A Mathematical Scheme for Checking the Bias of Relief in Determining Areas from Photo Points

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INTRODUCTION

IT IS known that relief affects the scale of aerial photographs and will affect the results using aerial photographs for determining areas of land.

The dot count is very popular for getting quick estimates of area. The effect of relief has been disregarded because tests by Wilson (2) and others by Moessner (1) do not indicate that the effect of relief in the tests used was serious. But these tests are limited to narrow ranges of elevation, or were tested in areas where land use patterns in higher elevation were of similar portions to those in lower elevations.

Therefore a mathematical scheme for testing these differences in elevation is proposed that will isolate the critical changes in elevation where bias may be a serious factor. Thus specifications can be quickly set up where bias can be expected. More precise methods of determining areas from photo plots can be used when these specifications are exceeded.

DESCRIPTION OF SCHEME

Development of formula for weighting photo plots within and between stereo pairs due to topographic relief.

The adjusted portion of a photo class (F_i) is equal to the weighted photo plots for the forest photo class divided by the weighted photo plots for all classes. Weighting is based upon the square of scale change due to relief. Scale change is based upon average scale for the total area interested divided by the scale of a point. Points having the same scale may be grouped. The development of the formula follows:

1. Scale of a photo point $=P_j/B_b$ where:
   P_j = parallax of a given point in mm.
   B_b = air base in feet between stereoscopic pair.
2. Average scale of all points $=\overline{P}/\overline{B}$ where:
   $\overline{P}$ = average parallax of all points
   $\overline{B}$ = average air base for all points
3. Ratio to scale change from average scale is:

$$\frac{\overline{P}}{\overline{B}} \div \frac{P_j}{B_b} = \frac{\overline{P}B_b}{P_j\overline{B}}$$

4. F_i is a portion of photo points in a photo class divided by the total number of photo points in all classes, thus:

$$F_i = \frac{\sum_{i=1}^{1_i} m_{ib_j}}{\sum_{i=1}^{1} m_{ib_j}}$$

   where m_{ib_j} is total number of photo points for several land classes identified by i subscript and several air base classes identified by b subscript and several parallax classes identified by j subscript.

If there were no relief involved the formula developed in point 4 would furnish valid estimates of portion of land classes. Where relief is involved, then the use of formula 5 would correct for bias caused by this factor.

5. $$\text{Adjusted } F_i = \frac{\sum_{i=1}^{1_i} m_{ibj}\left(\frac{\overline{P}B_b}{P_j\overline{B}}\right)^2}{\sum_{i=1}^{1} m_{ib_j}\left(\frac{\overline{P}B_b}{P_j\overline{B}}\right)^2}$$

6. Since mean parallax and mean air base are common to all products these values cancel: thus formula under point 5 becomes:

$$\text{Adjusted } F_i = \frac{\sum_{i=1}^{1_i} m_{ib_j}\left(\frac{B_f}{P_j}\right)^2}{\sum_{i=1}^{1} m_{ibj}\left(\frac{B_b}{P_j}\right)^2}$$

The *parallax* is measured for each photo plot with an engineer's ruler, parallax wedge, stereoscope bar or stereoscopic plotter, depending upon the precision required. The *air base* is constant for each stereo pair and is measured on a map showing location of photo centers. This map distance in feet times the map scale in RF is the ground distance between photo centers (B) in feet.

Example of Use

An example of the application of this technique is given below using two stereo pairs and 2,000 photo plots distributed by assumed air bases and parallaxes. Tests of the difference in F_i and adjusted F_i are made using the t test for significance. Other assumptions can be made for these same terms and theoretical specifications determined.

EXAMPLE OF COMPUTATIONS USED FOR COMPUTING ADJUSTED F_i AND THE t TEST OF SIGNIFICANCE

Photo class	m_{ibj}	B_b *feet*	P_j *mm*	F_i	$m_{ibj}(B_b)^2 \div (P_j)^2$	adj. F_i	t
1	50	6,800	85		320,000.0		
	50	6,800	90		285,428.0		
	10	6,100	95		41,229.2		
	10	6,100	100		37,210.0		
	120			.0600	683,867.2	.079094	3.983**
2	20	6,800	85		128,000.0		
	30	6,800	90		171,256.8		
	250	6,800	95		1,280,852.5		
	50	6,100	100		186,050.0		
	100	6,100	105		337,561.0		
	50	6,100	110		153,757.5		
	500			.2500	2,257,477.8	.261093	1.150*

(*Continued on next page*)

Photo class	m_{ibj}	B_b *feet*	P_j *mm*	F_i	$m_{ibj}(B_b)^2 \div (P_j)^2$	adj. F_i	t
3	100	6,800	85		640,000.0		
	150	6,800	90		856,284.0		
	250	6,800	95		1,280,852.5		
	150	6,100	105		506,341.5		
	150	6,100	110		461,272.5		
	100	6,100	115		281,356.0		
	900			.4500	4,026,106.5	.465648	1.411*
4	100	6,800	95		512,341.0		
	80	6,100	105		270,048.8		
	200	6,100	110		615,030.0		
	100	6,100	115		281,356.0		
– – –	480			.2400	1,678,775.8	.194162	4.805**
Total	2,000			1.0000	8,646,227.3	.999997	

To give some idea of how much elevation change occurs under this example it can be assumed that the air base is at 6,500 feet and the local-length of camera lens at 152 mm and also that 85 mm parallax is at sea-level elevation. Elevation for other parallaxes are shown below under these assumptions.

Parallax (mm)	*Elevation (feet)*
85	0,000
90	0,647
95	1,224
100	1,744
105	2,214
110	2,642
115	3,033

Literature Cited

(1) Moessner, K. E., 1957. How Important is Relief in Area Estimates from Dot Sampling on Aerial Photos? "Intermountain Forest and Range Experiment Station, U. S. Forest Service, Ogden, Utah. Research Paper 42.

(2) Wilson, R. C., 1949. "The Relief Displacement Factor in Forest Area Estimates by Dot Templets on Aerial Photographs." Photogrammetric Engineering Vol. XV, 225–236.

NEW PUBLICATION BY C. & G. S.

Readers of the analytic aerotriangulation paper in the March 1962 issue and authored by Harris, Tewinkel and Whitten will be interested in learning of the recently issued C. & G. S. Bulletin "Analytic Aerotriangulation." In addition to the published paper the Bulletin contains a revised report of the results and also twelve appendices explaining some of the derivations.