations (52) and (53) by m^2 to obtain

$$-h_M(2H_M - h_M) = (mx_{R1})^2 + b_M(mx_{R1}) + (my_{R1})^2 - b_M x_M - x_M{}^2 - y_M{}^2 \quad (55)$$

and

$$-h_M(2H_M - h_M) = (mx_{R2})^2 - b_M(mx_{R2}) + (my_{R2})^2 + b_M x_M - x_M{}^2 - y_M{}^2 \quad (55a)$$

The sum of these two equations may be expressed as

$$\begin{aligned} -h_M(2H_M - h_M) = \tfrac{1}{2}[(mx_{R1})^2 + (mx_{R2})^2 + b_M(mx_{R1} - mx_{R2}) + (my_{R1})^2 + (my_{R2})^2] \\ - (x_M{}^2 + y_M{}^2) \end{aligned} \quad (56)$$

The terms in mx_R and my_R are readily obtained as functions of the projection plotter settings after substituting Equations, (16), (29), (35), and (36) into Equations (21) through (25):

$$mx_{R1} = \frac{x_p - b_M H_p/2}{1 - H_p} \quad (57)$$

$$my_{R1} = \frac{y_p - \Delta y_p}{1 - H_p} \quad (58)$$

$$mx_{R2} = \frac{x_p + b_M H_p/2}{1 - H_p} \quad (59)$$

$$my_{R2} = \frac{y_p + \Delta y_p}{1 - H_p} \quad (60)$$

With these results, Equation (56) becomes

$$-h_M(2H_M - h_M) = \frac{x_p{}^2 + b_M{}^2 H_p{}^2/4 + y_p{}^2 + (\Delta y_p)^2}{(1 - H_p)^2} - \frac{b_M{}^2 H_p}{2(1 - H_p)} - (x_M{}^2 + y_M{}^2) \quad (61)$$

The last term in parentheses may be expressed directly in terms of the plotter parameters by substituting Equations (39) and (40), or (46) and (47). However, there seems to be no advantage to this further development, because the results are quite unwieldy. Since both x_M and y_M must be computed in any event, it is reasonable to consider them as being known when the computation of the elevation is undertaken. With this viewpoint, Equation (61) next is expressed as

$$(H_M - h_M)^2 = H_M{}^2 + \frac{x_p{}^2 + y_p{}^2 + (\Delta y_p)^2 - b_M{}^2 H_p(2 - 3H_p)/4}{(1 - H_p)^2} - (x_M{}^2 + y_M{}^2)$$

The target elevation is, therefore,

$$h_M = H_M \pm H_M \left[1 + \frac{x_p{}^2 + y_p{}^2 + (\Delta y_p)^2 - b_M{}^2 H_p(2 - 3H_p)/4}{H_M{}^2(1 - H_p)^2} - \frac{x_M{}^2 + y_M{}^2}{H_M{}^2}\right]^{1/2} \quad (62)$$

Since our concern in mapping is limited to targets at lower elevations than that of the aircraft, the negative sign is taken in front of the radical; then the target elevation is

$$h_M = H_M - H_M \left[1 + \frac{x_p{}^2 + y_p{}^2 + (\Delta y_p)^2 - b_M{}^2 H_p(2 - 3H_p)/4}{H_M{}^2(1 - H_p)^2} - \frac{x_M{}^2 + y_M{}^2}{H_M{}^2}\right]^{1/2} \quad (63)$$

PROBLEM 7

With the data of Problem 2 (specifying x_p, y_p, Δy_p, and h_p) and the computations of Problem 3 for the target location, compute its elevation when $H_M = 625$ mm.

The values are substituted into Equation (63) to obtain

$$\frac{h_M}{625} = 1 - \left[1 + \frac{(-727.215)^2 + (-513.204)^2 + (-55.770)^2 - (2,500)^2(0.028\ 4145)}{(625)^2(0.937\ 268)^2} - \frac{(-750.05)^2 + (-624.925)^2}{(625)^2}\right]^{1/2}$$

$$= 1 - \left[1 + \frac{528,842 + 268,378 + 3,110 - 177,591}{343,153} - \frac{562,575 + 390,531}{390,625}\right]^{1/2}$$

$$= 1 - \left(1 + \frac{617,739}{343,153} - \frac{953,106}{390,625}\right)^{1/2} = 1 - (0.36\ 024)^{1/2} = 0.39980$$

whence

$$h_M = 249.875 \text{ mm.} \tag{63a}$$

For the scale of 1:8,000, the actual height is computed to be

$$h = 1,999.0 \text{ m.} \tag{63b}$$

which is one meter less than the elevation of 2,000.0 m given in Problem 1.

Normalized Coordinates

Normalization of the height of the viewing screen in Equation (29) resulted in simplified relations that were more readily manipulated in subsequent steps. It is pertinent now to review the expressions for target position and elevation to determine whether additional normalization of the parameters would have computational advantages. This proves to be the case, the normalizing divisor being $b_M/2$ in the stereo space and $b_R/2$ in the film plane; consequently, we define

$$X_M \equiv \frac{2x_M}{b_M} \tag{64}$$

$$Y_M \equiv \frac{2y_M}{b_M} \tag{65}$$

$$X_p \equiv \frac{2x_p}{b_M} \tag{66}$$

$$Y_p \equiv \frac{2y_p}{b_M} \tag{67}$$

$$\Delta Y_p \equiv \frac{2\Delta y_p}{b_M} \tag{68}$$

$$H_M' \equiv \frac{2H_M}{b_M} \tag{69}$$

and

$$X_R \equiv \frac{2x_R}{b_R} \tag{70}$$

$$Y_R \equiv \frac{2y_R}{b_R} \tag{71}$$

For example, the equation for y parallax, Equation (48), is divided by $b_M{}^4/2$ to give

$$X_pY_p(\Delta Y_p)^2 + (1 - 2H_p)[(1 - H_p)^2 + Y_p{}^2 - X_p{}^2]\Delta Y_p + H_p(2 - 3H_p)X_pY_p = 0 \tag{72}$$

After division by $b_M/2$, the film-coordinates as a function of the viewing-screen position, Equations (57) through (60), become

$$X_{R1} = \frac{X_p - H_p}{1 - H_p} \tag{73}$$

$$Y_{R1} = \frac{Y_p - \Delta Y_p}{1 - H_p} \tag{74}$$

$$X_{R2} = \frac{X_p + H_p}{1 - H_p} \tag{75}$$

$$Y_{R2} = \frac{Y_p + \Delta Y_p}{1 - H_p} \tag{76}$$

Similarly, Equations (17) through (20), (33), and (34) express the location of the viewing screen in terms of the image coordinates:

$$H_p = \frac{X_{R2} - X_{R1}}{2 + X_{R2} - X_{R1}} \tag{77}$$

$$X_p = \frac{X_{R1} + X_{R2}}{2 + X_{R2} - X_{R1}} \tag{78}$$

$$Y_p = \frac{Y_{R1} + Y_{R2}}{2 + X_{R2} - X_{R1}} \tag{79}$$

$$\Delta Y_p = \frac{Y_{R2} - Y_{R1}}{2 + X_{R2} - X_{R1}} \tag{80}$$

The normalized map-coordinates of the target follow from Equations (39) and (40):

$$X_M = \frac{X_pY_p + (1 - 2H_p)\Delta Y_p}{X_p\Delta Y_p + (1 - 2H_p)Y_p} \tag{81}$$

$$Y_M = \frac{Y_p{}^2 - (\Delta Y_p)^2}{X_p\Delta Y_p + (1 - 2H_p)Y_p} \tag{82}$$

or, in place of the latter relation,

$$Y_M = \frac{Y_p - \Delta Y_p}{X_p + 1 - 2H_p}(X_M + 1) \tag{83}$$

Alternatively, Equations (46) and (47) express these coordinates as

$$X_M = \frac{(1 - 2H_p)X_p - Y_p\Delta Y_p}{(1 - H_p)^2} \tag{84}$$

$$Y_M = \frac{Y_p - \Delta Y_p}{X_p + 1 - 2H_p}\left[\frac{(1 - 2H_p)X_p - Y_p\Delta Y_p}{(1 - H_p)^2} + 1\right] \tag{85}$$

Equation (83) may be employed in preference to Equation (85) as a solution for Y_M.

The elevation of the target, Equation (63), is best put into the form

$$\frac{h_M}{H_M} = 1 - \left[1 + \frac{X_p^2 + Y_p^2 + (\Delta Y_p)^2 - H_p(2 - 3H_p)}{H_M'^2(1 - H_p)^2} - \frac{X_M^2 + Y_M^2}{H_M'^2}\right]^{1/2} \tag{86}$$

At first glance there appears to be no advantage to the normalized equations, since the input data must be divided and then the answers subsequently multiplied by appropriate factors. This statement is only partially correct, however, since the manuscript scale in general differs from that of the equipment itself, so that the results would have to be modified in any event. In other words, the only additional steps required to use these equations arises in the initial normalization. The reduction of computational effort that ensues more than compensates for this preliminary manipulation.

The Computational Program

The form of the computational program will be presented in very informal fashion in this section in order to indicate the nature of the demands on the digital computer. The general flow of information is illustrated in Figure 11, which shows the association of the projection plotter with the computer. The position of the viewing-screen, (x_p, y_p, h_p) enters the computer in digital form. The prismatic correction is determined by Δy_p, which is computed as frequently as possible.

When the viewing screen has been adjusted to eliminate x parallax, the operator presses a "READ" button, thereby terminating the repeated computation of Δy_p. The computational program then proceeds to evaluate the target coordinates at the manuscript scale. The output is fed to a tape recorder to facilitate additional data processing, or to a digital coordinate plotter, or to a device for the direct construction of raised relief maps. In all likelihood, the output will go to two or even three of these components simultaneously. Upon completion of the read-out step, the "READ" button automatically returns to the *off* position, and the operator moves the viewing-screen to another position, as the program is repeated.

Three special cases may be recognized as pertinent to the programming:

(a) $h_p = 0$. When the viewing-screen is at the level of the datum surface,

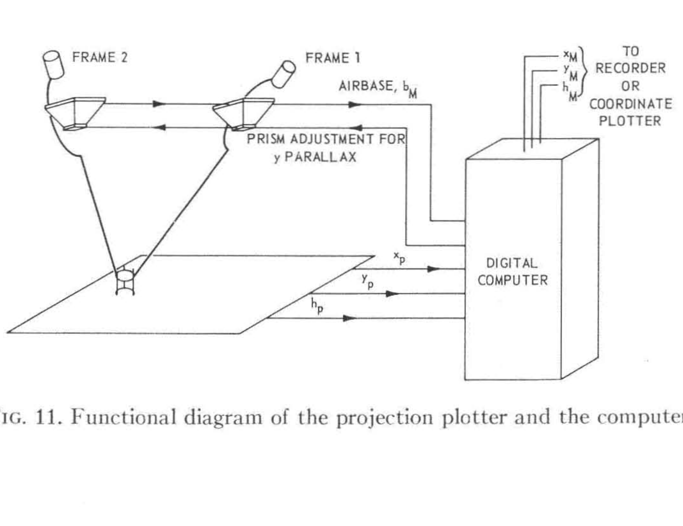

FIG. 11. Functional diagram of the projection plotter and the computer.

Equation (72) becomes

$$(X_pY_p\Delta Y_p + 1 + Y_p^2 - X_p^2)\Delta Y_p = 0$$

with roots

$$\Delta Y_p = 0$$
$$\Delta Y_p = -\frac{1 + Y_p^2 - X_p^2}{X_pY_p} \tag{87}$$

the latter solution being extraneous. Then Equations (81), (83), and (86) yield

$$X_M = X_p$$
$$Y_M = Y_p \tag{88}$$
$$h_M = 0$$

(b) $X_p = 0$. When the viewing-screen is on the perpendicular bisector of the line segment $\overline{N_1N_2}$, ΔY_p again is zero, and Equations (81), (83), and (86) give

$$X_M = 0$$
$$Y_M = \frac{Y_p}{1 - 2H_p}$$
$$\frac{h_M}{H_M} = 1 - \left[1 + \frac{Y_p^2 - H_p(2 - 3H_p)}{H_M'^2(1 - H_p)^2} - \frac{Y_M^2}{H_M'^2}\right]^{1/2} \tag{89}$$

(c) $Y_p = 0$. When the viewing-screen is on the line $\overline{N_1N_2}$, ΔY_p is zero, and Equations (84), (85), and (86) reduce to

$$X_M = \frac{(1 - 2H_p)X_p}{(1 - H_p)^2}$$
$$Y_M = 0 \tag{90}$$
$$\frac{h_M}{H_M} = 1 - \left[1 + \frac{X_p^2 - H_p(2 - 3H_p)}{H_M'^2(1 - H_p)^2} - \frac{X_M^2}{H_M'^2}\right]^{1/2}$$

The first operational step after inserting a pair of overlapping frames in the projectors is to set the distance between them equal to the airbase in the model space, b_M. This is best accomplished by means of well-defined control points, as in photogrammetric procedures. The value of $b_M/2$ is stored in the computer for use in normalizing the data. The aircraft altitude, H, and the distance between the lens and the datum surface, L_M, also are placed in permanent storage. The rest of the data is handled as in the flow chart of Figure 12.

The first box in the upper left part of this diagram symbolizes the introduction of a set of instantaneous values of the viewing-screen coordinates into the computer. When any one of these parameters is zero, Δy_p is known to be zero so that its computation is unnecessary. Otherwise, Δy_p is evaluated by means of Equations (72) and (49b). In either case, its value is fed out to the prismatic control in order to eliminate the y parallax in the stereo model. Then, if the READ button is not depressed, a new set of viewing-screen coordinates enters the computer, and the same steps are repeated. In this way the digital computer serves to eliminate the y parallax during the entire examination of the stereo model.

When the operator has removed the x parallax at a point, he presses a READ button on the mobile stand to which the viewing-screen is attached. The controls

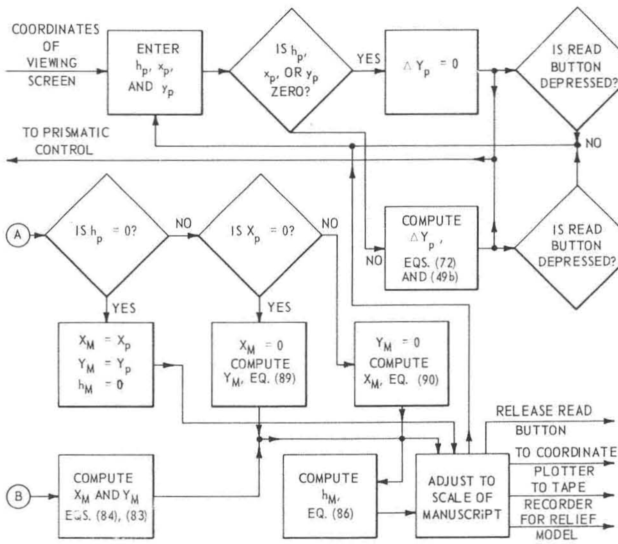

FIG. 12. Flow chart for the computer program.

lock and the button remains depressed during the period of the computation, until the target-coordinates at the manuscript scale are read out. When this button is initially depressed, the calculation of Δy_p is accomplished as described above, and the computer then evaluates the normalized map-coordinates X_M and Y_M. The procedure employed for the latter step depends upon whether or not h_p, X_p, or Y_p is zero, so that there are four alternate paths; that is, there is a different method for each of the following:

1. h_p is zero
2. X_p is zero
3. Y_p is zero
4. None of the viewing-screen coordinates is zero.

If h_p is zero, then the map-coordinates and the elevation are known without further computation, so that these values are entered directly into the last stage, which corrects for the manuscript scale. Otherwise the target elevation remains for computation, this step being accomplished by means of Equation (86), which is suitable for the remaining cases.[8] With all target-coordinates known, the final adjustment to manuscript scale is introduced, and the output fed to one or more recording or plotting devices. Also, the READ button is released, the controls unlocked, and the loop for computing Δy_p is reestablished.

With the association of the projection plotter and the computer described above, the mapping is achieved by taking a large number of spot elevations. This approach was adopted as being the simplest to explain, as well as offering a logical procedure for testing the accuracy obtainable with the radar stereo model. In standard operation of a projection plotter, however, it is common to trace complete contours in the stereo model. The same performance can be achieved with the radar model provided that a suitable computer program is employed. For this purpose, the height of the viewing-screen must be computed for a specified terrain elevation, so that h_p is an *output* of the computer rather than an input as in the application of Figure 12.

Further modifications based on other photogrammetric plotters are feasible, but

[8] In this description, one subroutine is used for computation of the elevation, even if X_p or Y_p is zero. Of course, an additional subroutine may be inserted to utilize the simpler relation given in Equation (89) or (90), if desired.

lie beyond the scope of the present paper. The essential feature of removal of y parallax can be accomplished in some standard instruments without modification, as contrasted to the projection plotter, which requires a special prism. Thus, because of its greater adaptability, initial tests of radar contouring are quite likely to be made on equipment such as the Wild Autograph. For the purposes of an initial analysis, however, the development and significance of the basic relations are more readily followed by means of the simple ray diagrams of the projection equipment described in this paper.

Notation

A = parameter in approximate analysis of parallax, Equation (7)
b_M = airbase at the scale of the stereo model
b_R = airbase at the scale of the radar photograph
h = elevation of a reflective object or surface
h_M = height of an elevated target at the scale of the stereo model, Equation (54)
h_p = height of the viewing-screen above the datum surface in a projection plotter
h_R = height of an elevated reflector at the scale of the radar photograph, Equation (41a)
H = aircraft altitude above the plane earth
H_M = aircraft altitude at the scale of the stereo model, Equation (54)
H_M' = normalized aircraft altitude, Equation (69)
H_p = normalized height of the viewing-screen, Equation (29)
H_R = aircraft altitude above the plane earth at the scale of the radar photograph, Equation (41a)
L_M = distance between lens of projection plotter and the datum surface of the stereo model, Figure 10
L_R = distance between the lens and the radar film in the projector of the projection plotter, Figure 10
m = magnification of the projection plotter, Equation (16)
m_R = scale of the radar photography, Equation (1)
N = nadir-point
N' = homologous image of a nadir-point
p = parallax vector, Figure 3
R = ground range to a reflective object
S = slant range to a reflective object
(x_M, y_M) = coordinate system in the plane of the datum surface of the stereo model, employed to express the target location, Equations (39), (40), (46), and (47)
(x_p, y_p) = coordinate system on the table of the projection plotter with the origin midway along the projected airbase. Positive x_p is in the direction of flight. This coordinate system is employed to express the location of the viewing-screen.
(x_R, y_R) = coordinate system for the radar photographs, with the origin midway between the nadir-points of two overlapping photographs. The x_R-axis passes through the nadir-points with positive x_R in the direction of flight. This coordinate system is employed for the apparent location of a reflective object, uncorrected for relief-displacement. [When the relief-displacement is removed, the coordinates are identified as (x_{RG}, y_{RG}).]
(x_{RG}, y_{RG}) = the actual target location in the (x_R, y_R) coordinate system, after correction for relief displacement, Figure 3
(X_M, Y_M) = normalized coordinates for the target location in the datum surface

of the stereo model, Equations (64) and (65)

(X_p, Y_p) = normalized coordinates of the location of the viewing-screen, Equations (66) and (67)

(X_R, Y_R) = normalized coordinates of the target in a radar frame, Equations (70) and (71)

$z_R = x_R + iy_R$

Δy_p = half the y parallax, as defined in Equation (34)

ΔY_p = normalized value of Δy_p, Equation (68)

θ = angle of the target in the (x_R, y_R) coordinate system, measured counterclockwise from the positive x axis

ρ = apparent radar ground range at the scale of the radar photograph, uncorrected for relief displacement

Descriptions and Airphoto Characteristics of Desert Landforms

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ABSTRACT: *A brief discussion of desert landforms and their characteristic appearance on aerial photographs is presented. Annotated air photos are included of areas currently being studied at the Terrestrial Sciences Laboratory, Air Force Cambridge Research Laboratories. Air photo interpretation is particularly suited for the assessment of desert surface conditions.*

INTRODUCTION

AERIAL photographic interpretation offers a most effective and efficient means by which terrain studies of desert regions may be undertaken. The Terrestrial Sciences Laboratory of the Air Force Cambridge Research Laboratories is currently conducting such studies on selected desert areas in the Middle East and in the southwestern United States with emphasis placed on geomorphology, structural geology, engineering geology, soils, hydrology and vegetation. In addition to increasing our knowledge of the natural processes at work in desert regions, the present study program should materially aid the Air Force in improving its ability to assess terrain in all environments.

During these studies problems have arisen on the interpretation of surface character of certain desert landforms. It is anticipated that current research in remote sensing will provide the capability to further the interpretability of desert surface conditions.

CAPABILITIES AND LIMITATIONS OF ARID REGION PHOTO INTERPRETATION

Desert lands allow a great amount of information to be gleaned from aerial photoggraphy; more so than any other climatic region. In deserts the barren rock surfaces clearly reveal fractures, bedding and contacts. The general sparsity of vegetation permits runoff to erode gullies that may reflect the grain size of the soil mantle. Vegetation is highly specialized and can provide a clue to the presence of salts or the depth to the water table. Seepage zones and springs reveal their presence by darker tones from increased moisture and resulting denser vegetation. Practically every variation in topography is evident with only a minimum of soil or vegetation to conceal it. The remote southern desert of Israel, the Negev, was mapped recently with aerial photographs. At the end of the program it was stated that this map would not have been possible without aerial photography (Bentor, Y. K., 1952, p. 157).

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