New Aerial Triangulation Techniques Employed on a Mapping Project in Nigeria[*]

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INTRODUCTION

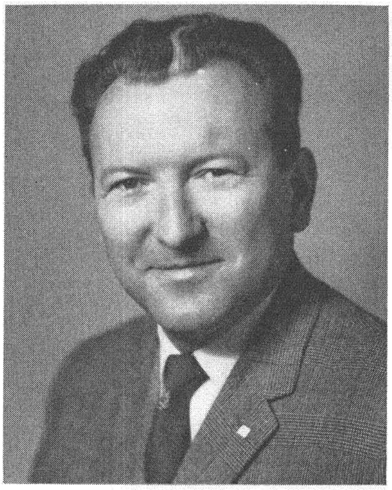
FOR many years, the requirements for ground control have been a major bottle neck in photogrammetric mapping.

The extension of control by the aero-polygon method of triangulation has some inherent limitations imposed by the unfavorable summation of errors. These limitations are present, whether the triangulation he carried out on a first-order instrument, such as the Wild A-7 or a Zeiss C-8, or by employing a Stereocomparator and analytical methods.

This unfavorable summation of error can be eliminated only when auxiliary instruments are used in conjunction with the aerial camera. These provide independent determination of the orientation of the aerial camera at the moment of exposure, and independent determination of a scale of each stereo-model. New auxiliary instruments have been developed recently and old ideas and constructions improved. These developments may effect a considerable reduction of the ground-control requirements in aerial triangulation.

Such auxiliary instruments have been employed recently by Canadian Aero Service Limited on a 37,000 square mile mapping project in Nigeria. The project comprised aerial photography, ground-control surveys, compilation and color separation scribing of topographical maps at a scale of 1:50,000 with 50 foot contour intervals.

This survey was sponsored by the Canadian Government under the Special Commonwealth Assistance Program for Africa, in response to an urgent need by Nigeria for accurate topographical maps. These were required for planning of transmission lines, roads, railways, oil and ore exploration. Other uses included forest inventories and management, agriculture, hydrological investigations, irrigation projects and water transport development of the Niger and Benue Rivers. (Figure 1)



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INSTRUMENTATION USED

Tropical terrain and difficult weather conditons demanded new approaches to the mapping to make the project economically feasible.

After evaluation of available auxiliary instruments, it was decided to employ the following system:

(1) Wild RC-9 aerial camera equipped with a Super-Infragon lens.
(2) Radan-Doppler navigational system.
(3) Wild Horizon Camera and Statoscope.

Principal obstacles to obtaining economical, high-quality photographs were low-lying clouds and Harmattan haze. By employing the superwide angle camera, it was possible to obtain photography at a scale of 1:40,000, while flying just beneath the heaviest layer of

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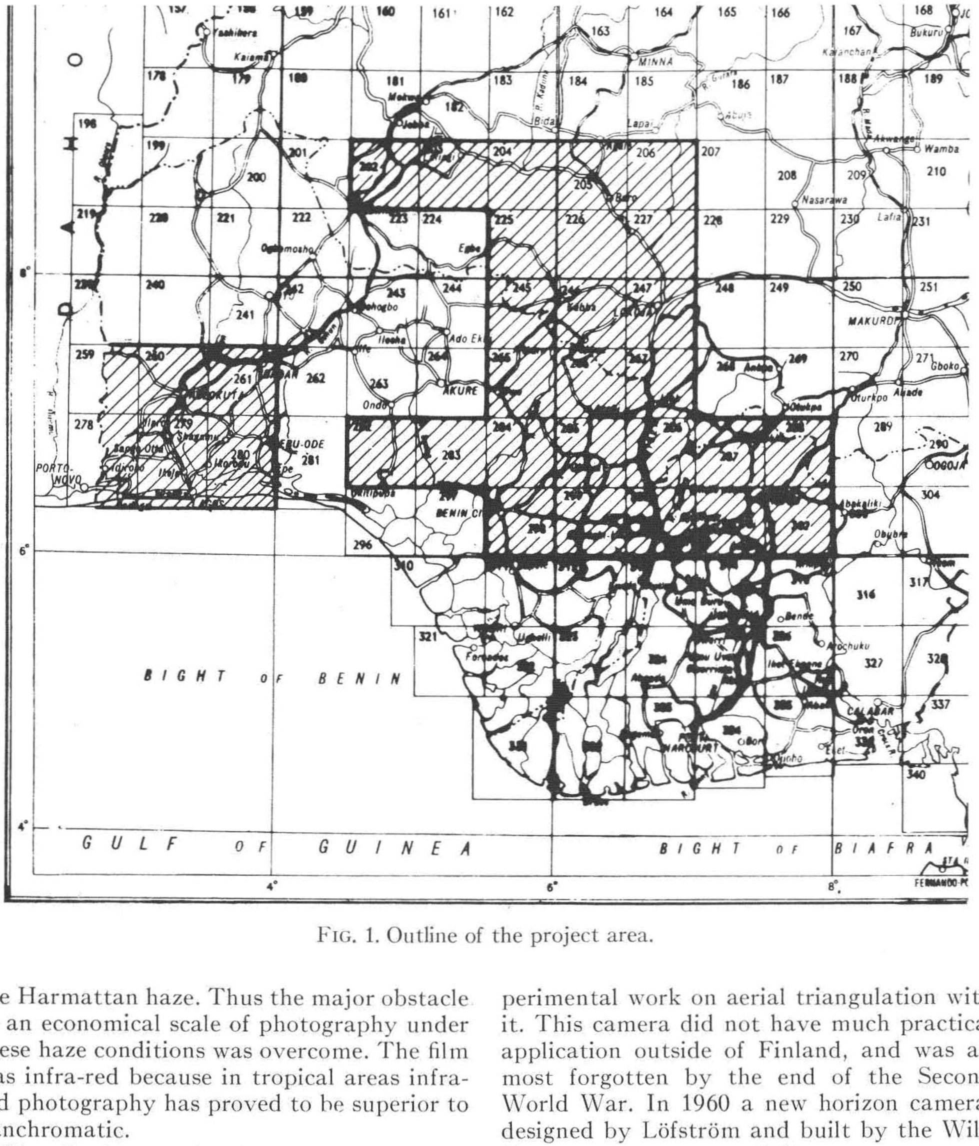

Fig. 1. Outline of the project area.

the Harmattan haze. Thus the major obstacle to an economical scale of photography under these haze conditions was overcome. The film was infra-red because in tropical areas infrared photography has proved to be superior to panchromatic.

The Radan navigation system controls flight in a predetermined direction, the overlap between flight lines, and the fixed distance between exposures. These features ensure accurate navigation and correct scaling of each stereo-model without reference to ground-control. (Figure 2)

The first horizon camera was developed in Finland by Nenonen in 1928 and was built by the Zeiss Company. With this camera the horizon was photographed in only two directions perpendicular to each other. This horizon camera was used almost exclusively in Finland, primarily for rectification of aerial photographs. But v. Gruber, Schermerhorn, Neumaier, Brucklacher and Löfström did experimental work on aerial triangulation with it. This camera did not have much practical application outside of Finland, and was almost forgotten by the end of the Second World War. In 1960 a new horizon camera, designed by Löfström and built by the Wild Company, became available.

The Wild Horizon Camera photographs the horizon in four directions, (forward, aft, left and right), on 35 millimeter film. The relative tips and tilts of the vertical photographs are determined from the horizon pictures, by measuring the difference in the position of the horizon line between a reference horizon picture and the successive horizon pictures (Figure 3). The φ is determined from the forward and aft pictures, and the ω from left and right pictures. It is not necessary to see the true horizon for this determination; any well identified line on the horizon can be used.

This new Wild horizon camera (see Figure 4 and Figure 5) is in the same mount as the

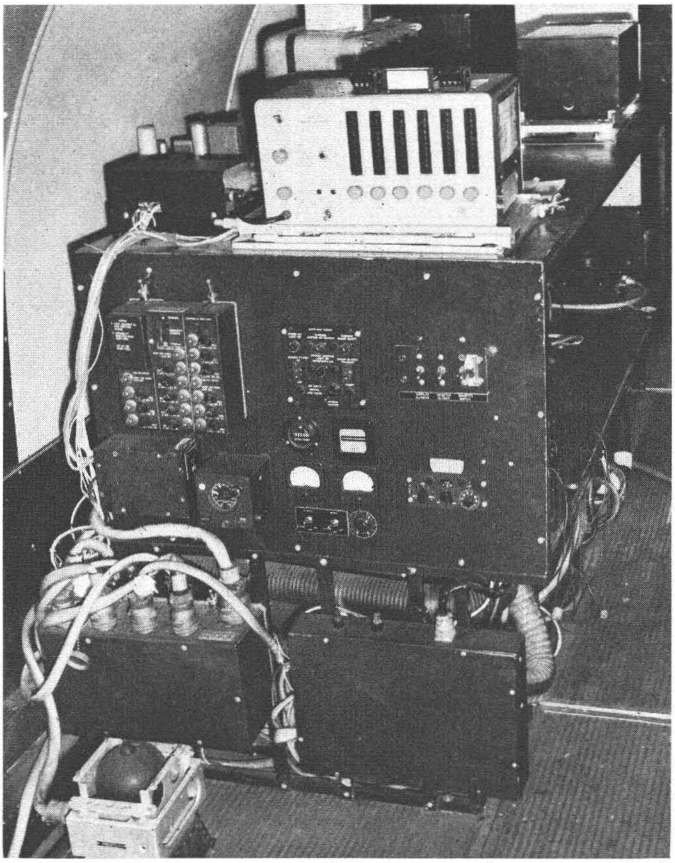

FIG. 2. Radan-Doppler installed in a DC-3 aircraft.

RC-9 camera. A parallellogram linkage system between the RC-9 and the horizon camera assures the swing of both cameras being identical at all times. The horizon pictures are synchronized with the vertical photographs in two ways:

FIG. 3. Typical horizon picture.

(a) By zeroing exposure counters on both the RC-9 and on the horizon camera before each flight. Both counters are operated by the same electrical impulse, and the same exposure number appears on both negatives (9″×9″ and 35 mm.).

(b) By recording the time on both negatives at the moment of exposure. The clocks on the vertical camera and on the horizon camera are synchronized before each flight.

The statoscope registers the difference in elevation between each exposure station relative to a reference barometric altitude. The differences in elevation sensed by the statoscope are registered on the 9″×9″ aerial negatives.

All these instruments have previously been

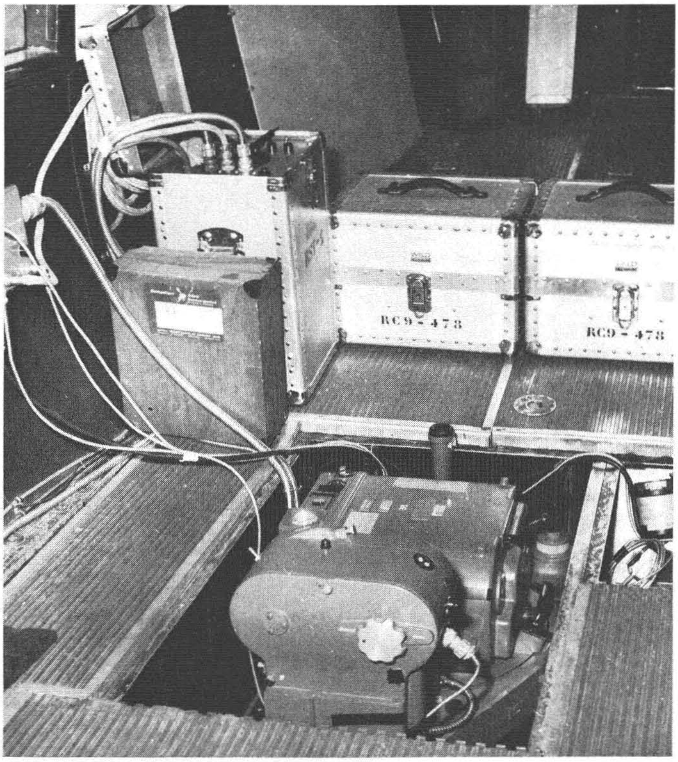

FIG. 4. Wild horizon camera, RC-9 aerial camera and statoscope in a DC-3 aircraft.

used separately. This project was the first time that they had been integrated as a mapping system. The size and difficulties of the project warranted the cost of acquiring and assembling this system.

To take full advantage of the auxiliary data provided by the system, the aerial triangulation had to be approached in a new manner. There was the choice of using the Wild A-9 or a Stereocomparator, or of developing new techniques using the Wild B-8.

Because of shortcomings for project purposes in the use of either the A-9 or the Stereocomparator, there was undertaken the development of a new aerial triangulation method around the Wild B-8 plotter and the

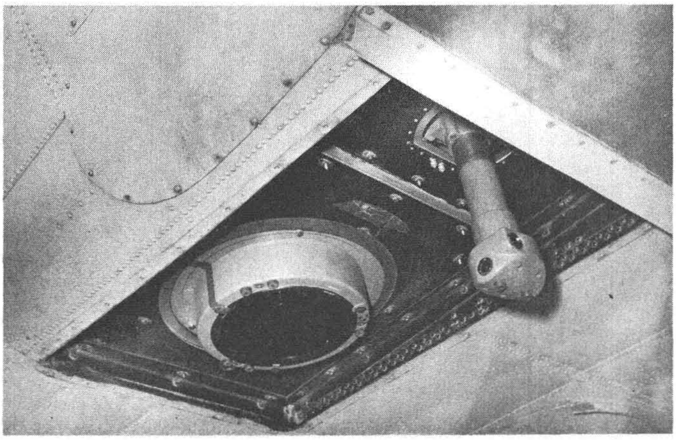

FIG. 5. Outside view of the RC-9 aerial camera and horizon camera.

auxiliary information provided by the horizon camera, statoscope and Doppler. The theoretical evaluation of the accuracies of each of the auxiliary instruments and of the B-8 plotter indicated that accurate aerial triangulation with independent pairs could be done on the B-8 by incorporating the auxiliary data.

Introduction of these data, particularly the horizon data, into the bridging procedure, made unnecessary the double summation of errors. This eliminated the need for vertical control in the center of the bridged strip. Permitted was a one-third reduction of the density of the vertical ground-control, compared with the control required in the aeropolygon method of triangulation.

The introduction of Doppler permits independent scaling of each stereo-model.

The introduction of statoscope in conjunction with horizon camera and Doppler permits the transferring of the vertical datum by means of computed flying height to any stereo-model of the same strip without instrumental triangulation. (Figure 10).

This aerial triangulation method employs the principle of bridging with independent pairs. Since Doppler triggers the aerial camera at predetermined distances, the base between two consecutive stereograms is known and is introduced into the B-8. Therefore, the scale of each model is determined from auxiliary information and does not depend upon ground-control or a photogrammetric process. The absolute orientation of the stereomodel is determined by the ϕ and ω derived from the horizon pictures; hence it is independent of any other stereo-model of the line. The vertical datum can be transferred by computing H (the flying height above a vertical datum) for each exposure with the help of statoscope data.

The instrument used for aerial triangulation is a Wild B-8 stereoplotter which was slightly modified. The modifications consist of the following:

(a) An extension to the plate holders to permit placing precise L-shaped bubbles.

(b) Adding specially designed L-shaped bubbles to allow accurate introduction of the tip and tilt in each camera (Figure 6). The attachment consists of two 20-second bubbles placed on a stable mount in the shape of letter L. The bubbles when placed on the plate holder are parallel to the x and y axis of the $9'' \times 9''$ diapositives. Each bubble can be tilted in the vertical direction. A drum is attached to the vertical screw to permit direct readings of ϕ and ω to an accuracy of 1^c (30 seconds). The B-8 is very carefully calibrated, using precise grid plates to determine the zero readings for ϕ and ω for both the left and the right plate holders and also to determine the zero value for general Φ.

AERIAL TRIANGULATION PROCEDURE

SELECTION OF PASS POINTS

The selection of pass-points in areas covered with rain forest and in areas without much clear planimetric detail is extremely difficult. The transferring of pass-points between flight lines can be a considerable source of errors if the utmost care is not exercised. To achieve maximum accuracy in the selection and transferring of pass-points, they are pre-selected and pricked on the glass diapositives, using the Wild PUG 2 point transfer device.

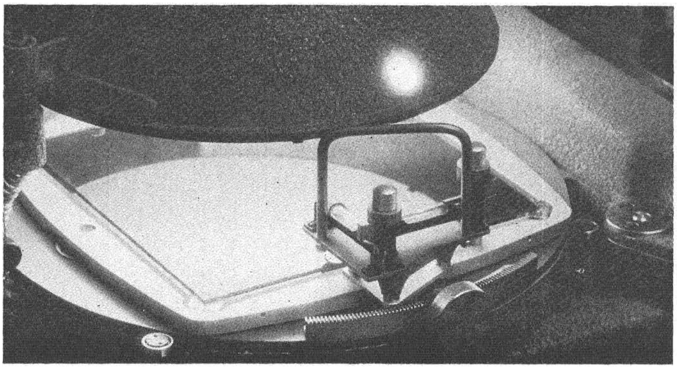

FIG. 6. L-shaped bubbles on the B-8 plate holder.

MEASURING OF HORIZON PICTURES

The horizon pictures are measured using a Wild stereomicroscope (Figure 7). Only the relative differences in ϕ and in ω are determined between individual photographs. The horizontal parallaxes of the central fiducial marks (cross $R_1R_2R_3$) and the horizontal parallaxes of the adjacent points on the horizon line ($h_1h_2h_3$) in corresponding horizon pictures are stereoscopically measured by using the floating mark, Figure 8. The differences of these parallax readings R_1-h_1, R_2-h_2, R_3-h_3, express the displacement of the horizon line in the measured horizon picture in relation to the horizon line in the corresponding reference horizon picture.

The equation:

$$\gamma = \frac{(R_1 - h_1) + (R_2 - h_2) + (R_3 - h_3)}{3} \cdot \frac{\rho^c}{f}$$

gives the relative difference in inclination ($\Delta\phi$, $\Delta\omega$) of the aerial camera between the measured picture and the reference picture.

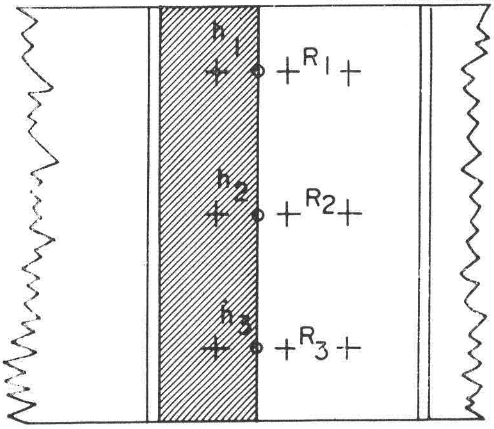

FIG. 8. Principle of the measuring of the horizon pictures.

DETERMINATION OF ABSOLUTE ϕ AND ω

To determine the absolute values of ϕ and ω it is necessary that at least one model in each flight line be fully controlled in elevation. The base distance given by Doppler is introduced into the B-8, and this model is levelled to the ground control. The ϕ and ω of the left and of the right pictures of the controlled model are read on the drums of the L-shaped bubbles. Then the absolute ϕ and ω of any subsequent photographs in the strip can be computed by adding to the absolute values the differences $\Delta\phi$ and $\Delta\omega$ determined from the horizon pictures. Vertical control is provided for the first and the last model of each flight line in order to check the determination of absolute ϕ and ω. The results are considered to be correct if a closure is obtained with an accuracy equivalent to that of the absolute orientation of the model.

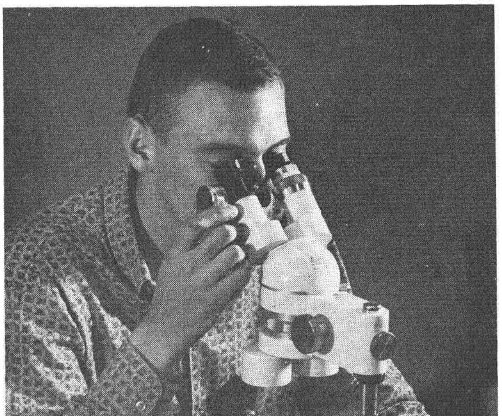

FIG. 7. Wild stereomicroscope used for measuring horizon pictures.

INSTRUMENTAL PROCEDURE

The absolute orientation of the stereomodel can be determined either by introducing the ϕ and ω on the left plate-holder or on the right plate-holder of the B-8 plotter. The absolute orientation of the model will differ depending on which is used, because of errors in relative orientation and errors in the horizon camera readings. General Φ, that is the inclination of the base, can also be determined from statoscope data. However, the experience has proved that a more accurate and more reliable determination of general Φ can be made using the ϕ determined from horizon pictures in conjunction with a relative orientation procedure, than by using the statoscope data. The statoscope data are used to transfer the vertical datum within a strip from one exposure station to another exposure station.

The aerial triangulation by the independent

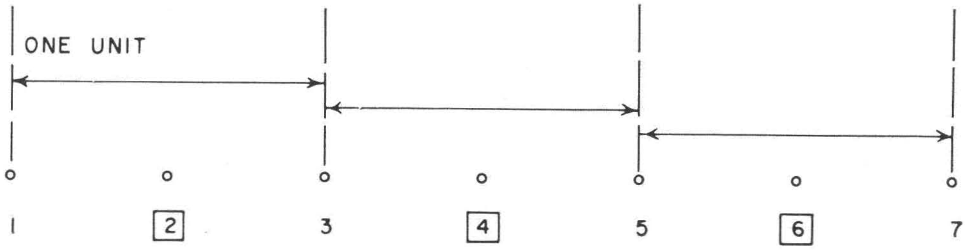

FIG. 9.

pairs methods can be carried out basically in three ways:

(a) The absolute values for ϕ and ω are introduced on every second vertical aerial photograph. Assuming that the ϕ and ω are introduced on all "even" pictures, that is, on picture #2 (for Models 1–2 and 2–3) and on picture #4 (Models 3–4 and 4–5), the absolute orientation of picture #3 will be determined twice, first based on picture #2 and second based on picture #4 (Figure 9). The difference between the two determinations of ϕ and ω gives an indication of the accuracy of the horizon data and of the accuracy of relative orientation of both models. These differences are also used in strip adjusting.

(b) The absolute value for ϕ and ω are introduced in the B-8 first on the left, and then on the right aerial photograph (plate holders). Then the average value for ϕ and ω for the left and for the right photograph is computed from both these determinations, and introduced in the plotting instrument to establish the absolute orientation of the stereo-model. The method (b) of determining absolute orientation produces somewhat better results. Its main advantage is that it minimizes the effect of the inaccuracies of the relative orientation on the absolute orientation. The adjusting is also simpler when method (b) is employed.

(c) When the statoscope data are combined with the data derived from the horizon pictures and Doppler, it is possible to plot any model within a strip without the necessity of bridging the entire strip. Figure 10 shows the principle of this method. Assuming that one model of a strip has sufficient control to determine the elements of absolute orientation, there can also be determined the flying height above a vertical datum of each individual aerial photograph within a strip. We know then the following elements of absolute

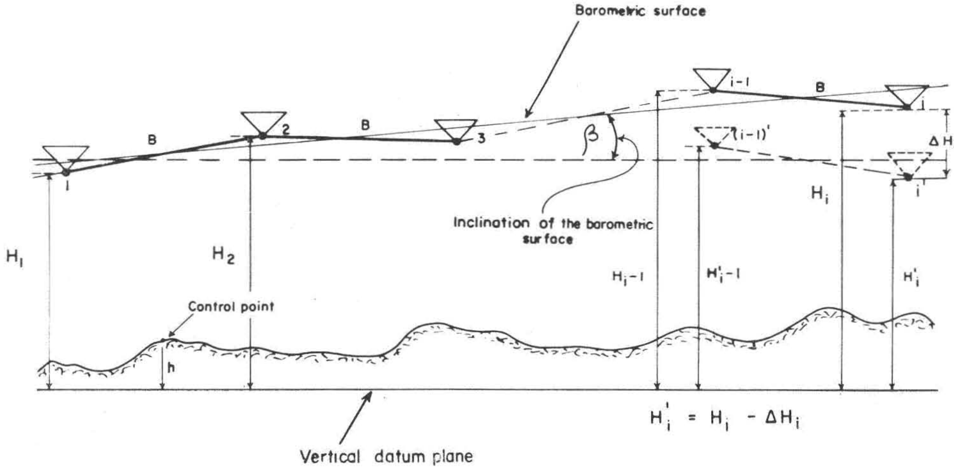

FIG. 10.

orientation of each stereo-model within a strip:

(1) Tip and tilt determined from the horizon camera.

(2) Flying height relative to a vertical datum determined with the help of statoscope data.

(3) The base distance between each exposure given by Doppler; consequently the scale.

The slope of the barometric surface can be determined either by taking a number of meteorological observations during the survey flight, or by having a known ground elevation at the end of the flight line and comparing the computed flying height with the one determined from the ground control (Figure 10).

The above information permits plotting any model within a strip at a correct scale, and a correct vertical datum without the necessity of bridging the entire strip in a stereoplotting instrument.

GENERAL REMARKS ON ADJUSTING PROCEDURES

The adjustment of strips when auxiliary data are introduced in the aerial triangulation procedure must be approached differently from that of the aero-polygon method. In that method, the adjusting procedure revolves around elimination of large errors in elevation caused by second summations of accidental and systematic errors, and also by the influence of the earth curvature. This is usually accomplished by means of second and third-order functions. The variations from this ideal function of errors are not taken into account although they undoubtedly exist.

When auxiliary data are utilized and aerial triangulation is carried out with independent pairs, the propagation of errors is quite different and generally follows a linear function. The double summation of errors does not take place; therefore, vertical control in the center of a strip has no real value. The closing errors before adjusting (Table 1) are considerably smaller than when the aero-polygon method is employed. However, there are, within a strip, undulations or local irregularities caused by inaccuracies in the auxiliary data and in the relative orientation. These must be considered in the adjusting procedure regardless of what types of auxiliary data are used.

Therefore, any method of adjusting strips triangulated with the help of the auxiliary data must provide means of eliminating these undulations in the surface of errors.

TABLE 1

Line	*Number of Stereo-Models Between Control Ground*	*Closing Errors in Elevation Before Adjusting (in Feet)*
43E	18	−14
44E	18	−24
45W	19	+ 2
46E	13	0
47W	14	+86
48W	14	+80
49W	12	+54
50E	17	−27
50AE	14	− 9

The information at our disposal for the strip and block adjustment is:

1. Closing error in elevation in the last stereo-model of the strip.
2. The difference in ϕ and ω determined for the common photographs from two adjacent models (for photograph i, from Model $i-1$, i and Model i, $i+1$).
3. The differences in elevation on common pass-points between adjacent strips.
4. The reliability of auxiliary data.
5. The general "behaviour" of the strip during aerial triangulation.

BLOCK ADJUSTMENT

The vertical block adjustment would be a relatively simple operation if it could be assumed that each strip is free from the influence of residual errors in ω. In this case, the discrepancies in elevations on the common pass-points between the adjacent strips could be adjusted by fitting in an average datum. However, there is always some ω influence left in each strip.

The discrepancies in elevation observed at the common passpoints between adjacent strips are mainly due to the following causes:

1. Residual error in ω direction.
2. Residual undulations of the strip in the φ direction or "datum" differences.
3. Errors in stereoscopic reading of the elevations of the pass-points.

   (NOTE: It was the experience on this project that the elevations of the outside pass-points could be read stereoscopically in the order of ±5 feet. However, in certain cases when the two adjacent flight lines were flown some weeks apart, this increased to ±8 feet. The reason for this, it was discovered, was that the vegetation grew several feet in the time interval between flights.)

4. Errors due to transferring of pass-points between flight lines.

Separation of these different errors is extremely difficult, if not impossible.

The block adjustment was done graphically. This permits a quick evaluation of differences between strips. However, the Jerie method of block adjustment could be employed to advantage. There is then a block comprised of independent stereo-models.

Extension of Horizontal Control

The horizontal control is extended by employing the well known stereotemplate method. This method is simple, permits quick evaluation of the quality of horizontal control, and produces excellent results.

Accuracy of the Method

To determine the accuracy of this method under the conditions of the project, a test area was established in Nigeria. This area measured approximately 34×34 miles and comprised a block of four 1:50,000 sheets. It was covered by nine flight lines of photography, averaging 15 stereo-models for each line. Ground-control was established at both ends of each line. In addition, 97 vertical points were established throughout the test area to provide a basis for evaluating the accuracy of of the above described method of aerial triangulation. The vertical control was established by third-order levels and stadia to an accuracy of not less than ±5 feet.

ACCURACY OF AUXILIARY DATA

The first test consisted of evaluating the the accuracy of φ and ω determined from horizon pictures, and the accuracy of the statoscope and the Doppler. To perform this evaluation a number of stereo-models were set-up using the ground-control. The orientation elements φ and ω for the right and left plateholder in the B-8 were then registered using the L-shaped bubbles. These elements were then compared with the ϕ and ω determined from horizon pictures. Also, the general Φ determined from the ground control set-up was compared with the general Φ determined from the statoscope data. The comparison established that the mean square error of the elements of absolute orientation ϕ and ω determined from horizon camera data and from statoscope (Φ) are as follows:

$d\phi = \pm\, 4^c (\pm 2')$ based on 30 comparisons

$d\omega = \pm\, 6^c\ (\pm 3')$ based on 35 comparisons

$d\Phi = \pm\, 7^c\ (\pm 5')$ based on 19 comparisons

The above errors are of the same order as the errors of the orientation elements resulting from errors in relative and absolute orientation of a stereo-model. The slightly higher error in ω is due to an instability of the Wild B-8 in this direction of approximately $\pm 2^c$. This test shows also that the general Φ can be established more accurately by using the horizon pictures and a relative orientation procedure than by using the statoscope.

The results obtained to date indicate that 95 per cent of the base distances determined by Doppler are accurate within ±0.3%. This was determined by comparison of the scale derived from the Doppler data with the scale from the stereotemplate laydown.

ACCURACY OF AERIAL TRIANGULATION

Each strip was adjusted separately utilizing vertical control at the beginning and at the end (at 15 models intervals). The residual errors in elevation after block adjusting based on 97 vertical controls points spread throughout the test area are as follows:

87% within ± 10' (1/5 of the countour interval)

92% within ± 12.5' (1/4 of the contour interval)

98% within ± 16.5' (1/3 of the contour interval)

There was one point in error by 18 feet, and another point by 21 feet. The mean square error in elevation is ±6.8'. It is of the same magnitude as the vertical reading accuracy, which under tht project conditions proved to be ±5'.

Conclusion

The results achieved to date indicate a great potential for the mapping system employed on this Nigerian project.

(1) The superwide angle infra-red photography has demonstrated that it is possible to obtain good quality pictures at an economical scale for 1:50,000 mapping in tropical areas, under adverse haze conditions.

(2) The horizon camera permits determination of the tip and tilt of the aerial camera at the moment of exposure with the same order of accuracy as the accuracy of relative orientation.

(3) The aerial triangulation can be carried out on a simple and relatively inexpensive instrument, such as a Wild B-8 plotter.

(4) The mapping system is independent of

the terrain (unlike the A.P.R. system).

(5) The density of the vertical-control is reduced by one-third as compared with the aero-polygon method.

(6) Because there is no rigid requirement regarding the location of the ground-control within a strip, such control can be established at points where the access is best and where good photo identification can be accomplished.

(7) The aerial triangulation need not be carried out in strips. It is possible to bridge across the block in any direction.

(8) The use of horizon camera, Doppler and statoscope developed a new concept in vertical aerial triangulation. It is no longer necessary to bridge the entire strip in a stereoplotting instrument in order to plot a number of selected stereo-models. Any stereo-model within a strip can be plotted independently because the scale is determined from the Doppler, the tilt and tip from the horizon camera, and the vertical datum from the statoscope.

This feature is of particular significance if maps of selected areas are urgently needed for special projects. They can be compiled before a whole block of map sheets is triangulated and adjusted.

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1963 Semi Annual Meeting

Time: Sept. 12–13

Place: Thousand Islands Club on Wellesly Island in Thousand Islands Area, New York State

Sponsor: Central New York Region

Chairman of Semi Annual Meeting Committee: Lewis Dickerson

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Technical Papers: Portion of program will be reserved for papers on analytical photogrammetry and its associated instruments, in keeping with Prof. Church's interests in these areas. For balance of program, papers on all phases of photogrammetry and photo interpretation are solicited. Anyone willing to contribute should contact Program Chairman. If offer accepted, prepare in form ready for publication in issue of PHOTOGRAMMETRIC ENGINEERING, following Publications Committee approval.

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