where:

$R_T$ = total resultant resolution
$R_a$ = resolution of non-optical elements being magnified
$R_b$ = resolution of optical elements not being magnified
$R_c$ = resolution of element not being magnified
$m_k$ = magnification at each step.

The limit of the equation can be written:

$$\lim_{\substack{m_1 \to \infty \\ M=1}} \frac{1}{R_T} = \frac{1}{R_a} + \frac{2}{R_b} + \frac{1}{R_c}$$

Where

$$M = m_1 \cdot m_2 \cdots m_n$$

The direct consequence of this relationship for the case under consideration is the theoretical possibility to perform an indefinite number of photographic reproductions by projection duplication and obtain only that image degradation that would be achieved in duplicating one such generation.

ACKNOWLEDGMENT

Acknowledgment is made to Mr. Leo Narodny for many stimulating conversations regarding the principles of the duplication process and especially to Mr. Wayne Tatro who painstakingly performed the experiments.

# *Photogrammetric Control in Quebec Using the Bi-Camera Method**

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ABSTRACT: *The National Research Council of Canada has demonstrated that it is possible to measure the y-error that accumulates in aerotriangulated strips by analyzing lines constructed on oblique photographs that cover the same terrain as those photographs used in the bridging.*

*A description of the application of the method to mapping a large area in the Province of Quebec, Canada, by the Canadian Topographical Survey is given. Bridging was further strengthened by Airborne Profile Recorder data.*

*Results indicated that corrections based on Bi-Camera data were less reliable than those based on A. P. R. data, and that the Bi-Camera method is less favorable to production than the A. P. R. method.*

## I. INTRODUCTION

THE original idea for the Bi-Camera method was conceived around 1954 by Mr. P. E. Palmer, who at that time was heading the Canadian Topographical Survey. The production of topographic maps was being vastly accelerated by utilizing photogrammetric control, and a great potential lay in the high precision of first-order plotters, if only ways could be found to measure the

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systematic errors in the $x$, $y$ and $z$ bridging coordinates. Excellent controls for the scale ($x$ error) and height ($z$ error) had been promised by successful research performed by the Canadian National Research Council utilizing A.P.R., the Airborn Profile Recorder.

Mr. Palmer's idea was directed at the $y$-error. He visualized a photo which would be a view of the terrain covered by many vertical photos used in aerotriangulation. Such a photo was available in the high-oblique which covers all terrain from the aircraft to the horizon. At the present date, with super-wide angle cameras and orbiting vehicles available, the situation is easily visualized whereby a photo would cover a strip as shown in Figure 1. As a result of aerotriangulation, no error would exist at control points $A$ and $B$ but some $y$-error would exist elsewhere, likely accumulating to some maximum towards the central part of the strip.

Now if a line was drawn on the small-scale photo in such a position as to cross the terrain covered by the strip, it would seem that all points on that line should lie on a line of almost constant azimuth. If it were possible to transfer images on the line to the large-scale photos, then the aerotriangulated $y$-coordinates of these points would conform to that line if no $y$-error existed. Any departure from the line would therefore be a measure of $y$-error in the strip.

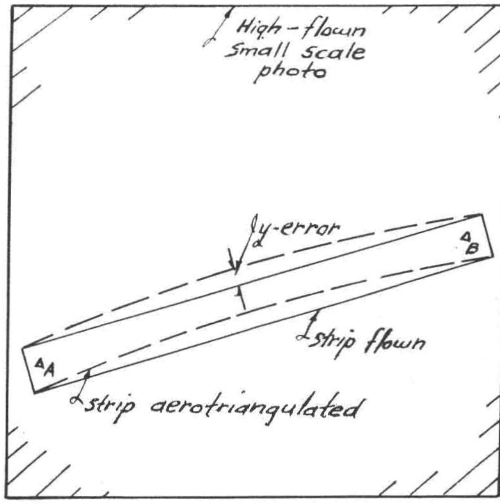


FIG. 1. Relationship of a theoretical small-scale photo and a strip of large-scale photos to be aerotriangulate.

The same argument would exist if the high-flown photo were replaced with a high-oblique. Mr. Blachut of the Canadian National Research Council was asked to investigate this proposal. The theory was developed and Mr. Helava ran four tests which indicated that residual errors of less than 100 feet could be anticipated in lines up to 180 miles long. This would be most satisfactory for mapping

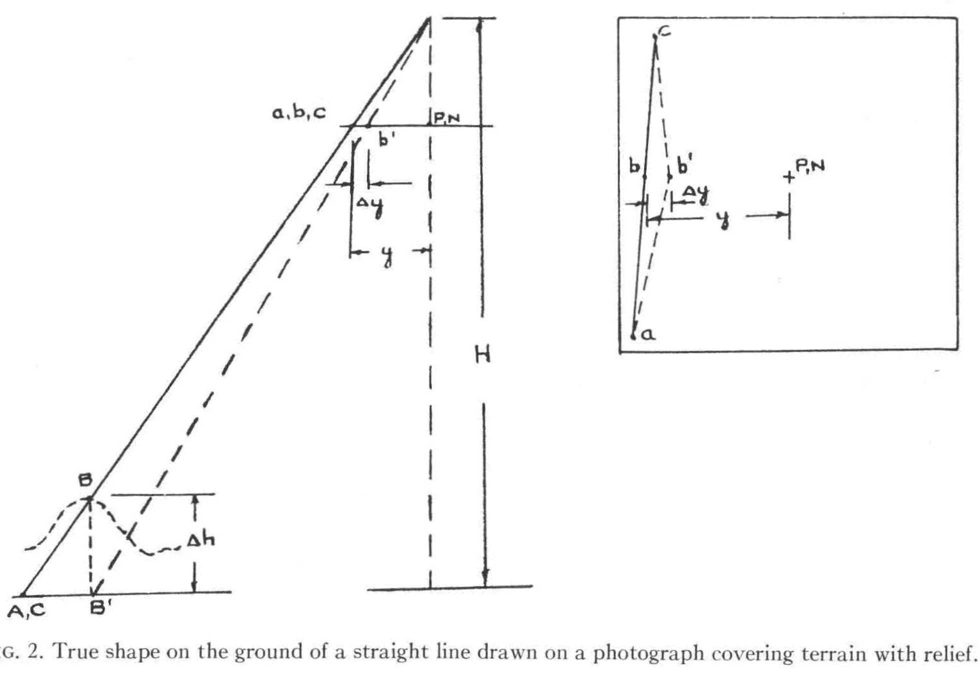


FIG. 2. True shape on the ground of a straight line drawn on a photograph covering terrain with relief.

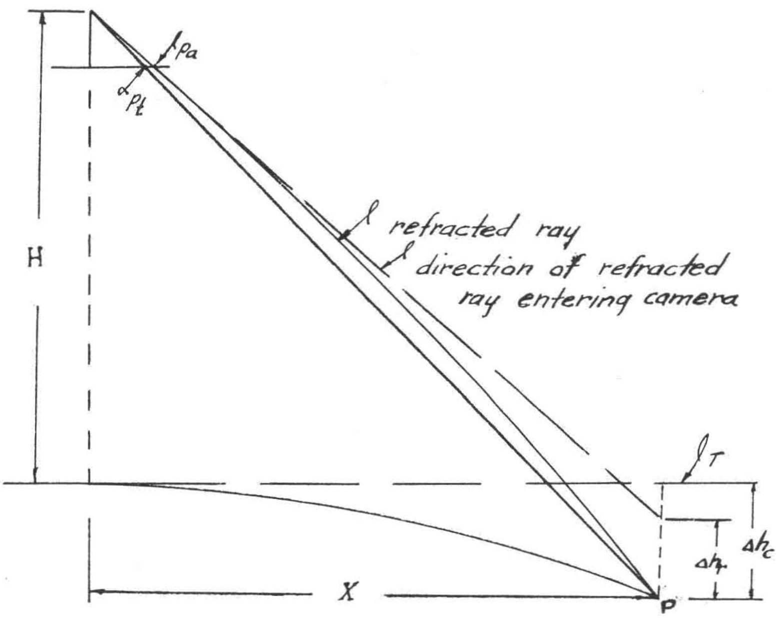


FIG. 3. The effect of Earth curvature and atmospheric refraction on the position of points on the photograph.

at scales of 1 inch = 1 mile and smaller.

## II. THEORY

The theory that the straight line on the oblique photo is a straight line on the terrain would hold true only if the terrain were flat and that lens distortion and atmospheric refraction would not shift images from their expected plane position. Tilt would have no influence inasmuch as a line in a tilted plane is projected as a line in any other plane.

Figure 2 shows the effect of relief in the terrain. Three points $a$, $b$ and $c$ lie on the line where the elevations of $A$ and $C$ are equal and $B$ is higher. The true position of $B$ is $B'$, and the true shape of the line on the terrain is represented by $a$, $b'$, $c$. The further the line is away from the nadir-point and the greater the difference in elevation, the greater is the correction $\Delta y$ from $b$ to $b'$ in accordance with

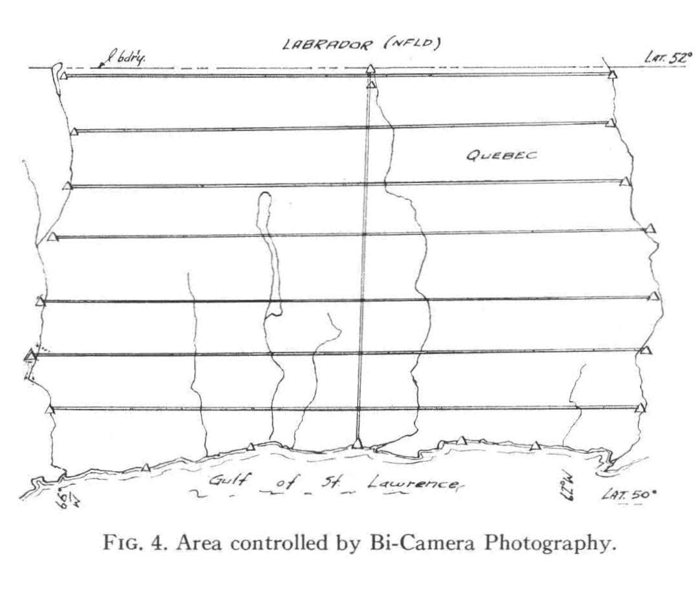


FIG. 4. Area controlled by Bi-Camera Photography.

Formula (1)

$$\Delta y = \frac{y\Delta h}{H} \qquad (1)$$

This is the familiar relief displacement formula, which is modified to such an extent as to reflect only the displacement perpendicular to the direction of flight; with the $x$ or direction-of-flight component being of no concern.

The effect of curvature and refraction is illustrated in Figure 3. Due to curvature the point $P$ is lower than the plane $T$ by the amount $\Delta h_c$, given approximately by the formula

$$\Delta h_c = \frac{D^2}{2R} \qquad (2)$$

Due to refraction, the apparent height of the point is higher than its true height by the amount $\Delta h_r$, so that the apparent image $p$ would appear at $p_t$ if refraction were not present. The magnitude of $\Delta h_r$ can be obtained from a graph plotted according to the formula,

$$\Delta h_r = \frac{49.4H}{K}(\log H - 3) + \frac{49.4X^2}{KH}(\log H - 3) \qquad (3)$$

where $K$ is a constant for the prevailing atmospheric conditions. To get the final position corrected for relief, curvature and refraction; $\Delta y$ is computed by combining all relief effects in accordance with the formula:

$$\Delta y = \frac{y}{H}(\Delta h + \Delta h_r - \Delta h_c) \qquad (4)$$

Both radial and tangential distortion must be reduced to their $y$-component for each image on the line, but aerial cameras are not usually calibrated for tangential distortion. Distortion corrections were not computed for reasons to be discussed later.

## III. Application

In 1955 an area was chosen for the Bi-Camera project in the Province of Quebec north of the St. Lawrence River, as illustrated in Figure 4. Two geodetic triangulation nets 150 miles apart would control the ends of the strips, and as a check, a middle north-south flight was flown over terminal geodetic positions. High-oblique infra-red photographs were taken in the direction of flight, concurrently with the vertical photos to be used to establish the photogrammetric mapping control. The relationship of oblique and vertical photos is illustrated in Figure 5a. In addition, Airborne Profile Recorder data were taken to strengthen the bridging scale and to permit evaluation of the height of terrain. This area was considered ideal not only because of control distribution, but also because of numerous water bodies which serve well as identifiable points and as good reflecting surfaces for the A.P.R.

Paper and glass prints of the vertical and oblique as well as the A.P.R. profiles with their 35 mm. photos were procured. The A.P.R. data were immediately evaluated for terrain elevations and differences in flying height, the latter to be introduced as the relative orientation element $bz$ during aerolevelling. Trial lines were run on the paper obliques to find lines which would stay on the vertical strip. It was at first thought that a suitable line could be located simply by laying out the 75-odd vertical photos with approximate orientation and drawing a line down the center; then by using conspicuous image-crossing, transfer that line to the obliques. It was soon obvious that photo swings and variations in flight-line azimuth were such that there was no guarantee that this line transferred to the obliques would stay on the verticals. Lines finally had to be drawn by trial and error on the paper obliques until a suitable location was found. Inasmuch as $\Delta y$ is proportional to the distance of the line from a central location, it was disappointing to see the line take such extreme swings from a cen-

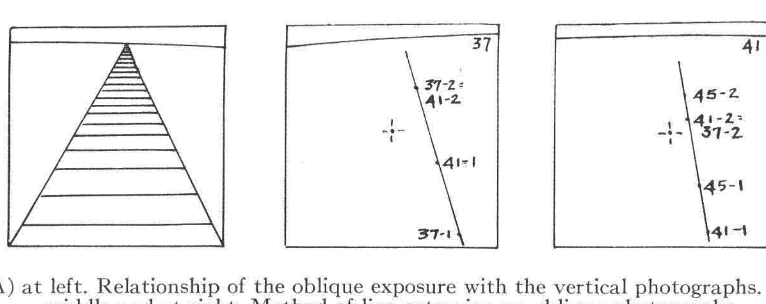


FIG. 5. (A) at left. Relationship of the oblique exposure with the vertical photographs. (B & C) in middle and at right. Method of line extension on oblique photographs.

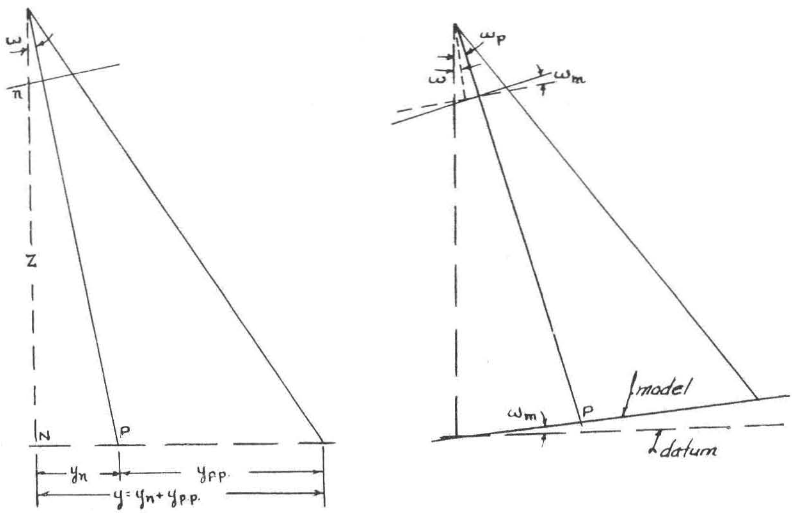


FIG. 6. The correction to measured $y$-distances to the line due to $\omega$. A, at left, is distance between principal point and nadir B, at right, is actual $\omega$ at the time of flight.

tral location. Also, some of the flights were broken because of weather which further weakened the solutions.

The line found was then transferred to the glass prints of the obliques using the sharpest scribing tool and straightest straight edge available in the manner shown in Figure 5b. Instead of using every oblique every 4th or 5th may be used, since points in the background of photo 37 establish the orientation of the line in the foreground of photo 41. It would be noted by checking the scale of a high-oblique photo of $f=6$ inches, $H=20{,}000$ feet, that if a point one inch above the principal point is missed by .01 mm., this would represent an error of 3.4 feet on the ground. Two inches above the principal point, identification is being attempted on a point 20 miles away where .01 mm. represents 5.6 feet. Those who have made long-distance theodolite ob-

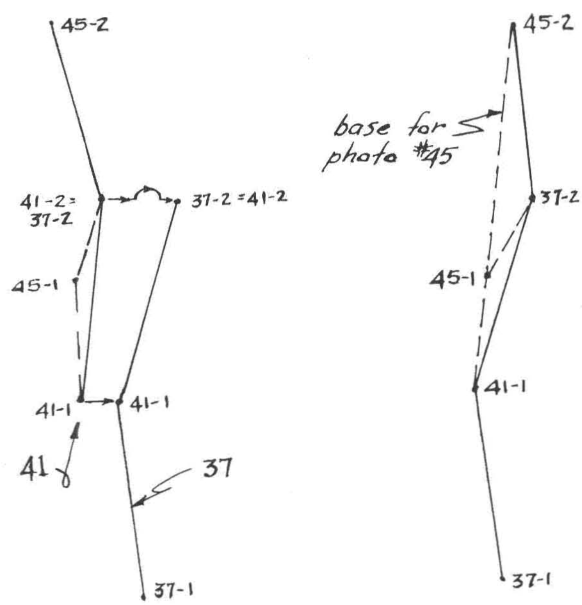


FIG. 7. Method of constructing final shape of the line on the ground.

servations will realize that what is being done here is extremely delicate. If points 41-1 and 37-2 are $3\frac{1}{2}$ inches apart and the line is drawn through 37-2 with an error of .01 mm., then point 54-2, 7 inches along the line has the error doubled, with the problem of the transfer of that point to photo 45 still to be faced. The critical nature of the line construction led to the decision to bypass the work involved in correcting for distortion, which was believed to be a much smaller source of error.

With the line completed on the obliques, all points used in construction were transferred to the vertical diapositives, pass points were picked and aerolevelling performed reading $x$, $y$ and $z$ coordinates of all points with an electrical recording device. $\omega$, in particular, was noted in each projector. The absolute orientation of the first model involved a scaling to approximate flying height, levelling on a suitable water body and setting an optional azimuth.

The distance $y$ from the principal-point to the line may be obtained directly from machine $y$-coordinates if the photo is truly vertical. The correction $y_n$ illustrated in Figure 6a must be applied, which is the distance between the principal-point and the nadir. The actual $\omega$ at the time of the time of flight is seen in Figure 6b to be

$$\omega = \omega_p - \omega_m \tag{5}$$

Where $\omega_v$ is read at the projector and $\omega_m$ is measured on suitable water bodies along the strip then

$$y = y_{p.p.} + Z \operatorname{Tan} \omega \tag{6}$$

The distance $X$ to each point, which determines $\Delta h$ for curvature and refraction, is determined by taking the $X$ coordinate of the exposure station and applying it to the $X$ coordinate of each point on the oblique. Formulas (2) and (3) or a suitable graph give $\Delta h_r$ and $\Delta n_c$. All influences of height are now combined and $\Delta y$ is computed for each point on each oblique.

Recalling Figure 5, it is obvious that points 41-1 and 37-2 which appear on both obliques, will each have two distinct values $\Delta y$ because of variation in $X$ distance to points and nadir distance to line. Accepting the values $\Delta y$ in photo 37, the line on photo 41 will have to be shifted an amount equal to the difference in $\Delta y$ for point 41-1 and rotated about that point to make the computed value of $\Delta y$ for point 37-2 to agree with the accepted value of that point as shown on Figure 7. The required shift and rotation must be combined with $\Delta y$ for the other points on the line (45-1 and 45-2) to get their final values. Every segment is likewise added.

A comparison of the machine $y$-coordinates and those computed will have no apparent similarity until the first computed point is shifted an amount to make it equal to the machine coordinate of that point, and the last computed point is rotated the amount required to make it equal to the machine coordinate, with, of course, the same shift applied.

All intermediate points get the same shift and such a rotation component as their distance along the strip demands. The difference between the computed $y$-coordinates and the machine $y$-coordinates is a measure of the $y$-error in the strip.

## IV. Conclusion

The corrected machine coordinates of all photogrammetric mapping control points were then transformed to Transverse Mercator coordinates on all the strips, and comparisons were made for common points at the N-S and E-W flight intersections. The hope was that the A.P.R. controlled Eastings of E-W flights would check the Bi-Camera controlled Eastings of the N-S flights, and vice versa for the Northings. Since better comparisons existed for Eastings, it was concluded that the A.P.R. correction of $X$-coordinates of the strips was better than the Bi-Camera correction, and the final adjustment was made whereby strip $X$-coordinates were given the greater weight. In spite of the enthusiasm expressed by technicians for the trouble-free stereotemplate laydowns accomplished by the Bi-Camera control, the author anticipates that future ground surveys will reveal errors up to 300 feet in absolute position inasmuch as errors up to 900 feet were adjusted.

## V. Acknowledgments

The writer would like to acknowledge the aid and encouragement given by the Canadian Topographical Survey and the Division of Applied Physics of the National Research Council of Canada for the preparation of this paper.

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1. Helava, U. V., "Use of Aerial Photographs to Control the Bend of Aerial Triangulation," National Research Council of Canada Report #5396, September 1959.