*An Analytical and Experimental Study of Stereo for Radar**

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ABSTRACT: *The similarities and differences of the geometry of radar and aerial photos were analyzed to determine optimum flight configurations for radar bearing aircraft. Parallax and error curves were plotted to determine modifications necessary to adapt present stereo-plotters for use with radar imagery. A radar simulator was used to prepare stereo-pairs which were used in experiments with a number of observers to evaluate the visual effect produced. Finally, the significance of the results are summarized and optimum flight configurations for stereo-radar are given.*

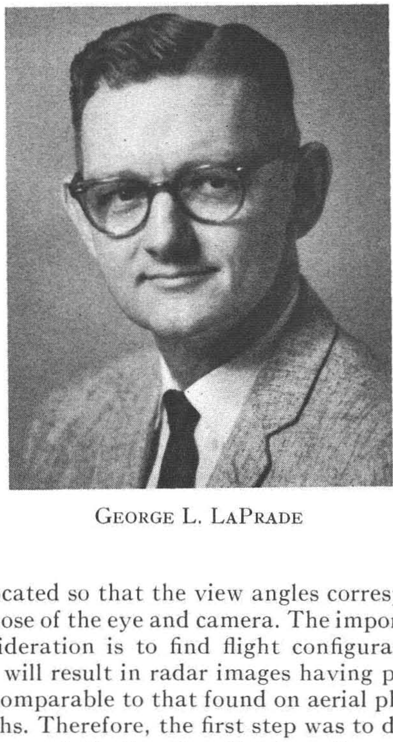
I. INTRODUCTION

SINCE the early years of PPI radars, attempts have been made to obtain the same visual stereo effect with radar imaging as has been developed using aerial cameras. Usually these efforts have consisted of examining random pairs of radar photographs taken for some completely unrelated purpose. No particular attention was given to the flight configurations which would be most favorable for stereo viewing. This is roughly analogous to taking scenic snapshots from a Piper Cub with a Brownie Camera, and then trying to use the pictures in a stereopticon. It is not surprising that past results have not been too encouraging.

In the study to be described in this paper a different approach was used. Instead of making random trials which might work, an attempt was made to discover the techniques which should work. This investigation was a small part of a general study on "The Extraction of Mapping Detail From Airborne Radar Photography," done for GIMRADA,‡ an agency of the U. S. Army Corps of Engineers.

A camera and the human eye record an image in almost exactly the same manner. Therefore if two cameras are located with the same relative geometry as the eyes, a satisfactory stereo-model will result. The fact that radar records its image in an entirely different manner is often overlooked. Thus it is not necessary, or even desirable, for the radar to be located so that the view angles correspond to those of the eye and camera. The important consideration is to find flight configurations that will result in radar images having parallax comparable to that found on aerial photographs. Therefore, the first step was to determine analytically the flight paths which would satisfy these conditions. Next, this configuration was tested with a radar simulator to see if the required visual stereo effect was produced. Finally the available radar imagery was searched for actual samples which conformed to the theoretical specifications.

The top diagram in Figure 1 compares the



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imaging geometry of a side-looking strip mapping radar and a camera. The elevated point "a" is projected down to "n" by the camera. The radar locates objects in the range direction by measuring their distance from the antenna and, with a ground range sweep, will display point "a" at "m." The base of the feature with height h, and all other points on the datum plane will be imaged at the correct location by both devices. Obviously, it would be possible to make "m" and "n" coincide by relocating the radar. The objective of the geometrical analysis was to determine radar positions that would make such points coincide over an appreciable portion of the image for all values of h.

II. Radar Parallax

In diagram (B) of Figure 1, the imaging geometry of two radars is shown. Point "a" is projected down to "g" and "d" resulting in a radar parallax of ΔP_r. If there is to be any hope of simulating the images formed by conventional techniques, a flight configuration must be found which will produce a practically constant value of ΔP_r over an appreciable proportion of the range.

Using simple geometric relationships it can be shown that

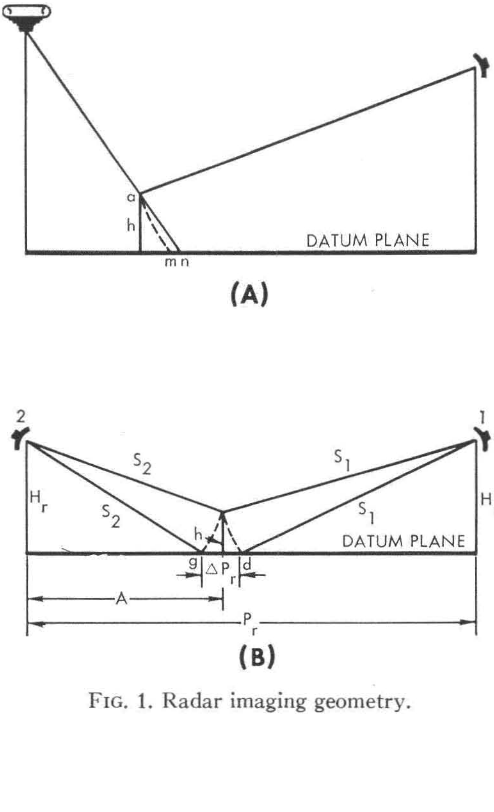

FIG. 1. Radar imaging geometry.

$$\Delta P_r = P_r - \sqrt{[A^2 + (H_r - h)^2] - H_r^2} - \sqrt{[(P_r - A)^2 + (H_r - h)^2] - H_r^2}$$

For a given flight configuration, H_r and P_r are constant so that curves can be obtained for the parallax, ΔP_r, as a function of A, the distance from the aircraft ground track to features with an altitude h. This equation was programmed into our computer in order to obtain a wide variety of plots. The most favorable configurations occurred when the altitude of the aircraft was approximately one fifth of the distance between their ground tracks.

Figure 2 shows one of the graphs obtained. In this case the aircraft were 11.2 nautical miles apart and flying at an altitude of two nautical miles. Five terrain elevations are shown: the datum plane, $h=0$, and .05 and .025 n. mi. above and below this. The graph is symmetrical, of course, so only one-half is shown. As can be seen, there is a strip one to two miles wide on either side of the origin where the parallax is relatively constant. This seems promising and encouraged us to make the error analysis discussed below.

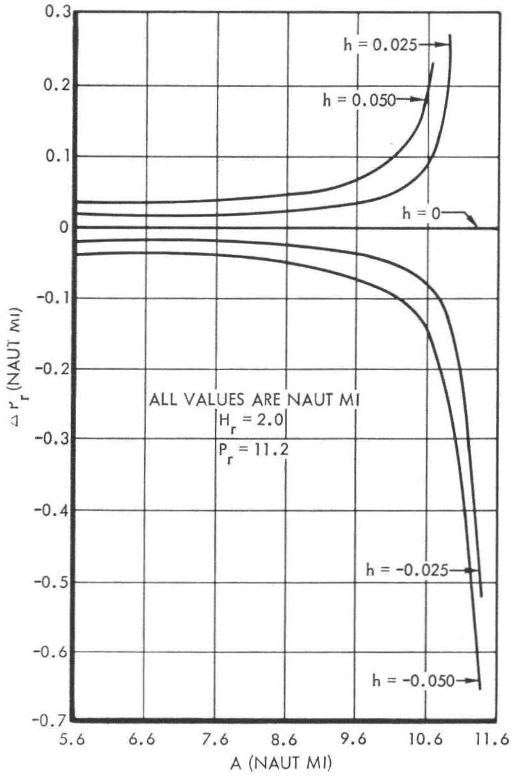

FIG. 2. Radar parallax.

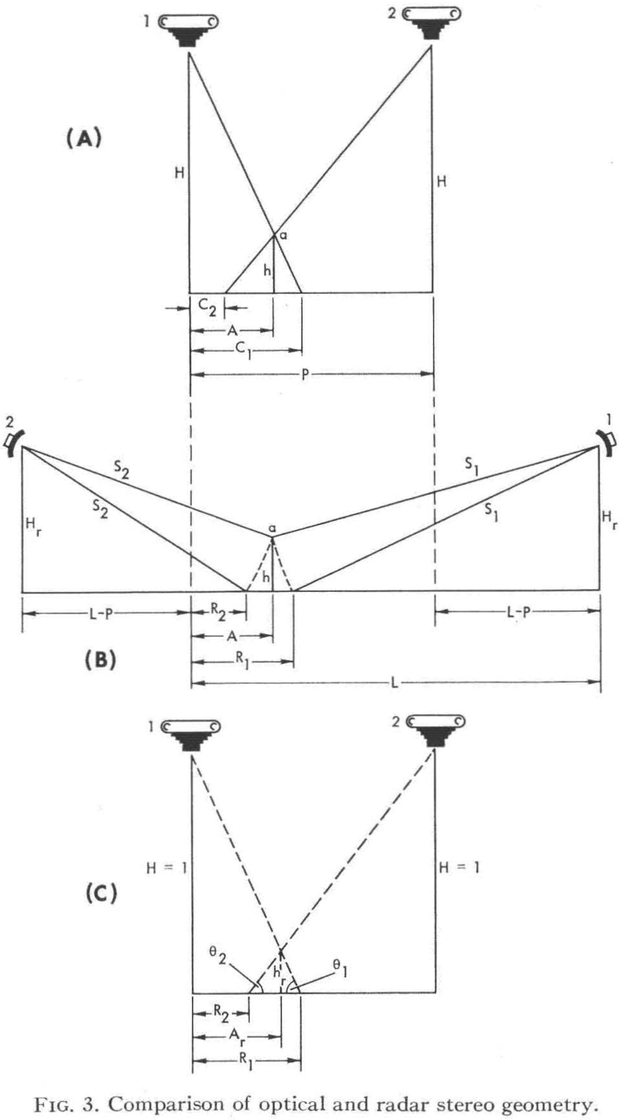

FIG. 3. Comparison of optical and radar stereo geometry.

III. ERROR ANALYSIS

Figure 3A shows a diagram of the parallax resulting when a feature with elevation h is photographed by an aerial camera. In the radar case (3B) there will, in general, be a slightly different parallax. Note that the radar on the right simulates the image of the camera on the left and vice versa, so that the placement of the two radar photographs in a stereo-viewer would be just reversed from that of conventional aerial photos. This corresponds to the images produced by the aerial cameras with some other object having the elevation h_r as shown in 3C. The difference between h and h_r represents the error which would result if the radar images were placed in a conventional stereo-plotter.

A relatively simple but somewhat lengthy derivation using the geometrical relationships between these three diagrams gives a rather involved equation for h-h_r in terms of the flight configuration parameters and the location of the feature with respect to the aircraft ground track. Fortunately such awkward mathematical expressions can be quite readily handled by a digital computer once they have been programmed for it. Curves were thus plotted for a large number of typical radar

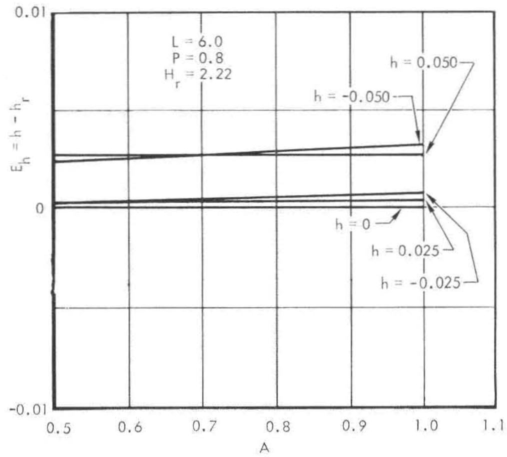

FIG. 4. Error curves for stereo radar.

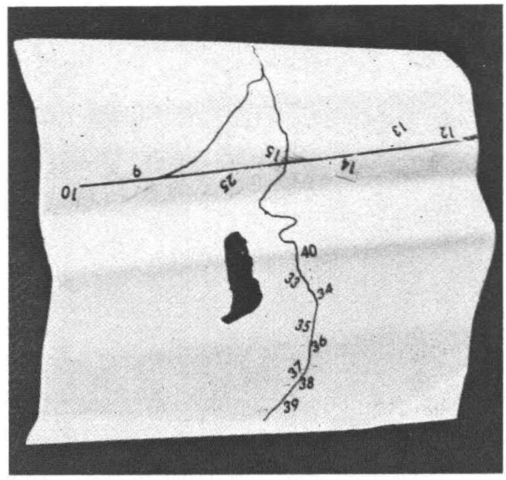

FIG. 6. Terrain model.

configurations. As might be expected, most of these gave extremely erratic error curves indicating they would not produce suitable stereo models.

However, there were a few radar locations which gave promising results. The curves for one of these is given in Figure 4. The aircraft are separated by 11.2 nautical miles. It can be seen that elevations equidistant on either side of the datum plane require approximately the same constant correction factor. The average correction factor for a terrain elevation spread from plus 300 feet to minus 300 feet is 3%.

Figure 5 presents the same data in a different form. The solid line represents the actual terrain profile, while the dotted line shows the corresponding profile developed by the radar stereo-model. At the highest and lowest elevations the error is approximately 5% while along the dashed datum line it is zero. For contouring purposes of course, these discrepancies could be compensated for by the use of calibration curves or an adjusted scale on the stereo-plotter.

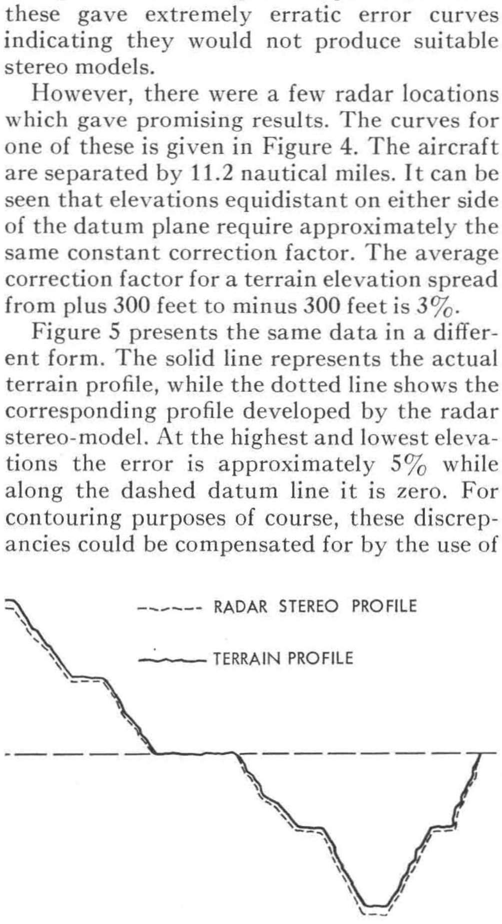

FIG. 5. Radar stereo error profile.

IV. SIMULATED RADAR IMAGES

To verify the geometrical analysis above, simulated stereo radar photographs were made using the GAC developed radar simulator. This device simulates exactly the geometry of radar images. However, since it uses visible light, the reflectivity and illumination intensity are somewhat different than those of radar. Figure 6 shows one of the models used. The difference in elevation between point 39 and 40 was just over $\frac{1}{2}$ inch, the maximum feasible with the small-scale simulator available. The dark area representing a lake was deliberately placed on top of the ridge to see if such a false clue would confuse the observers.

Figure 7 shows the stereo-pair produced by the simulator. A group of observers were able to correctly locate the relative elevation of all the numbered points. In addition an optical height finder and lens stereoscope were used to determine the relative elevations indicated by parallax differences. These were found to conform with the results given by the geometrical analysis including the predicted type of errors.

Without taking space to go into details, it can be said that this complete procedure was also followed for slant range sweep radars with similar results.

V. SAME SIDE RADAR STEREO

In the situation discussed so far, the airborne radars have been traveling parallel

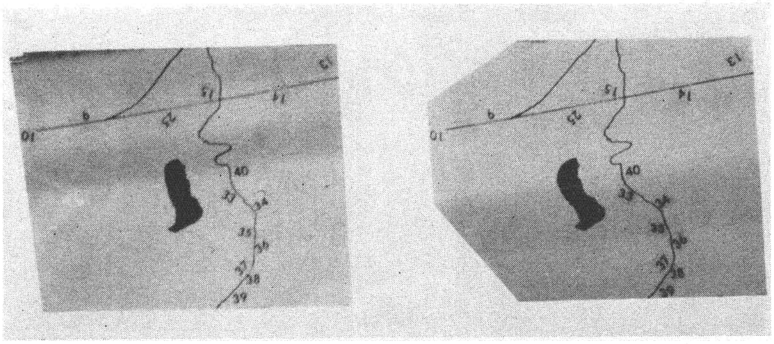

FIG. 7. Stereo pair from radar simulator.

paths with their beams directed toward the ground between them. Two radars flying parallel paths but both looking to the *same* side will also result in parallax. This is comparable to using the section of a conventional stereo-model beyond the center points of both the aerial photographs. For civilian uses this technique is normally avoided but it does provide useable results in emergencies and in the military situation has special significance because it permits the obtaining of mapping information for enemy terrain from behind the front lines.

Again an equation was derived for radar parallax and plotted against the distance of the terrain feature from the nearer aircraft. Figure 8 shows one such plot along with a diagram of the flight configuration. At near ranges the parallax curves for points above and below the datum plane are widely divergent, meaning the apparent height of mountains and depths of valleys increase radically. At far ranges, where the parallax is more nearly constant, its values are too small to give satisfactory results.

However, the situation can be modified if it is remembered that there is nothing sacrosanct about keeping the flight paths at the

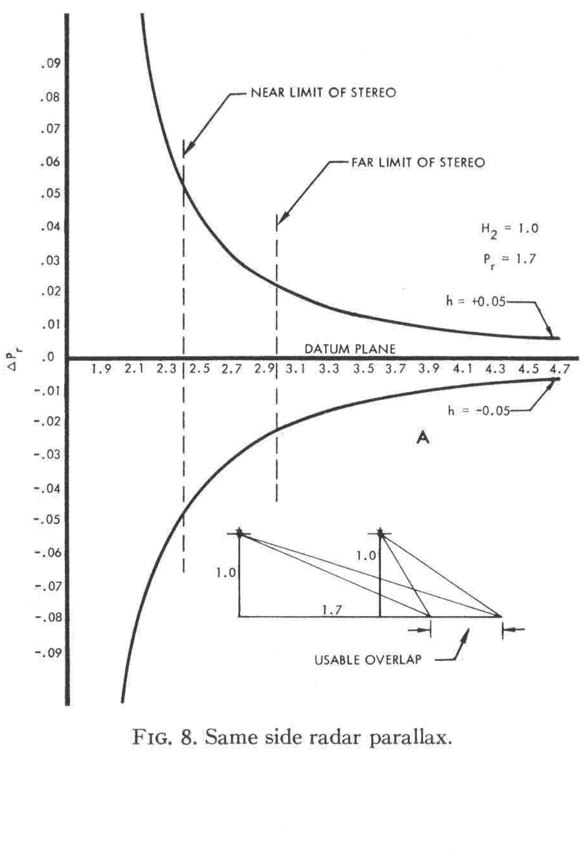

FIG. 8. Same side radar parallax.

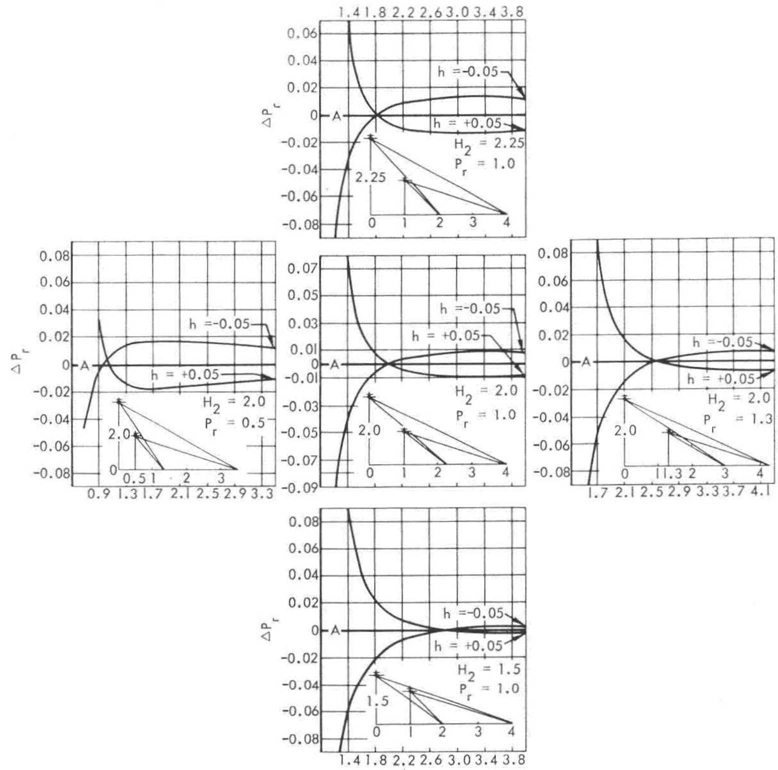

FIG. 9. Effect of flight configuration on radar parallax.

same altitude. The objective is *not* to have *flight paths* analogous to those of aerial cameras but to produce *images* with comparable parallax displacements. Figure 9 shows the effect of varying the altitude and the distance between the groundtracks of the two airborne radars. The points where the parallax curves cross correspond to points on the terrain which are approximately on a line through the two aircraft. On opposite sides of this crossover the relative displacements of an elevated point reverse, indicating the necessity of reversing the location of the stereo-pair under the stereoscope.

The most favorable configuration for same side stereo is probably that shown in the top diagram. The portion at very near range is still unsatisfactory but beyond the crossover the parallax is relatively constant and of sufficient magnitude to produce good results.

VI. Examples of Stereo Radar Imagery

Having determined the flight configurations which would produce radar photographs with the same parallax as an aerial camera, a search was conducted of the available radar imagery to find examples which satisfied the required conditions. More than 3,500 feet of radar imagery was inspected, representing over 100,000 square miles of terrain. Not a single case was discovered where opposite side flights had been flown with anything approaching the proper configuration. Occasional pairs seemed to produce the enhancement of depth reported by other observers. However, they failed to produce a pseudoscopic effect when their position was reversed, indicating that true stereo was not present.

For the same side stereo one short flight was discovered that provided several samples which produced both conventional and pseudoscopic effect. Unfortunately they cannot be presented here because of security regulations. The flight configuration was among the least optimum, approximating that shown in Figure 8. In addition the right-hand image was somewhat blurred. The fact that a stereo-image can be obtained at all under such adverse conditions suggests that a fairly reliable stereo model could be produced by controlled flights flown with the configurations prescribed by the geometrical analysis given earlier.

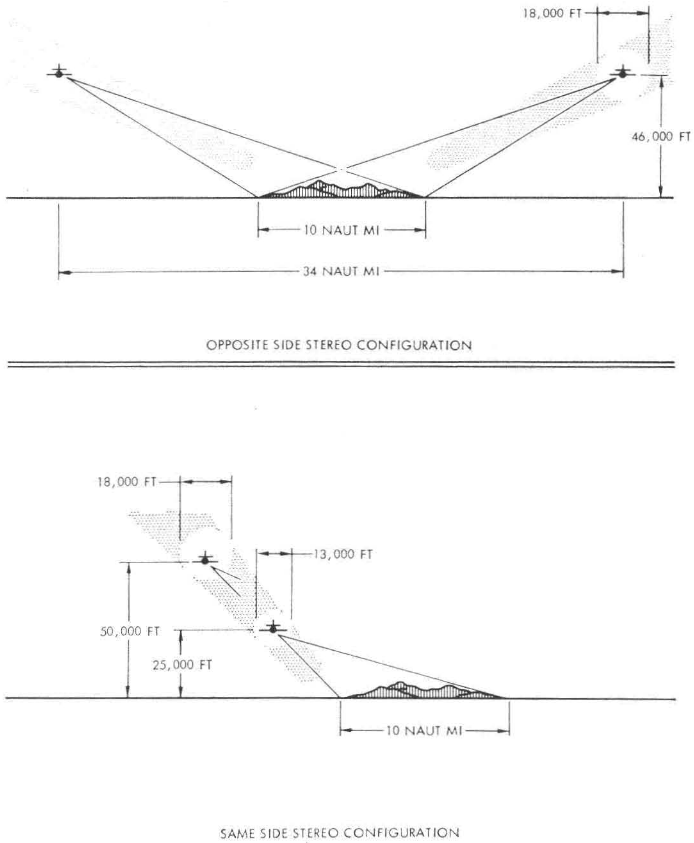

FIG. 10. Permissible radar stereo flight configurations.

VII. CONCLUSIONS

In the discussion given here, only the geometrical aspects of stereo-radar have been covered. In the original study such factors as shadows and resolution were also examined. For example, radar transmits its own energy instead of using that of the sun, so that a radar stereo-pair corresponds to two aerial photographs taken at different times in the day. As with aerial cameras the accuracies of elevation measurements are going to be determined by system resolution. These are some of the factors whose effect on the visual stereo-model should be investigated further. The most urgent need is for a representative supply of imagery flown with the required configuration. If this were available, it is possible that operational standards could be established similar to those now in effect for conventional stereo. At the present time we can say with assurance that it is possible to obtain a bona fide visual presentation with radar stereo, there is a potential contouring capability, and strictly prescribed flight configurations must be maintained for successful results.

Figure 10 summarizes the results of the geometrical analysis. If the depicted relative geometrical location of the aircraft is maintained, their flight lines can be anywhere within the shaded area and still give a satisfactory visual stereo-model for recognition of detail. The circles indicate the allowable error the aircraft may have with respect to each other. For contouring purposes the results are similar except that the location of the flight lines must be known to a somewhat greater degree of accuracy.