Maximization of Resolution in Photographic Duplication

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ABSTRACT: *A photographic reproduction process which incorporates magnification and minification of the transferred information is investigated mathematically. As a result, it is theoretically indicated that aerial photography can be reproduced by projection through any number of generations and incur only that image degradation which would normally result from one generation duplication. The results of a series of experiments performed to examine the validity of this theoretical mathematical model are presented and the resulting implications discussed.*

INTRODUCTION

FOR many years considerable effort has been expended to increase the resolution of duplicated copies of aerial photographs. It is well recognized that a continuing loss of information is inherent in the process of photographic duplication as progressive generations are reproduced. By using various techniques of reproduction (e.g. point source contact printers), the transfer of information, or resolution, between successive generations of duplicated photography has reached an advanced state. However, the resolution loss remains significant especially in aerial photography where the image contrast is low.

This paper continues theoretical investigations previously begun[1] on the creation of a mathematical model for the duplication process within a system incorporating a number of photo-optical equipments. A mathematical approximation for the resolution transfer of aerial photography by optical projection incorporating magnification is developed. The limiting case of this model theoretically indicates that it is possible to reproduce an indefinite number of duplicate generations and incur only that image degradation which would occur from one generation of such a process. A series of experiments were carried out to investigate this hypothesis.

A THEORETICAL MATHEMATICAL APPROXIMATION

The performance of a photo-optical system in terms of the way in which information, or resolution, is attenuated is commonly approximated by heuristic relationship:[2]

$$\frac{1}{R_T} = \frac{1}{R_1} + \cdots + \frac{1}{R_n} \tag{1}$$

where:

R_T = total resolution

$R_1 \cdots R_n$ = resolution of the individual components of the system.

It must be freely admitted at the outset that this relationship is believed by many authorities to be an over simplification. It is generally considered to be a broad brush treatment of such concepts as the point spread function, molecular cascading, contrast transfer function, etc. These arguments notwithstanding, it is the

[1] E. Yost, PHOTOGRAMMETRIC ENGINEERING, Vol. XXVII, No. 5 (1961).
[2] A. H. Katz, *J. Opt. Soc. Am.*, 38: 604 (1948).

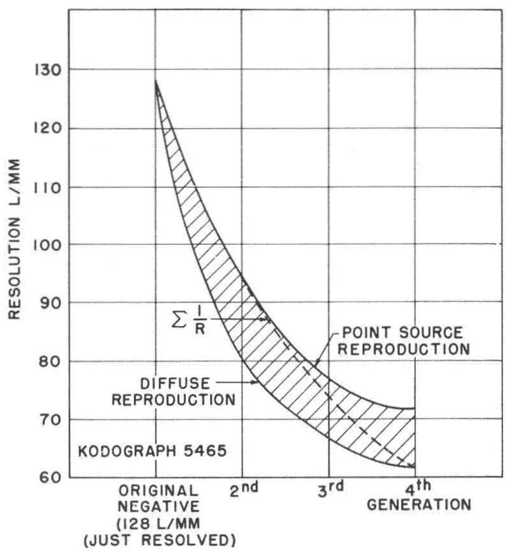

FIG. 1. Reproduction of Resolution Targets Through Four Generations.

contention here that this heuristic transformation is a sufficient approximation to the reproduction of low-contrast, high-resolution aerial photography to allow analysis of the photo duplication process.

In order to investigate this contention, data recently published[3] is presented in graphical form in Figure 1. This figure is a composite of data obtained from a simulated aerial photograph containing low contrast resolution targets. The contrast transfer function (*CTF*) in both photographs was such that in each case the last microdensometrically perceivable target was 128 l/mm.

Upon examination of Figure 1, it can be seen that when the low-contrast image is contact printed on a point light source printing, frame, a higher resolution is achieved than reproduction using a diffuse contact-reproduction process. The analytical approximation of these curves using equation (1) shown as the $\sum 1/R$ dashed line. A "reasonable" approximation between this curve and the area of resolution variability of the two reproduction processes can be seen.

Thus, given that Equation (1) represents a "reasonable" approximation for the attenuation of information for the photographic duplication process, it is possible to theoretically extend this concept to an optical projection system incorporating variable magnification.

For a lens, the angle of a point source subtended by the airy disc is

$$\theta = \frac{1.22\lambda}{D}$$

where:

θ = the angle subtended (in radians)
λ = the wavelength of light used
D = diameter of the aperture

Since

$$\theta = \frac{1}{RF}$$

where:

R = resolution element
F = focal-length

we can write

$$\frac{1}{R} = \frac{1.22\lambda F}{D}$$

For a light of specific wavelength, it is convenient to write the above equation as:

$$\frac{1}{R_T} = K\frac{F}{D}$$

[3] A. W. Berg, *Phot. Sci. Eng.*, 5: 321 (1961).

But since D/F is in reality the ratio of exit pupil to image distance, it is possible to write

$$\frac{D}{F} = \frac{1}{\text{f/no. } (m + 1)}$$

where m, is the magnification, thus accounting for the change in effective F number as the image object-distance varies.

From the above it follows that an estimate information transmission (resolution) of a lens approaching the defraction limit can be expressed as:

$$\frac{1}{R_T} = \frac{(m + 1)}{R} \tag{2}$$

where:

R = measured resolution with an ∞ object-distance (and as such includes the constants of the preceding equations).

Considerations of the information content of the projected image due only to the effect of scale change, makes it intuitively obvious that the information content is indirectly proportional to the magnification, or

$$\frac{1}{R_T} = \frac{m}{R} \tag{3}$$

Thus, if the resolution content of an image is considered (e.g. projected through a fixed magnification), from Equation (1) it is seen that the resolution on the image side of the lens would be:

$$\frac{1}{R_{\text{object}}} + \frac{1}{R_{\text{lens}}}$$

and that the information density in terms of line-per-millimeter resolution decreases with an increase in scale. Thus, if the image is magnified twice, the information content of the original input is spread over twice the distance in any given direction in the image.

Finally, the recording emulsion itself can be considered to follow the relationship expressed in equation (1), that is:

$$\frac{1}{R_T} = \frac{1}{R} \tag{4}$$

Therefore, it is possible to rewrite Equation (1) for the case where magnification is incorporated and one step reproduction is performed, as follows:

$$\frac{1}{R_T} = \frac{M}{R_a} + \frac{M + 1}{R_b} + \frac{1}{R_c} \tag{5}$$

where:

R_a = Resolution of non-optical elements involved in magnification
R_b = Resolution of optical elements involved in magnification
R_c = Resolution of element not being magnified
M = Magnification

Equation (5) can be expanded to cover the general case where a number of successive generations are duplicated by:

$$\left\{\left[\left(\frac{m_1}{R_a}+\frac{m_1+1}{R_b}+\frac{1}{R_c}\right)m_2+\frac{m_2+1}{R_b}+\frac{1}{R_c}\right]m_3+\frac{m_3+1}{R_b}+\frac{1}{R_c}\right\}m_4\cdots \tag{6}$$

where:

$$m_1m_2m_3m_4\cdots = M$$

This relationship can be expressed in more concise mathematical terms as a general expression for the transformation in a photo-optical system, incorporating magnification, for any number of duplicated generations as:

$$\frac{1}{R_T}=\left(\frac{1}{R_a}+\frac{1}{R_b}\right)\prod_{k=1}^{n} m_k+\left(\frac{2}{R_b}+\frac{1}{R_c}\right)\sum_{j=2}^{n}\prod_{k=j}^{n} m_k \tag{7}$$

Even remembering that such an equation is only an approximation of the results to be expected, it is possible for such a "rough," mathematical expression for a 3-step contact-printing from an original negative (assuming a *CTF* representative of aerial photography). Thus, from Equation (7) we have:

$$\frac{1}{R_T}=\frac{1}{R_a}+\frac{3}{R_c} \tag{8}$$

Since no magnification is involved, $m=1$, and since no optical projection is used, the R_b does not appear. By substituting values of 128 1/mm for the resolution of the original photo (R_a) and 350 1/mm for the resolution of the film (R_c) a curve identical with the dashed curve $\sum 1/R$ in Figure 1 can be obtained.

If the same input was projection printed, an estimate of the resulting resolution would be:

$$\frac{1}{R_T}=\frac{1}{R_a}+\frac{6}{R_b}+\frac{3}{R_c}$$

where R_b is approximately 1600/f/no. in the case of a diffraction limited lens. Even if an f/2 diffraction limited lens is used, it can be seen that the output resolution must be less than contact printing.

We now determine the effect of magnification upon Equation (7) by examining optical projection with a manipulation of the variable of magnification. For the previously considered case of three successive generations from an original negative, Equation (7) can be expanded as follows:

$$\frac{1}{R_T}=\left[\left(\frac{m_1}{R_a}+\frac{m_1+1}{R_b}+\frac{1}{R_c}\right)m_2+\frac{m_2+1}{R_b}+\frac{1}{R_c}\right]m_3+\frac{m_3+1}{R_b}+\frac{1}{R_c}$$

$$=\frac{m_1m_2m_3}{R_a}+\frac{m_1m_2m_3}{R_b}+\frac{m_2m_3}{R_b}+\frac{m_2m_3}{R_c}+\frac{m_2m_3}{R_b}+\frac{m_3}{R_b}+\frac{m_3}{R_c}+\frac{m_3}{R_b}+\frac{1}{R_b}+\frac{1}{R_c} \tag{9}$$

where the total magnification $M=m_1m_2m_3$, the magnification performed in each successive generation.

The above equations reduce to:

$$\frac{1}{R_T}=\frac{M}{R_a}+\frac{M}{R_b}+\frac{M}{m_1R_b}+\frac{M}{m_1R_c}+\frac{M}{m_1R_b}+\frac{M}{m_1m_2R_b}+\frac{M}{m_1m_2R_c}+\frac{M}{m_1m_2R_b}+\frac{1}{R_b}+\frac{1}{R_c}$$

But if the constraints of the problem are such that scale of the photograph in the last generation is the same as the initial photo, $M=1$. Simple inspection indicates that as the magnification of the first generation (m_1) approaches a very large number, all terms with m_1 in the denominator become quite small and these terms can be neglected. Thus the specific limit of the above equation and the general limit of Equation (7) for the above constraints can be written as:

$$\frac{1}{R_T} = \frac{1}{R_a} + \frac{2}{R_b} + \frac{1}{R_c} \tag{10}$$

It should be noticed that this equation expresses the identical relationship achieved with one generation projection duplication.

As long as the resolution of optical systems (R_b) is appreciably greater than the resolution of films (R_c), projection printing with very large magnification in the first generation, Equation (10), will theoretically produce higher information-content (greater resolution) than will contact printing devices, Equation (8).

Experimental Evaluations

A series of experiments were conducted to empirically investigate the relationship of the variable of magnification and resolution within the photo-optical process. The most readily available apparatus with defraction limited resolution and precise focus was a Swift microscope mounted on an optical bench. The expected relationship of the variables for one configuration of this apparatus is shown in Figure 2.

A photographic negative containing 143 lines-per-millimeter resolution and having a contrast transfer function considered to be representative of "medium" altitude aerial photography was used as the first generation. Current United States Air Force practice was utilized duplicating through three additional generations; that is, starting with a first generation negative and proceeding to the fourth generation positive.

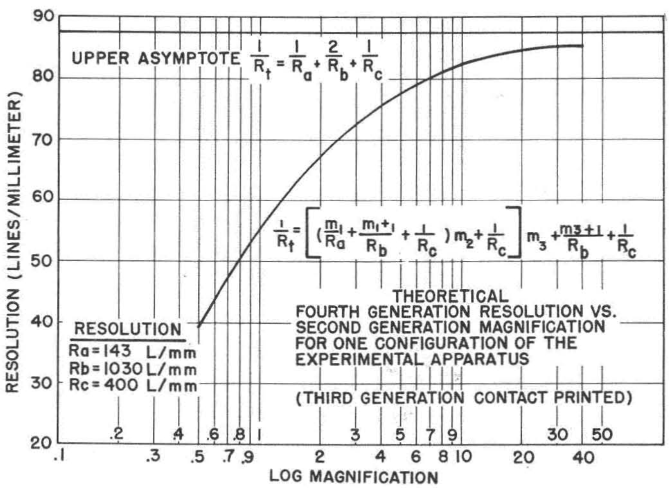

FIG. 2. Relationship of Resolution and Magnification Showing the Upper Asymptote.

The experimental controls were performed at the outset and consisted of two parts. Initially the first generation negative was contact printed through four generations using a diffused light source. Following this, the same first generation negative was projection duplicated through the fourth generation using an equivalent f/10 defraction limited objective. After each generation, the image-object planes were reversed to avoid any slight scale change in the final image. The results of these controls are shown in Figure 3.

The first portion of the experiment consisted of projection printing a second gen-

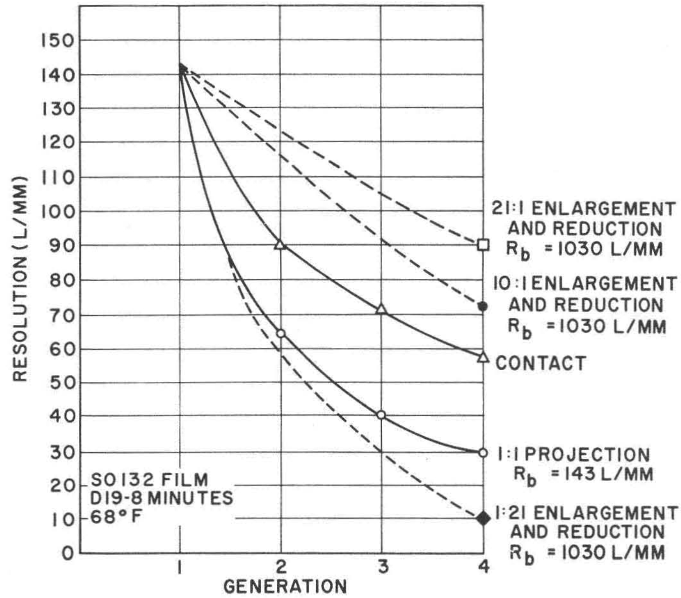

FIG. 3. Experimental and Control Results Reproduction Through Four Generations.

eration positive at 10× magnification using a defraction limited objective with a numerical aperture of .25. The third generation negative was then contact printed and the fourth generation positive placed in the original image plane of the apparatus and reduced by 10× using the original objective. The second portion of the experiment consisted of repeating the first part using a 21× objective of .25 numerical aperture.

The final part of the experiment consisted of reversing the sequence used in the first portion. Initially a 10× reduction was performed followed by contact printing and 10× magnification to obtain the fourth generation positive. SO 132 emulsion developed in D-19 for 8 minutes at 68°F. was used as a recording medium throughout the experiment.

The results of the experiment are presented in comparative graphical form in Figure 3. Figure 4 presents 40× enlargements of each of the fourth generation positive for each of the five methods of reproduction as well as the original "aerial" photographic negative.

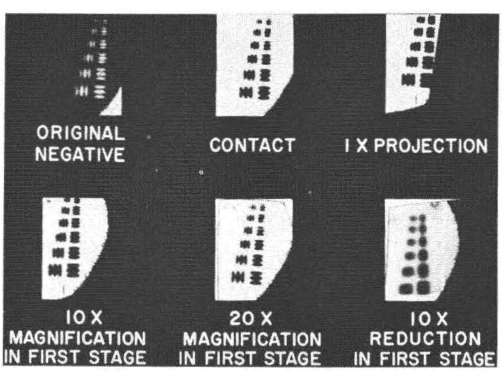

FIG. 4. Comparison of Experimental Results—40× Enlargement of Fourth Generation Image.

CONCLUSION

A mathematical model has been presented of the information transmission of aerial photography in a photo-optical system incorporating magnification in terms of resolution, which can be expressed in the following form:

$$\frac{1}{R_T} = \left(\frac{1}{R_a} + \frac{1}{R_b}\right) \prod_{k=1}^{m} m_k + \left(\frac{2}{R_b} + \frac{1}{R_c}\right) \sum_{j=2}^{n} \prod_{k=j}^{n} m_k$$

where:

R_T = total resultant resolution
R_a = resolution of non-optical elements being magnified
R_b = resolution of optical elements not being magnified
R_c = resolution of element not being magnified
m_k = magnification at each step.

The limit of the equation can be written:

$$\lim_{\substack{m_1 \to \infty \\ M=1}} \frac{1}{R_T} = \frac{1}{R_a} + \frac{2}{R_b} + \frac{1}{R_c}$$

Where

$$M = m_1 \cdot m_2 \cdots m_n$$

The direct consequence of this relationship for the case under consideration is the theoretical possibility to perform an indefinite number of photographic reproductions by projection duplication and obtain only that image degradation that would be achieved in duplicating one such generation.

ACKNOWLEDGMENT

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*Photogrammetric Control in Quebec Using the Bi-Camera Method**

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ABSTRACT: *The National Research Council of Canada has demonstrated that it is possible to measure the y-error that accumulates in aerotriangulated strips by analyzing lines constructed on oblique photographs that cover the same terrain as those photographs used in the bridging.*

A description of the application of the method to mapping a large area in the Province of Quebec, Canada, by the Canadian Topographical Survey is given. Bridging was further strengthened by Airborne Profile Recorder data.

Results indicated that corrections based on Bi-Camera data were less reliable than those based on A. P. R. data, and that the Bi-Camera method is less favorable to production than the A. P. R. method.

I. INTRODUCTION

THE original idea for the Bi-Camera method was conceived around 1954 by Mr. P. E. Palmer, who at that time was heading the Canadian Topographical Survey. The production of topographic maps was being vastly accelerated by utilizing photogrammetric control, and a great potential lay in the high precision of first-order plotters, if only ways could be found to measure the

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