*Analytical Triangulation with Small or Large Computers**

ROBERT A. MATOS,
Photogrammetry Division,
GIMRADA, Ft. Belvoir, Va.

ABSTRACT: *The Corps of Engineers has a requirement to investigate the feasibility of recapturing the original orientation of a camera station and for the determination of supplemental control in sparsely controlled areas by the use of automatic and semi-automatic Analytical Triangulation procedures adapted to electronic computation.*

Two approaches to the Analytical Triangulation problem are now under study and test by the U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency (GIMRADA). The first approach is through the use of a small computer system to triangulate and to adjust individual strips and then by multiple strip adjustment to obtain large block solutions. The second approach employs the use of a large computer system to simultaneously triangulate and to adjust, in a single operation, all overlapping strips comprising a large block problem. Development of both procedures is rapidly advancing through in-house testing and contractual studies coordinated by GIMRADA.

INVESTIGATIONS of analytical aerial triangulation were initiated in 1954 by the Topographic Engineering Department of the U. S. Army Engineer Research and Development Laboratories. This department was later reorganized and integrated into the U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency (GIMRADA), where investigations were continued under the subproject, "Mapping with Minimum Ground Control."

Early results indicated that the usefulness of analytical triangulation was not restricted to conditions of minimum ground-control, but also extended to conditions of abundant control. This is readily apparent when upgrading maps of fair reliability and in handling situations where abundant, but non-standard control is given, such as base-line information or exposure-station orientation data. The full potential of this technique for military use is just beginning to unfold in such applications as improvement of available control, supplying supplemental control, extending control, feature location, supplying orientation data for positioning automatic and semi-automatic map compilation instruments, and for the rapid solution of large numbers and non-standard types of photographs.

It is apparent that one method or one computer system cannot efficiently handle the many situations which will arise. Consequently, for our investigation, each situation is assigned within two general classifications, and this assignment is determined by the computational requirements of the specific problem. For instance, can the particular situation be computed efficiently using a small computer or will it require a large computer system? In this respect, it can be anticipated that the computer would not only be used to make the problem computations, but would also be used for storage and retrieval of basic information and to make decisions as to what basic data are available and what operational groups should be utilized for best results. Although a complete computer application is being considered, this paper will be limited to the developments and actual results accomplished in our investigation of analytical aerial triangulation techniques first if using a small computer system, and second if using a large computer system. But first we must define what we mean by a small or large computer.

The terms "small, medium or large" are of little value in describing electronic computers

* Presented at the 1962 St. Louis ACSM—ASP Meeting.

since, in most cases, they imply physical size rather than operational capability. For instance, the physical size of the basic console of the IBM 650 computer is approximately 10 times larger than the equivalent unit on the Recomp II computer, but the internal memory which is a major factor in determining the magnitude and speed of computer operation, is only one-third to one-half the capacity of the Recomp II computer. For the purpose of this paper, a small computer may be thought of as one in which the internal storage is at least 8,000 to 10,000 10 decimal-digit-words, and the physical size is such that two men can manually transport the complete computer system.

During the past year, tests have been completed of four significantly different techniques of single strip triangulation on a small computer. Two of the methods were coded for the IBM 650 computer, and two were coded for computation on the Recomp II computer.

The system of support instrumentation and computer programs as developed and used in these tests is diagrammed in Figure 1. Basic input data consist of known control, photographic diapositives and camera calibration data. Photo coordinate measurements were made using the Nistri TA/3 stereo-comparator. A computer program was designed to correct the raw X and Y photo coordinate measurements (1) by translation and rotation of the instrument coordinate system to the photographic coordinate system, (2) to apply a film distortion correction based on the calibrated distance between the side fiducials, (3) to apply a lens distortion correction based on the average lens distortion curve, and (4) to assemble the corrected photo coordinate data in the prescribed format for input into the particular triangulation program to be tested.

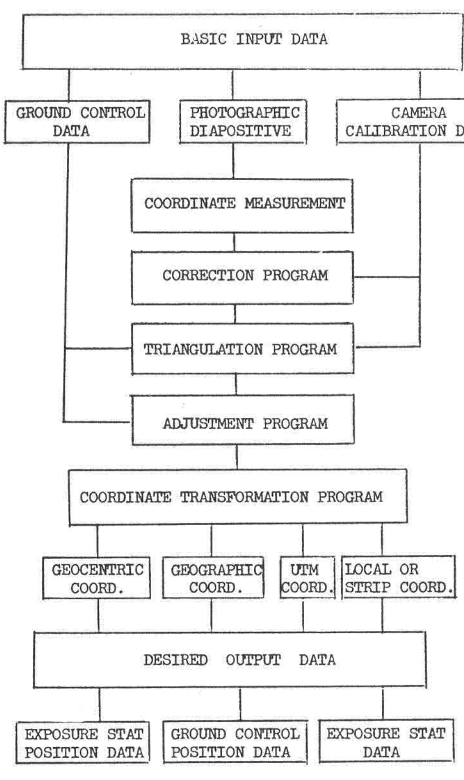

FIG. 1. Analytical system flow summary.

A third computer program was designed to accept the unadjusted coordinate output of the triangulation program, and to adjust these values using a second or third-order curve defined by the available control in the area. In normal operation, it is assumed that three control positions are known at the beginning, middle and end of each strip; however, it is possible to use the program when the known control is not located in the described positions or if more or even less control is available. Finally, the adjusted coordinates are printed out as geographic latitude and longitude with the elevation data given in feet. Through use of additional programs, this output may be converted to Geocentric, Local, or UTM coordinates as desired.

Initial tests of each of the triangulation programs were conducted using fictitious data representing a twelve photo strip flown at 40,000 feet with a six-inch focal-length lens (Figure 2). Up to twenty-five points were intersected on each photograph. By comparison, each of the tested triangulation methods resulted in very nearly the same absolute accuracy with a Root Mean Square Error of less than two feet in horizontal position and less than five feet in vertical position. No attempt to adjust these results was made since the error could be assumed to represent a minor systematic build-up attributable to round off during the computer computation and associated coordinate transformations.

Additional tests using fictitious and real data were made to test the effect on the final results of varying the known control input configuration and increasing the number of pass-points per photo. It was found that if a minimum number of controls were used at the beginning of the strip, better results could be obtained in later standard adjustment procedures than if all input controls were scattered throughout the entire length of the

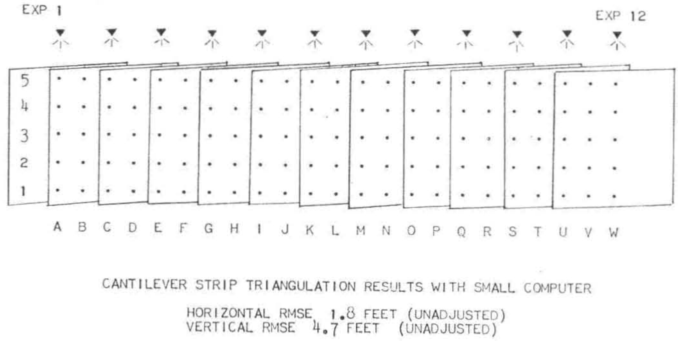

FIG. 2. Twelve photo fictitious data test block.

strip. It is believed that this was caused by the larger systematic errors taking precedence over the smaller random errors during strip triangulation, resulting in an uninterrupted error build-up forming a smooth curve for later matching with the second or third-order adjustment curve. When the known control was spread throughout the strip, the resulting curves were irregular and did not adjust as well using the standard techniques.

It was also found that a minimum number of nine pass-points per photo or six points per model were adequate to control the triangulation; the only requirement being that the measured points selected be the best available points in the model area and that the operator error be minimized through repeated measurements and other procedures to obtain the best possible point coordinate data for each set of points measured. No significant improvement in the triangulation was experienced when using up to 30 points per photograph.

From these investigations, it was found that if absolutely correct data are used in the triangulation procedures, the result of the strip computation, regardless of the method used, is more or less exact depending on the sophistication of the computer program and significant digits maintained during computation. It is also true when using real data, that the accuracy obtained by all methods tested is very much the same. The apparent major difference between methods was not in the accuracies achieved but in their speed of convergence. Also, it was found that the major portion of this speed difference could not be attributed to the difference in the computers or coding of the methods, but was a distinct factor of the mathematical technique used for triangulation. A method developed by Mr. Schut of the National Research Council proved, on the average, to be approximately 5 to 10 times faster than the other methods tested, requiring approximately two minutes-per-photo for triangulation and adjustment.

Typical real data bridge triangulation results from all methods tested are shown in Figure 3.

Further analysis of all real data results obtained by the various methods investigated were made in order that a more direct comparison could be made with expected triangulation results from Multiplex and higher order triangulation instruments. The statistical index of accuracy utilized for this evaluation was the standard deviation values for horizontal and vertical positioning of each run. The horizontal index was computed as the square root of the product of the horizontal standard deviations, and was expressed as a fraction of the distance extended between known control positions. The vertical value was reduced to a fraction of the nominal flight-height. The overall average of all strips had resultant planimetric errors (expressed as a fraction of distance extended) ranging from 1/2250 to 1/3800. For similar type runs, an expected Multiplex average value would be 1/1400, ranging possibly from 1/700 to 1/2100. The overall average of vertical position errors for the runs varied from 1/390 to 1/1120 (expressed as a fraction of flight altitude). Expected accuracy for Multiplex is 1/200. In these tests the horizontal accuracy of even the poorest run was superior to the expected accuracy of a Multiplex extension.

Figure 4 lists the results of three strips, each triangulated using "first-order" plotters and also by analytical techniques. As can be seen in the table, the results of the analytical triangulation compare very favorably with the results of the instrument bridges performed on "first order" analog instruments. The re-

Photography			ϕ		λ		*h*	
Type	*Scale*	*No. of Photos*	*Mean (Ft.)*	*RMSE*	*Mean (Ft.)*	*RMSE*	*Mean (Ft.)*	*RMSE*
KC-1	1:20,000	14	−4	6	2	10	2	3
KC-1	1:20,000	12	−4	11	−6	9	2	4
KC-1	1:20,000	13	−2	4	−1	4	1	3
KC-1	1:20,000	13	0	4	0	6	1	6
KC-1	1:20,000	14	1	5	−2	5	−2	3
KC-1	1:20,000	16	−4	7	0	3	−1	3
KC-1	1:20,000	13	1	3	−5	6	0	5
KC-1	1:20,000	13.6	−1.7	6.2	−1.7	6.6	0.4	4

FIG. 3. Analytical bridge triangulation results with small computer

sults, strictly speaking, are not directly comparable since the instrument bridges extend over 8 models while the analytical bridges extend only 6 models between controls. However, by reducing the 8 models to an equivalent 6 model result, it was found that the resultant value of the overall average was not significantly changed and that the best results of the analytical bridges were still superior to the best results reported for the instrument bridge.

In future testing of analytical triangulation with small computer, it is planned to use the FADAC computer (Figure 5) which was developed by the U. S. Army Ordnance Corps for field computation of artillery problems. This computer will meet the basic storage, speed, and weight requirements previously mentioned in this paper and therefore has been selected for incorporation in an initial field system for analytical triangulation.

NETWORK ADJUSTMENT SMALL COMPUTER

In addition to the single-strip triangulation method on a small computer, a method was developed at NRC for multiple strip adjustment suitable for small computer computation of the large block adjustment problem. This method coded for the IBM 650 was tested by this Agency using a seven strip, 102 photograph block, (Figure 6). The block consisted of KC-1 photography at a scale of 1:20,000 flown over the Arizona Test Area. For the seven-strip network adjustment, seven ground-control points were used as basic-control, and 53 points were used as ties between strips. In all, 150 check points were computed for final analysis.

In the X direction, the errors were distrib-

Triangulation Technique	*No. of Models*	*Flight Line*	*No. of Runs Averaged*	*Elevation Acc*		*Horizontal Acc*	
				RMSE (Ft.)	*RMSE/H*	*RMSE (Ft.)*	*RMSE/H*
C-8 Stereoplanigraph	16	2	3	6.66	1/1502	20.01	1/495
		3	3	7.69	1/1300	13.24	1/752
		4	3	8.78	1/1139	9.82	1/1018
SERIES AVERAGE				7.71	1/1297	14.41	1/694
720 Plotter	16	2	3	7.11	1/1406	16.97	1/589
		3	3	5.81	1/1721	17.52	1/571
		4	3	5.75	1/1739	15.45	1/648
SERIES AVERAGE				6.22	1/1608	16.65	1/601
INSTRUMENTAL OVERALL AVERAGE				6.97	1/1434	15.53	1/644
Analytical	13	2	2	3.5	1/2857	8.25	1/1212
	11	3	2	10	1/1000	10.75	1/930
	12	4	1	3	1/3333	4.5	1/2222
ANALYTICAL OVERALL AVERAGE				5.5	1/1818	7.0	1/1277

FIG. 4. Bridge triangulation results with instruments and analytical techniques

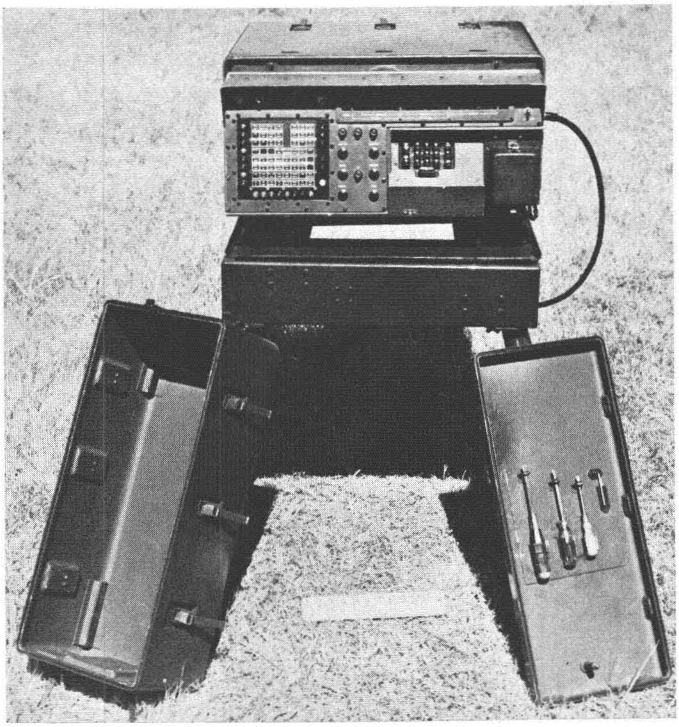

FIG. 5. FADAC computer.

uted about a mean of −1 foot with a RMSE of +5.7 feet. The maximum error in X was 22 feet. In the Y direction, the errors were distributed about a mean of 1.5 feet with a RMSE of +5.6 feet. The maximum error in Y was 27 feet. Twelve iterations were performed, each requiring about 4 minutes or a total machine time of about 50 minutes to complete the entire block adjustment. Further development of this method is planned to include vertical adjustment.

ANALYTICAL TRIANGULATION WITH LARGE COMPUTER

Although it has been shown that it is possible to complete large block solutions with a small computer, a major drawback to this system is the inherent requirement for many control points to obtain a reasonable solution. On the other hand, it was found that the control requirement can be greatly reduced if a simultaneous adjustment of all photos is performed, based on all input controls, in a

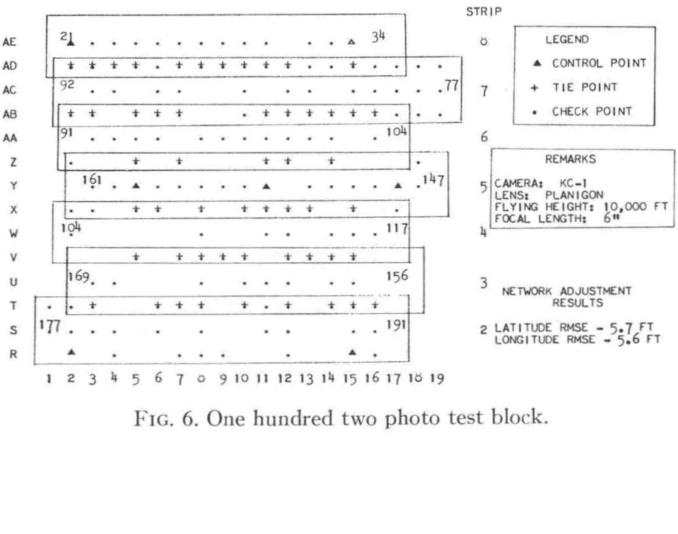

FIG. 6. One hundred two photo test block.

one-shot computation. This procedure would require the use of a large computer having a great amount of storage and extreme speed of computation, which in turn would allow a high degree of sophistication in method and program development. Such is the case of a method currently under development and test by this Agency. The technique is an outgrowth of work initiated in 1958 in a contract with Cornell University.

The original method has been modified and revised in cooperation with the Geological Survey, General Kinetics, Inc., the Massachusetts Institute of Technology and the National Bureau of Standards. The technique nowincorporates the fundamental intersection equations by Herget, supplemental condition equations by Dodge, and a method of individually weighting the input parameters by MIT based on work reported by Brown. The combined method is a simultaneous triangulation and adjustment technique referred to as a General Program for Analytical Aerial Triangulation and Adjustment.

Initial input data to this program as in the small computer methods is derived from sequences of overlapping aerial photographs about which certain information of varying reliability is known. An initial estimate is required for each camera position including latitude, longitude, flying height, x and y-tilt, and heading. The estimates are transformed into an initial orientation matrix representing the spatial relation of each set of camera coordinates. The orientation matrix (3×3) provides the direction cosines of the angles between the photographic and geocentric coordinate axes. The condition equations relating successive camera positions in terms of the rotational and positional axes form a system of simultaneous linear equations which are solved as a part of an iterative process. Iterations continue until the errors between successive values are reduced to the required accuracy level. Condition equations are also included to enforce adjustment to available combinations of the following known control conditions:

a. Complete geodetic stations giving latitude, longitude and elevation, either of exposure stations or ground positions, where each element is considered of equal reliability.
b. Complete geodetic stations either of the exposure station or ground type that are not of equal reliability.
c. Geodetic positions as individual elements either of exposure station or ground control.

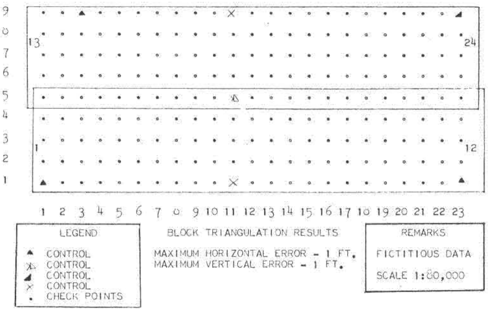

FIG. 7. Twenty-four photo test block.

d. Any combination of geodetic positions either as exposure station or ground control.
e. Points of known equal elevation such as points along shorelines or individual lakes. In all, 17 different types or combinations of given control may be used to position the given photography.

The method was originally coded and tested on the IBM 704 computer but has since been modified to run on the IBM 7090 computer. It is intended that a final system be designed to simultaneously triangulate and adjust single strips or combination of overlapping strips of up to 100 aerial photographs. The present program will solve medium-size blocks of aerial photographs in a reasonable computing time within National Map Accuracy Standards. A few of the more significant investigations and test results experienced with the 704 program are summarized as follows:

Initial tests were conducted to determine the absolute accuracy characteristics of the technique using the previously described fictitious data in strip and block configurations up to a maximum two strip, 24 photo block (Figure 7). The fictitious data problem required 45 minutes of computer time to triangulate, to adjust and to output the final results. The point and control configuration shown resulted in the solution of 207 ground points, each computed to a final accuracy within one foot of its true horizontal and vertical position.

In addition to the fictitious data runs, tests using real photography of both single strips and multiple strips have been completed. One such test was a 15 photo block consisting of three overlapping strips of five photos each taken with the KC-1 camera at 10,000 feet over the Arizona Test Area. Photo coordinate measurements were made using the Nistri Stereocomparator. The problem configuration, shown in Figure 8, resulted in the computation of 21 checkpoints distributed

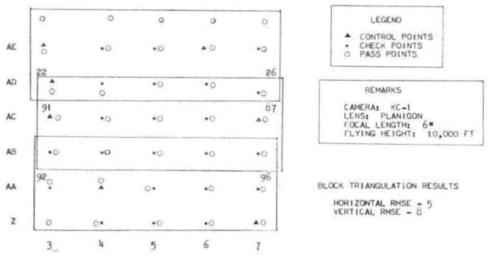

FIG. 8. Fifteen photo test block.

throughout the area with a maximum positional error of 9.5 feet in latitude, 7 feet in longitude and 17 feet in elevation. The RMSE was 5 feet in horizontal position and 8 feet in vertical position.

Another test was made to investigate the feasibility of computing 25 degree convergent photography. This test was made in conjunction with tests of the Halcon Plotter procured by GIMRADA to test the feasibility of the Halcon System. Since the plotter does not provide an aerotriangulation capability, the only procedure available for establishing supplemental control is by radial-templet assembly or by analytical triangulation. This triangulation was performed on a 24 photo block consisting of three strips of eight photographs each. Pass and control points were selected as diagrammed in Figure 9. The selection of points was limited to the identification and measuring of ground-control checkpoints, thus somewhat limiting the accuracy of the final analytical solution to a solution based on the ability of the operator to identify the unmarked ground-control points and to obtain their photo-coordinates. This procedure is not normally recommended for high accuracy requirements; however, it was done in this case as a matter of expediency because of the large number of check-points included. The results of this test are shown in Figure 10.

Based on the results of this test, it was concluded that:

a. Horizontal control can be rapidly and accurately established over a 110-square mile area using 30,000-foot, 12-inch focal-length, Halcon photography with sufficient accuracy to permit the preparation of 1/25,000 and larger scale maps that meet national map accuracy standards.

b. Vertical control can be rapidly computed with sufficient accuracy to establish spot elevations within 50 feet.

c. No inherent weaknesses were apparent in the Halcon system which would serve to limit the effectiveness of extending both horizontal and vertical-control using analytical triangulation procedures.

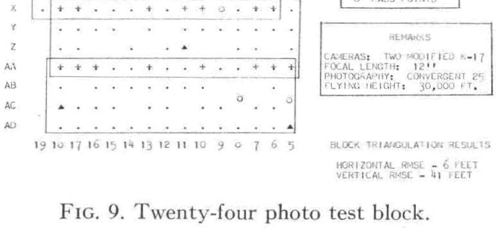

FIG. 9. Twenty-four photo test block.

	Mean Error (Feet)	RMSE (Feet)
Latitude	1	6
Longitude	1	6
Elevation	19	41
		Feet
50% CPE $(1.1774\sqrt{\sigma_x \sigma_y})$		7.06
Min 90% rad $(2.146\sqrt{\sigma_x \sigma_y})$		12.88
50% CPE/Flying Height		1/4249
Max Map Pub Scale		1/7728

FIG. 10. Twenty-five degree convergent photography results

The final test to be mentioned is the most recent test made in this series. This test (Figure 11) was the triangulation of a two-strip, 30 photo block of real photography at 1:20,000 scale, taken over the Arizona Test Area. Only pass-points and selected control positions were used as basic orientation data. In application, this test probably comes closer to an actual field problem than any of the previous tests mentioned. Point coordinate measurements were again made on the Nistri Stereocomparator and initial photo-coordinate correction techniques were applied as previously described. A total of 68 check points were computed. The results of this tri-

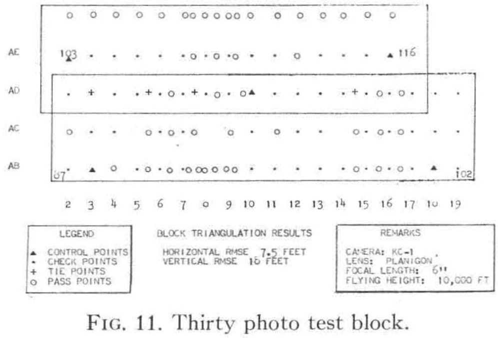

FIG. 11. Thirty photo test block.

angulation are summarized below:

	Max Error (*Feet*)	*RMSE* (*Feet*)
Latitude	16	7
Longitude	26	8
Elevation	47	18

Total 704 computer time required was 50 minutes.

Future development tests of this technique and program will be conducted using the IBM 7090 computer since this computer has replaced the IBM 704's in the Washington area. Eventual application of the block solution technique on a large computer is planned for the Army FIELDATA, INFORMER computer. This computer is one of a family of militarized computers developed by the Army for field use. The computer is van mounted, has paper-tape or keyboard entry and exit, contains 4096 internal core-storage and auxiliary disc file-storage of about three million thirty-eight binary digit words. The speed of computation is roughly equivalent to the IBM 704. The minimum configuration of this computer can be greatly expanded if necessary to include additional storage units of core, drum, or magnetic tapes, and the input-output capability can be expanded for the simultaneous computation of several problems in the same real time frame.

All of our past test results indicate the feasibility of analytical triangulation techniques. Since these results were obtained usually on a one-run basis using prototype techniques, there is reason to assume that greater accuracy can be achieved through effective weighting and special programming techniques, revised methods of photo-coordinate measurement, and new developments in aerial cameras and data acquisition techniques especially designed for analytical triangulation.

Our investigations also indicate that the accuracy of analytical triangulation is not necessarily a basic characteristic of a particular mathematical technique but is a function of the basic input data and original problem configuration; that is, the closeness of conformity of the data to the exact data. On the other hand, a refinement in the mathematical techniques to eliminate unnecessary computation while not necessarily improving the final accuracy may in itself decrease the actual computer computation time which in turn could be a significant factor in the economy of any system's operation. Therefore, GIMRADA is continuing its efforts in this field toward development of two independent systems, one for small computer application and the other for large computer application.

Bibliography

Bodnar, B. J., *Analytical Aerial Triangulation*, USAERDL, First Interim Report, 13 March, 1958.

Collins, J. G., *Test and Evaluation of the Interim Halcon Mapping System*, GIMRADA, Technical Report 3-TR, 11 December 1961 (Appendix B, Exhibit 2 by R. A. Matos)

Dodge, H. F. *Geometrical Foundation for Aerotriangulation*, USGS, December 1959.

Lyon and McNair, *Cantilever Extension of Ground Control in Strips of Convergent Photographs by Analytical Aerotriangulation*, Cornell University, USAERDL Contract No. DA-44-009 ENG 2986, Phase I, Interim Report, 30 January 1958.

McNair and Shu, *Summary of Investigations of Analytical Aerial Triangulation Including an Initial Differential Error Analysis for Vertical Photography*, Cornell University, USAERDL Contract No. DA-44-009 ENG 2435, Final Technical Report, 10 April 1956.

Schut, G. H., *A Program for Analytical Aerial Triangulation by the NRC Method on the IBM 650*, National Research Council of Canada, September 1959.

Orton, J. N. "Aerotriangulation with the LGP-30 Computer," PHOTOGRAMMETRIC ENGINEERING, Vol. XXVI, No. 5, pp. 739–742.